IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

A joint study by the Australian Bureau of Statistics, the Department of Employment and Industrial Relations, the Department of Environment, Housing and Community Development, the Department of Industry and Commerce and the Industries Assistance Commission

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ENGEL CURVES AND DEMAND SYSTEMS : DEMOGRAPHIC EFFECTS ON CONSUMPTION PATTERNS IN AUSTRALIA

by

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$$(H - 2)\ \mathrm{var}\ \hat{\mu} = \Sigma\ (v - \hat{\mu}\ y)^2/\Sigma\ y^2 = (\Sigma v^2/\Sigma y^2) - (\Sigma vy/\Sigma y^2)^2\ . \qquad (10)$$

Substitution of (8), (9) and (10) into (7) yields, after some manipulation, equation (5) .

QED

ENGEL CURVES AND DEMAND SYSTEMS : DEMOGRAPHIC EFFECTS ON CONSUMPTION PATTERNS IN AUSTRALIA*

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1. INTRODUCTION

In order to capture adequately the effects of structural changes in an economy it is necessary to use a disaggregated model.[1] Empirical implementation of economy-wide models at a disaggregated level, however, requires a formidable input of data and parameter estimates. In this paper we are concerned solely with providing such estimates for consumer behaviour. In particular, we are interested in measuring the effects of family size and age of household head on expenditure patterns of Australian households. These estimates form part of the input into the SNAPSHOT model of the Australian economy[2] being constructed as part of the IMPACT project.

If demographic effects on consumption patterns are to be measured adequately cross-section data must be used. The data used here

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1. Some of the more important disaggregated economy-wide models of recent years are : Johansen (1974), Adelman and Robinson (1975) and those contained in Blitzer, Clark and Taylor (1975).
2. See Dixon, Harrower and Powell (1976).

* I am indebted to Ellen Hope for efficiently carrying out all the computational work reported here.

is the 1966-68 Survey of Consumer Expenditures and Finances undertaken by Macquarie University. As is usual with a single cross-section study, no information is available on price variation across consumers. However, estimates of both income and price responsiveness are required. To obtain the price estimates we resort to using an additive utility function in deriving our estimating equations.[1] Expenditure systems obtained from the Klein-Rubin (1947-48) utility function are considered : Stone's (1954b) Linear Expenditure System (LES) and Lluch's (1973) Extended Linear Expenditure System (ELES). Demographic effects are captured by fitting the systems separately to households of differing family sizes and ages.

In a cross-section context in which all consumers are assumed to face identical prices LES reduces to a set of Engel curves, i.e., expenditure on each commodity is a simple linear function of total expenditure. Two difficulties arise in obtaining estimates of the parameters of the Klein-Rubin utility function from Engel equations. First, OLS estimates of the Engel equation parameters are inconsistent. Second, it is not possible to identify fully the Klein-Rubin parameters from estimates of the corresponding Engel equation parameters. It will be shown, however, that these two problems and their solutions are interdependent. The use of ELES, in which income replaces total expenditure as the explanatory variable in the Engel equation, avoids both problems, but only if the income data is measured without error. A further limitation of Engel curves is the linearity assumption. The empirical importance of this is explored through the use of spline functions.

1. With an additive utility function all price responses can be written as a function of "income" responses and one other parameter -- the expenditure elasticity of the marginal utility of expenditure or Frisch parameter.

$$\text{var}\ \tilde{\beta} = \text{var}\ (\hat{\beta}_i / \Sigma\ \hat{\beta}_j^*)$$

$$= \mu^{-2}\ (e - \tilde{\beta}_i\ \iota')\ \textstyle\sum_{\beta^*} (e' - \tilde{\beta}_i\ \iota)\ , \qquad (6)$$

where

e is an $n \times 1$ vector with unity in the i^{th} row and zero elsewhere ;

ι is an $n \times 1$ vector of units ; and

$\sum_{\beta^*}$ is the $(n \times n)$ variance-covariance matrix of $\hat{\beta}^*$.

Noting that $\text{var}\ \hat{\mu} = \iota' \sum_{\beta^*} \iota$, equation (6) may be written as[1] :

$$\text{var}\ \tilde{\beta}_i = \mu^{-2}\ [\text{var}\ \hat{\beta}_i^* + 2\ \tilde{\beta}_i\ \Sigma_j\ \text{cov}\ (\hat{\beta}_i^*, \hat{\beta}_j^*) + \tilde{\beta}_i^2\ \text{var}\ \hat{\mu}]\ . \qquad (7)$$

Now terms appearing on the RHS of (7) may be expressed as follows :

$$(H - 2)\ \text{var}\ \hat{\beta}_i^* = \Sigma(v_i - \hat{\beta}_i^*\ y)^2/\Sigma\ y^2 = (\Sigma\ v_i^2/\Sigma\ y^2) - (\Sigma\ v_i\ y/\Sigma\ y^2)^2 \qquad (8)$$

$$(H - 2)\ \Sigma_j\ \text{cov}(\hat{\beta}_i^*, \hat{\beta}_j^*) = \Sigma\ \hat{u}_i\ \hat{u}_j/\Sigma\ y^2 = \Sigma\ v_i\ \hat{u}/\Sigma\ y^2\ , \qquad (9)$$

where $\hat{u}_i$ is the residual on the i^{th} ELES equation, $\hat{u} = \Sigma\ \hat{u}_i$ is the residual of the ELES aggregate consumption function, and we have used the fact that $\Sigma\ \hat{u}_i\ \hat{u}_j = \Sigma\ v_i\ \hat{u}_j$.

1. Also $\iota' \sum_{\beta^*} \iota = (H - 2)^{-1}\ [(\Sigma v^2/\Sigma y^2) - (\Sigma vy/\Sigma y^2)^2]$, i.e., the variance of $\hat{\mu}$ may be obtained from either $\Sigma\ \beta_j^*$ or from the ELES aggregate consumption function.

Appendix

Equivalence of variances of β_i from IV estimation of LES and OLS estimation of ELES

In deriving relationships between estimated variances we follow the notation of section 2 except that all variables are now deviated from sample means. For convenience the household subscript is omitted, but all summations are over households unless otherwise stated.

The basic relationships between the point estimates of LES and ELES may be summarized as follows :

ELES $$\hat{\beta}_i^* = \Sigma v_i y/\Sigma y^2 \qquad (i = 1, \ldots, n) \qquad (1)$$

ELES $$\hat{\mu} = \Sigma v y/\Sigma y^2 = \sum_j \hat{\beta}_j^* \qquad (2)$$

LES/ELES $$\tilde{\beta}_i^* = \hat{\beta}_i^*/\hat{\mu} = \Sigma v_i y/\Sigma v y \qquad (i = 1, \ldots, n) . \qquad (3)$$

Estimation of LES by IV yields :

$$\text{var } \tilde{\beta}_i = s^2 \Sigma y^2/(\Sigma v y)^2 , \quad \text{where } s^2 = \Sigma(v_i - \tilde{\beta}_i v)^2/(H - 2) \qquad (4)$$

$$= [\Sigma v_i^2 - 2\tilde{\beta}_i \Sigma v_i v + \tilde{\beta}_i^2 \Sigma v^2]/[\mu(H - 2)\Sigma v y] . \qquad (5)$$

Now consider the expression for the variance of $\tilde{\beta}_i$ under ELES :

The plan of the paper is as follows. In section 2 we explore at a theoretical level alternative methods of obtaining estimates of the parameters of the Klein-Rubin utility function, namely, marginal budget shares and "subsistence" expenditures. The theoretical relationships between LES and ELES estimators is examined in some detail. Section 3 considers the use of spline functions to fit piece-wise linear relations for LES/ELES. Section 4 contains a brief description of the data followed by empirical estimates of Engel curves. Estimates of "subsistence" expenditure are derived in section 5 using LES/ELES estimates and a priori information. Section 6 summarizes the key findings and makes suggestions for further research.

2. ESTIMATION OF THE PARAMETERS OF THE KLEIN-RUBIN UTILITY FUNCTION USING CROSS-SECTION DATA

The Klein-Rubin (additive) utility function is given by

$$U = \sum_{j=1}^{n} \beta_j \ln(q_j - \gamma_j) , \qquad (1)$$

where U is an index of utility of a consumer or household, and q_j , $j = 1, \ldots, n$, represents quantities of goods purchased. The γ_i parameters may be thought of as "subsistence quantities," utility being defined only when $q_j > \gamma_j$, for all j. The β_i parameters $(\Sigma \beta_i = 1)$ are shown below to represent marginal budget shares.

Lluch's (1973) intertemporal maximization of (1) subject to a wealth constraint yields an expenditure system, ELES, which under the assumption of constant commodity prices, p_i, across consumers, may be written as[1] :

$$v_{ih} = \alpha_i^* + \beta_i^* y_h + u_{ih} \tag{2}$$

where

$$\alpha_i^* = \gamma_i^* - \beta_i^* \sum_{j=1}^{n} \gamma_j^* \quad , \tag{3}$$

$$\beta_i^* = \mu \beta_i \quad , \tag{4}$$

$$\gamma_i^* = p_i \gamma_i \quad , \tag{5}$$

v_{ih} is expenditure on the i^{th} good ($i = 1, \ldots, n$) by the h^{th} household ($h = 1, \ldots, H$), y_h is the income of the h^{th} household[2], u_{ih} is an error term, μ is the aggregate marginal propensity to consume, β_i^* is the marginal propensity to consume the i^{th} good ($\Sigma \beta_i^* = \mu$) and γ_i^* is "subsistence" expenditure on the i^{th} good at prices prevailing during the budget survey (assumed constant across consumers).[3]

Summing (3) over commodities yields a Keynesian aggregate consumption function :

$$v_h = \alpha^* + \mu y_h + u_h \tag{6}$$

where

$$\alpha^* = \sum_j \alpha_j^* = (1 - \mu) \sum_j \gamma_j^* \quad , \tag{7}$$

1. Estimation of ELES from cross-section data is discussed in more detail in Belandria (1971), Betancourt (1973), Howe (1974), Powell (1973, 1974) and Williams (1975).
2. In Lluch's (1973) general formulation the income term is a permanent income measure. In this paper we will consider only current income, which in Lluch's formulation is equivalent to assuming that the present value of expected changes in labour income is zero.
3. Korean results in Williams (1975) suggest that if cross-section price variations are of the order of 5 to 10 per cent then assuming them away does not affect parameter estimates materially.

to use the fact that under LES/ELES, ω is uniquely determined by the marginal and average propensities to consume.[1] Thus the problem of obtaining reliable estimates of ω from cross-section data is essentially the problem of obtaining estimates of the marginal propensity to consume that are suitable for use in projecting over time. Just as the way out in aggregate consumption analysis is to introduce concepts of permanent income, so the solution to the problems of demand analysis probably lies in using Lluch's general formulation of ELES which employs a measure of permanent income.

1. The precise relationship is

$$\omega = - \frac{(1 - \mu)}{\mu} \cdot \frac{(v/y)}{(1 - v/y)}$$

of total "subsistence" expenditure, which were based on time-series estimates of the Frisch parameter[1]. These were combined with the IV estimates of LES to obtain estimates of "subsistence" expenditures on individual commodities.

The empirical estimates of the Klein-Rubin parameters may be summarized as follows. Marginal budget shares are influenced by age of household head but not so much by family size within a given age class. The strongest results are that young households yield higher values of β_i for durables and lower values for housing and recreation than do old households. "Subsistence" expenditures, however, are influenced by both the age and size of the family. "Subsistence" expenditure on food is higher for both old and large families. Family size exerts a strong positive effect on γ_i-estimates for clothing. "Subsistence" expenditure on housing is estimated to be higher for young households than for old households.

Even though some care has been exercised in obtaining the γ_i^* estimates they still remain less reliable than those for β_i. In obtaining a priori estimates of $\Sigma \gamma_j^*$ from time-series estimates of the Frisch parameter we have assumed that ω is a homegeneous function of household expenditure and family size. It is quite likely that "income" and family size have differing effects on ω but little positive information is available on this[2]. A useful approach to this problem would seem to be

1. Insofar as estimates of the β_i and ω enable calculation of price elasticities we are employing time-series estimates to assist in estimating price elasticities from cross-section data. This is a long established procedure dating from the work of Stone (1954a).

2. For family-size effects on ω for developing countries see Lluch, Powell and Williams (1977).

and $v_h = \sum_j v_{jh}$, i.e., total consumption expenditure, and

$u_h = \sum_j u_{jh}$.

Under the conventional atemporal maximization of the Klein-Rubin utility function (1) subject to a budget constraint total expenditure is assumed exogenous. The resulting expenditure equations, LES, again assuming no price variation across households, are :

$$v_{ih} = \alpha_i + \beta_i v_h + \varepsilon_{ih} \qquad (8)$$

where

$$\alpha_i = \gamma_i^* - \beta_i \sum_{j=1}^{n} \gamma_j^* , \qquad (9)$$

and the only new notation is the error term ε_{ih}. It is now apparent that β_i represents the marginal budget share for the i^{th} good. Notice, however, that equations (8) - (9) may also be obtained by eliminating the income term, y_h, from the ELES equations (3) - (7) . Under this derivation, however, the error term on (8) becomes

$$\varepsilon_{ih} = u_{ih} - \beta_i u_h . \qquad (10)$$

It follows that if ELES is the maintained hypothesis then ordinary least squares (OLS) estimation of (8) would yield inconsistent parameter estimates since the error term, ε_{ih}, is then correlated with the explanatory variable, v_h,

$$\text{i.e.,} \quad \text{plim}\left\{(\sum_h v_h \varepsilon_{ih})/H\right\} \neq 0 , \qquad (i = 1, \ldots, n) .$$

The above result is formally equivalent to that obtained by Summers (1959) working outside a demand systems framework. He was concerned solely with the biases involved in using total expenditure rather than income in estimating Engel curves. Liviatan (1961) extended Summers' work by arguing that even if income theoretically is the correct explanatory variable it is inappropriate to use it as an explanatory variable in Engel curve analysis (assuming estimation is by OLS) as it is likely to contain measurement error. Liviatan's solution was to use total expenditure as the explanatory variable but estimate by instrumental variables (IV), using measured income as the instrument. In the context of LES/ELES, this amounts to estimating equation (8) by IV, using y_h as instrument, and thus obtaining consistent estimates of α_i, β_i. Exactly the same estimates of β_i and their standard errors would however be obtained from estimating ELES (3) by OLS and calculating β_i from[1] :

$$\beta_i = \beta_i^* / \sum_j \beta_j^* = \beta_i^* / \mu .$$

The proof for the point estimates is simple :

$$\tilde{\beta}_i = \frac{\sum_h v_{ih} y_h}{\sum_h v_h y_h} = \frac{\sum_h v_{ih} y_h}{\sum_h y_h^2} \Bigg/ \frac{\sum_h v_h y_h}{\sum_h y_h^2} = \hat{\beta}_i^* / \hat{\mu} , \qquad (11)$$

where "~" denotes an IV estimate and "^" an OLS estimate[2], $\hat{\mu}$ is the OLS estimate from (7) and for convenience variables are expressed as deviations

1. OLS estimation of the consumption function (6) yields an identical value of μ and its standard error to that obtained by summing the OLS estimates of β_i^* in (2) .

2. This convention is retained throughout the paper.

6. CONCLUDING REMARKS

This paper has been concerned with obtaining reliable estimates of the parameters of the Klein-Rubin utility function for use in economy-wide models which specifically allow for demographic effects. The statistical relationships which exist between methods of estimating two expenditure systems derived from the Klein-Rubin utility function, namely, LES and ELES, have been exploited in a manner which permits the choice of the method of estimating the subsistence parameters, γ_i^* , to be delayed to a half-way stage in the estimating procedure. This research methodology is of considerable advantage given the difficulties involved in obtaining reliable estimates of the γ_i^* . The procedure adopted is first to obtain IV estimates of Engel curves, which may be interpreted in the context of LES/ELES. At this stage the ELES estimates of "subsistence" expenditure are examined to see whether they are satisfactory on both statistical and economic grounds. If they are not, this may be due to either model limitations or data deficiencies, particularly with respect to income data. The most likely model limitations[1] are the assumptions of additivity and linearity. The former was not tested, the latter was investigated using linear spline functions. No significant nonlinearities were detected. Low estimates of both the marginal and average propensities to save provided some grounds for querying the accuracy of the income data. In any event, some of the implied ELES estimates of total "subsistence" expenditure were unacceptable on theoretical grounds. It was therefore decided at this stage to take _a priori_ estimates

1. Byron (1976) has developed a test of LES/ELES against the alternative hypothesis of an unrestricted linear expenditure equation, that is, one in which all prices appear in all equations with unrestricted coefficients.

"Subsistence" expenditure on food increases with family size and ranges from around \$800 in 1966-7 prices for young households with no children to around \$1200 per household for old households with four children. In part, however, this result is built into the model,[1] and with one exception (old households with no children) "subsistence" expenditure on food is around 45 per cent of total "subsistence" expenditure for all types of households. The γ_i^*-estimates for clothing increase noticeably with family size for old households, and to a lesser extent for young households. Estimates of "subsistence" expenditure on housing tend to be lower for households without children, particularly for young households. The estimates of γ_i^* for transport decrease with family size for young households although this may in part reflect the lower incomes of large families; the corresponding values for old families are positively correlated with household income. For cigarettes and alcohol the γ_i^*-estimates among young households are remarkably similar (around \$145) across family sizes. For the remaining categories (durables, medical, recreation and other) no systematic family-size effects emerge.

The combination of IV estimates of Engel curve parameters and independent estimates of total subsistence expenditure obtained using time series estimates of ω has thus provided quite acceptable estimates of "subsistence" expenditures on individual commodities. Age of household head and number of children in the family both appear to be important causes of variation in "subsistence" expenditure which cannot be neglected.

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1. The <u>a priori</u> estimates of ω and $\Sigma \gamma_j^*$ ensure that at least <u>total</u> "subsistence" expenditure increases with family size.

from sample means. A similar result was obtained by Liviatan outside a demand systems framework. The proof for the standard errors is more complex and is given in an appendix.

Thus we have two equivalent ways of obtaining consistent estimates of marginal budget shares, β_i : IV estimation of LES (8) or OLS estimation of ELES (2) and unscrambling. The OLS estimates of β_i^* and μ considered separately are consistent only in the absence of measurement error in income. The practical advantage of the IV approach is that standard errors of $\hat{\beta}_i$ are obtained directly. On the assumptions that the errors on (2) have zero mean, are uncorrelated across households, have constant variances and covariances, that is

$$E(u_{ih}) = 0 ,$$

$$E(u_{ih}u_{jh}) = \sigma_{ij} ,$$

$$E(u_{ih}u_{jk}) = 0 , \qquad h \neq k ,$$

and are normally distributed, the IV estimates of $\hat{\beta}_i$ will be asymptotically efficient and normally distributed with asymptotic variances given by the usual IV formula. This follows from the fact that on the above error assumptions OLS estimates of ELES (2) are maximum likelihood estimates and there is a one to one correspondence between these and IV estimates of LES.[1]

Consistent estimates of the intercept term, α_i , in the Engel equations may be obtained using IV. Whilst these estimates are of interest <u>per se</u>, it is also desired to obtain estimates of the subsistence parameters,

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1. Since within LES and within ELES the same explanatory variables appear in each equation of the respective system, estimation by OLS on a commodity-by-commodity basis is equivalent to systems maximum likelihood estimation. The outlined estimation method also ensures that $\sum_j \tilde{\beta}_j = 1$ as required.

γ_i^* , which are related to α_i as given in (9). It is well known, however, that estimates of the α_i (i = 1, ..., n) are not sufficient to permit estimation of the γ_i^* (i - 1, ..., n)[1]. Essentially this is because the parameter estimates (either IV or OLS) are such as to ensure that the budget constraint holds globally, i.e., $\sum_i \hat{v}_{ih} = v_h$ for all households. In particular, $\sum_i \tilde{\alpha}_i = 0$ and there are only (n - 1) independent linear expressions in the n different γ_i^* . One additional piece of information is required to unscramble the γ_i^* , i.e., one of the γ_i^* or their sum must be given a priori. For example, within the context of LES/ELES total "subsistence" expenditure is related to the expenditure elasticity of the marginal utility of expenditure or Frisch "parameter," ω , as follows :

$$\Sigma \gamma_j^* = v(1 + \omega^{-1}) \quad . \qquad (12)$$

If a suitable estimate of ω is available then it can be combined with mean expenditure to yield an estimate of $\Sigma \gamma_j^*$, thus permitting the individual γ_i^* parameters to be unscrambled[2].

The advantage of ELES in a cross-section context is that it does permit unscrambling of the individual γ_i^* . The $\hat{\alpha}_i^*$ values of equation (3) do not satisfy any adding-up constraint and $\hat{\gamma}_i^*$ may be unscrambled using :

1. See Howe (1974), Powell (1973, 1974) and, more recently, Muellbauer (1974, 1975).

2. An alternative approach is to incorporate a priori estimates of ω , and therefore of $\Sigma \gamma_j^*$, into the estimation routine. Such an approach is used by Ryan (1976).

Table 6

ESTIMATES OF "SUBSISTENCE" EXPENDITURES, γ_i^* , BY AUSTRALIAN HOUSEHOLDS, IN 1966-7 PRICES[1]

Commodity	HOUSEHOLD TYPE[2]								
	< 35				≥ 35				
	0	1	2	3	0	1	2	3	4
Food	789 (124)	926 (69)	887 (87)	1059 (72)	968 (83)	952 (55)	1171 (68)	1188 (58)	1219 (106)
Cigs., Alcohol	142 (62)	149 (39)	141 (53)	146 (44)	68 (55)	168 (69)	130 (67)	151 (33)	115 (52)
Clothing	-16 (102)	84 (38)	1 (59)	144 (38)	9 (50)	72 (54)	79 (48)	154 (35)	237 (80)
Housing	455 (164)	574 (72)	598 (90)	595 (61)	332 (68)	557 (68)	331 (75)	351 (58)	537 (85)
Durables	0 (306)	17 (132)	-51 (148)	-148 (145)	-44 (143)	80 (117)	386 (129)	158 (106)	104 (211)
Medical	94 (167)	244 (41)	171 (59)	195 (64)	226 (53)	140 (50)	194 (40)	93 (46)	150 (69)
Transport	382 (128)	319 (64)	301 (84)	260 (66)	77 (85)	325 (61)	357 (54)	351 (47)	193 (97)
Recreation	-108 (163)	27 (52)	105 (124)	97 (51)	22 (91)	-184 (81)	-93 (81)	37 (39)	138 (86)
Other	42 (53)	-308 (76)	21 (52)	114 (34)	12 (68)	73 (44)	-54 (58)	107 (46)	119 (59)
Sum	1779	2032	2174	2463	1670	2183	2502	2590	2812

1. Approximate standard errors are given in parentheses. Column sums may not add owing to rounding error.

2. < 35 and ≥ 35 relate to age of household head in years; 0, 1, 2, 3, 4 refer to number of children in household. Only households with working heads and non-working children are included.

Values of the individual γ_i^* are reported in table 6, together with estimates of their asymptotic standard errors.[1] About 60 per cent of the γ_i^*-estimates are significant, for both young and old households, where significance is defined as occurring when the point estimate exceeds twice the estimated standard error in absolute size. All nine estimates of "subsistence" expenditure on food are significant, whereas only one estimate is significant for durables and for recreation. Negative values of γ_i^* are obtained for durables, recreation and other, but they are significant in only two cases (recreation for old families with one child, "other" for young families with one child). The γ_i^*-estimates lose their "subsistence" interpretation when they are negative, and imply own-price elasticities which are greater than one in absolute value.[2] They are therefore most likely to occur for non-necessities, as is the case here.

Age of the household head appears to be an important determinant of "subsistence" expenditure. Comparing old and young households of a given family size, "subsistence" expenditure on food is always higher for old families, reflecting the older age of children, whereas "subsistence" expenditure on housing and "other" is always higher for young families. If households with no children are ignored, the γ_i^*-values for durables and transport are higher for old families, but the γ_i^*-values for recreation are higher for young families.

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1. The estimate of $\Sigma \gamma_j^*$ is treated as a constant in calculation of standard errors, that is

$$\text{var}(\text{est } \gamma_i^*) = \text{var}(\text{est } \alpha_i) + 2\,(\text{est } \Sigma \gamma_j^*)\, \text{cov}(\text{est } \alpha_i, \text{est } \beta_i) + (\text{est } \Sigma \gamma_j^*)^2\, \text{var}(\text{est } \beta_i) \quad ,$$

where "est" is short for "estimate of." It follows that the reported standard errors will underestimate the "true" values.

2. See Powell (1974), p. 38.

$$\gamma_i^* = \alpha_i^* + [\beta_i^* (1 - \sum_j \beta_j^*)^{-1} \sum_j \alpha_i^*] \qquad (i = 1, \ldots, n). \qquad (13)$$

Such estimates of γ_i^* are of course only consistent if income is measured without error.

Because of uncertainties about the quality of income data the procedure adopted here is to first obtain IV estimates of the Engel equation coefficients α_i and β_i in (8). Then the individual γ_i^* values will be unscrambled using both available alternatives : estimates of the subsistence sum, $\Sigma \gamma_j^*$, from (i) OLS estimation of the aggregate consumption function (6) - (7), and (ii) <u>a priori</u> information. In unscrambling we exploit the fact that ELES decomposes into two parts : LES and an aggregate consumption function. Thus using the LES equation (9) :

$$\tilde{\gamma}_i^* = \tilde{\alpha}_i + \tilde{\beta}_i \Sigma \hat{\gamma}_j^* \qquad (i = 1, \ldots, n) , \qquad (14)$$

where alternative values of the sum of subsistence expenditure, $\Sigma \hat{\gamma}_j^*$, will be investigated. Equations (13) and (14) yield the same estimates of γ_i^* if $\Sigma \gamma_j^*$ in (14) is obtained from the aggregate consumption function (6) - (7). Before proceeding to empirical results, however, some discussion of the linearity assumption is necessary.

3. SPLICED ENGEL CURVES

Demographic effects on consumption patterns are to be measured by fitting LES/ELES separately to demographically homogeneous groups of households, that is, households of a given family size and age class.[1] By so subdividing the sample we reduce the variability of income across the data points to which the model is fitted. Nevertheless, it is still desirable to check for the appropriateness of the linearity assumption. The option of further subdividing the sample by income class must be ruled out on the grounds that it would leave insufficient degrees of freedom for estimation. Instead we make use of linear spline functions.[2] This amounts to piece-wise linear approximations of the underlying relationships between consumption expenditures and "income." Suppose, for example, that the relationship between expenditure on commodity i and total consumption expenditure can be represented by three linear segments which join at total expenditure levels of \$a and \$b, where a < b. The estimating equation for LES is now

$$v_{ih} = \alpha_i + \beta_i v_h + \rho_i (v_h - a)_+ + \lambda_i (v_h - b)_+ + u_{ih} \qquad (15)$$

where "+" denotes that negative values are taken as zero. The coefficients ρ_i and λ represent changes in the i^{th} marginal budget share between successive

1. Thus separate sets of estimates of all parameters are obtained. The alternative is to assume that some parameters, usually the β_i, are constant across more than one household type - - for applications see Betancourt (1973, 1977), Howe and Musgrove (1977) and Kakwani (1976).

2. The key reference on spline functions is Poirier (1976).

The relevant value of X for this paper is per capita GNP in Australia in 1966-7 measured in 1970 US dollars. The calculated value of X is 2486[1] and the associated ω-estimate is - 2.16, which is similar to values obtained in other studies using Australian time-series data[2]. All that remains is to modify (17) to take account of the fact that the data used in this paper relate to household expenditure rather than GNP shares. This is done by forcing the ω-estimate to be - 2.16 when evaluated at mean per capita total expenditure for the whole sample. The latter figure is \$1157, so to replace X by v/f, where f is household size, requires the scale factor to be lowered such that :

$$\omega = -27.3 \, (v/f)^{-.36} \qquad (18)$$

Values of ω for each of the nine household types calculated from (18) are given in row 5 of table 5. They are smaller or larger than 2.16 in absolute value according to whether per capita total expenditure is larger or smaller, respectively, than the overall mean. The associated values of $\Sigma \gamma_j^*$, obtained using (12), are given in row 6 of table 5. It is these γ^*-sum estimates which are combined with the IV estimates of α_i and β_i (tables 2 and 3) to obtain estimates of "subsistence" expenditure on individual commodities using (14). In turn, these γ_i^*-values may be used to calculate the responsiveness of demand to price changes, although this will not be pursued here[3].

1. The method used is that adopted in Lluch, Powell and Williams (1977), chapters 3 and 4 : project the estimate of GNP per capita in 1970 (in US dollars) back to the base year using average rates of growth. Both figures are taken from the World Bank Atlas, 1972.
2. See Powell (1966), Tran Van Hoa (1968), Lluch and Powell (1975) and Lluch and Williams (1975).
3. The relevant price elasticity formulas may be found, for example, in Powell (1974), p. 41.

Table 5

ESTIMATES OF THE MARGINAL PROPENSITY TO CONSUME, μ, TOTAL SUBSISTENCE EXPENDITURE, $\Sigma \gamma_j^*$, AND THE FRISCH PARAMETER, ω, AUSTRALIA, 1966-8

	HOUSEHOLD TYPE[1]								
	< 35				≥ 35				
	0	1	2	3	0	1	2	3	4
1. Mean Expenditure ($)	4150	3967	3834	4198	3731	4410	4697	4495	4725
ELES Estimates[2]									
2. μ	.3537 (.0892)	.3281 (.0449)	.2202 (.0492)	.4972 (.0939)	.3181 (.0503)	.4505 (.0548)	.1870 (.0311)	.4003 (.0430)	.3499 (.0836)
3. $\Sigma \gamma_j^*$	3913 (1930)	3830 (408)	3785 (365)	4518 (1149)	3621 (416)	4107 (672)	4561 (305)	4274 (492)	4760 (2215)
4. ω	-17.5	-29.1	-79.2	+13.1	-33.9	-14.6	-34.6	-20.3	+136.2
Time-Series Estimates									
5. ω	-1.75	-2.05	-2.31	-2.42	-1.81	-1.98	-2.14	-2.36	-2.47
6. $\Sigma \gamma_j^*$	1779	2032	2174	2463	1670	2183	2502	2590	2812

1. < 35 and ≥ 35 relate to age of household head in years; 0, 1, 2, 3, 4 refer to number of children in household. Only households with working heads and non-working children are included.

2. Standard errors are given in parentheses.

"income" classes. Thus in equation (15) the marginal budget share for households with total consumption expenditure less than \$a is β_i; the marginal budget share for households with total consumption expenditure between \$a and \$b is $(\beta_i + \rho_i)$; and for expenditure greater than \$b the marginal budget share is $(\beta_i + \rho_i + \lambda_i)$.

Linear spline functions ensure that the linear segments form a continuous curve, but in so doing they implicitly impose relationships between slopes and intercepts, which in the context of our demand system means restrictions between subsistence expenditures and marginal budget shares. If we consider the three segments of (15) as separate LES equations, the intercept terms are α_i, $(\alpha_i - a\rho_i)$, and $(\alpha_i - a\rho_i - b\lambda_i)$, for the three "income" classes, respectively. Undue restrictiveness of the model can be avoided, however, by suitable choice of the number and location of the join points.

The arguments of section 2 still apply in deciding on the most appropriate method of estimating the parameters of the Klein-Rubin utility function. The method decided upon in section 2 may be used with spline functions, i.e., IV estimation of the coefficients of (15) and independent estimates of the sum of subsistence expenditures used in unscrambling the individual γ_i^*. The ELES aggregate consumption function is now of the form :

$$v_h = \alpha + \mu y_h + \eta(y_h - a)_+ + \xi(y_h - b)_+ + u_h \qquad (16)$$

where η and ξ measure changes in the marginal propensity to consume across income classes. The successive intercept terms, from which it is possible to unscramble estimates of the subsistence sums, are α, $(\alpha - \eta a)$ and $(\alpha - a\eta - b\xi)$.

Strictly speaking, if ELES is the correct model then the same join points (a, b) should only be used for both ELES and LES if $v_h \equiv y_h$. If a and b are the join points in ELES, then the appropriate join points in LES (obtained after eliminating y_h) are functions of (a, b) and parameters of the aggregate consumption function. Since in this paper we require estimation methods which do not necessarily assume that consistent estimates of the latter are available we will for convenience use the same join points in (15) and (16) and use $(y_h - a)_+$ and $(y_h - b)_+$ as instruments in (14).

4. EMPIRICAL ESTIMATES OF LINEAR AND SPLICED ENGEL CURVES

The 1966-8 Survey of Consumer Expenditures and Finances was carried out in two stages.[1] First, expenditure and income information was obtained from around 5,500 urban households. Second, detailed financial information was obtained from about half the original households. The income data collected at the second round is the more comprehensive and reliable and primarily for this reason we confine ourselves to this data. But, in addition, it enables us to use imputed values for consumption of owner-occupied houses and motor vehicles. In estimation the sample has been confined to households composed of married couples and married couples with non-working children. Households where the head is not working are excluded. This reduces the effective sample size to 1405.

1. For descriptions of the survey, definitions of variables, etc., see Drane (1969, 1970/71), Podder (1971) and Podder and Kakwani (1975).

5. "SUBSISTENCE" EXPENDITURES

In section 2 two approaches to unscrambling estimates of "subsistence" parameters, γ_i^*, were outlined. One approach is to use an ELES estimate of the "subsistence" sum, $\Sigma \gamma_j^*$, obtained from OLS estimation of the aggregate consumption function (6) - (7). Such values are given in row 3 of table 5. These estimates will be consistent only if income is measured without error. There is some evidence from table 1 that income is underestimated in the survey : for large families household expenditure exceeds disposable income. Estimates of the marginal propensity to consume are low (row 2, table 5). Furthermore, the obtained estimates of the "subsistence"-sum are theoretically unacceptable in two cases where they exceed actual expenditure (see table 5). In the other seven cases the γ^*-sum estimates are sufficiently close to actual expenditure to yield unacceptably high estimates of the Frisch parameter in absolute terms (line 4, table 5). These ELES estimates are therefore rejected and the second approach to unscrambling the γ_i^* terms, namely, using *a priori* information, is now explored.

Equation (12) sets out the relationship between total "subsistence" expenditure and the Frisch parameter, ω. An estimate of ω, and therefore of $\Sigma \gamma_j^*$, may be derived from Lluch, Powell and Williams (1977, chapter 4). Here, using time series data to estimate LES and ELES for different countries, ω was found to vary with per capita GNP, X (measured in 1970 US dollars) in the following manner :

$$\omega = -36 X^{-.36} . \qquad (17)$$

In table 4 the β_i-values measure marginal budget shares for households with total expenditure below $4000 per year in 1966-7 prices. The ρ_i-estimates measure the change in the slope of the Engel curves at the join points. Estimates of the marginal budget shares for households with total expenditure above $4000 are thus given by $(\hat{\beta}_i + \hat{\rho}_i)$. If the ρ_i-estimate is not significantly different from zero then the assumption of linearity over the whole income range may be retained. In only 9 of the 45 estimates of ρ_i does the point estimate exceed the estimated standard error[1], and in no case is the estimate significant at the 5 per cent level. No "t-values" exceed one in the case of households with no children, even though the sample size of 232 is relatively large. The only result of note is that the marginal budget share for recreation tends to increase with "income." The ρ_i-value for recreation is positive and exceeds its standard error for families with one, three and four children. In a number of cases marginal budget shares are negative for one of the segments, although the average values for the two segments tend to be similar to those given in table 3. It may be concluded that there is no evidence of the linearity assumption being inappropriate within our subgroups of consumers. Thus the Engel curve estimates presented in tables 2 and 3 will be used in unscrambling estimates of "subsistence" expenditures, γ_i^* .

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1. The cases are : 1 child, housing and recreation ; 2 children, housing, durables and medical ; 3 children, food, recreation and other ; 4 children, recreation.

In order to capture age effects the sample is partitioned into "young" and "old" households according to the age of the household head. A break-point of 35 years is chosen, primarily because families at this stage in the life cycle would not have children beyond the secondary school level. Family size effects are measured by fitting the system separately for households with 0, 1, 2, 3 and, for the older group only, 4 children.[1]

A nine commodity classification of expenditure is adopted : Food, Cigarettes & Alcohol, Clothing, Housing, Durables, Medical Goods, Transport[2], Recreation, and Other. The income variable is personal disposable income including imputed rent. Both the expenditure and income definitions correspond fairly closely to those used in the Australian National Accounts.

Characteristics of the sample data are given in table 1. Mean household income and expenditure for a given family size is lower for young households except for households with no children. Both the mean household income and the average savings ratio for young households decline uniformly with the number of children. This is because the percentage of working wives is higher in small households. For old households, income and the savings ratio reach a peak for households with two children. The most noticeable features of the figures for average budget shares, v_i/v, are that the values for food increase with family size for both young and old households, and the values for durables are much higher for young households than for old households.

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1. There are insufficient observations in other classes.
2. Includes computed value of services provided by motor vehicles rather than actual cash outlays on purchases. The imputations were constructed by the author from the market value of cars owned by households. A consumption rate of 20% was assumed. This value is suggested by the work of Swan (1971), p. 15, and Industries Assistance Commission (1974), p. 40.

Table 1

CHARACTERISTICS OF THE SAMPLE DATA, AUSTRALIAN HOUSEHOLDS, 1966-8

	HOUSEHOLD TYPE[1]								
	< 35				≥ 35				
	0	1	2	3	0	1	2	3	4
No. observations	82	149	187	98	232	148	256	166	87
Mean Exp. ($)	4150	3967	3834	4198	3731	4410	4697	4495	4725
Mean Income ($)	4582	4246	4005	3875	3966	4779	5287	4826	4661
Av. Prop. Save	.094	.066	.043	-.083	.059	.077	.112	.069	-.014
Average Budget Shares									
Food	.234	.259	.295	.297	.264	.267	.286	.307	.321
Cigs., Alcohol	.041	.041	.044	.045	.054	.056	.045	.043	.037
Clothing	.071	.061	.067	.064	.065	.080	.073	.077	.080
Housing	.169	.173	.177	.159	.178	.156	.164	.157	.154
Durables	.187	.159	.122	.159	.109	.106	.109	.110	.099
Medical	.066	.083	.078	.082	.082	.081	.077	.074	.081
Transport	.123	.124	.114	.106	.129	.121	.106	.108	.109
Recreation	.076	.058	.066	.052	.078	.077	.076	.063	.063
Other	.034	.042	.037	.035	.040	.056	.062	.062	.057

1. < 35 and ≥ 35 relate to age of household head in years; 0, 1, 2, 3, 4 refer to number of children in household. Only households with working heads and non-working children are included.

Table 4

IV ESTIMATES OF SPLICED ENGEL CURVES, HOUSEHOLD HEAD AGED 35 YEARS OR MORE, AUSTRALIA, 1966-8[1]

	Food	Cigs.	Cloth.	Hous.	Dur.	Med.	Trans.	Rec.	Other
0 children									
α_i	975.1	-358.6	-108.8	596.8	-756.5	408.4	319.8	-793.4	-273.6
	(635.6)	(456.7)	(393.3)	(659.7)	(1179.0)	(468.1)	(817.4)	(817.2)	(558.2)
β_i	.0011	.1777	.0881	-.0286	.3456	-.0491	-.0064	.3428	.1287
	(.2254)	(.1620)	(.1395)	(.2339)	(.4181)	(.1660)	(.2899)	(.2899)	(.1979)
ρ_i	.0159	-.2345	.0541	.3923	-.2611	.1816	.4189	-.4394	-.1277
	(.4705)	(.3381)	(.2911)	(.4884)	(.8728)	(.3465)	(.6051)	(.6050)	(.4132)
1 child									
α_i	553.1	529.0	-75.2	-559.3	-1384.2	247.5	391.1	409.5	-111.6
	(560.1)	(682.2)	(538.0)	(674.5)	(1256.5)	(522.1)	(637.1)	(815.2)	(444.7)
β_i	.1539	-.0937	.0885	.3504	.4939	.0029	.0138	-.0916	.0819
	(.1641)	(.1999)	(.1576)	(.1976)	(.3681)	(.1530)	(.1867)	(.2388)	(.1303)
ρ_i	-.0651	.1609	.0468	-.3608	-.3965	.1174	.0992	.4040	-.0059
	(.2010(	(.2449)	(.1931)	(.2421)	(.4510)	(.1874)	(.2287)	(.2926)	(.1596)
2 children									
α_i	642.7	131.4	-461.9	1150.0	-1501.5	548.1	69.1	-451.0	-126.9
	(650.1)	(639.8)	(470.3)	(764.0)	(1254.4)	(411.2)	(511.4)	(769.7)	(552.0)
β_i	.1710	.0112	.1872	-.1666	.5426	-.0749	.0994	.1617	.0684
	(.1817)	(.1789)	(.1315)	(.2136)	(.3507)	(.1149)	(.1430)	(.2152)	(.1543)
ρ_i	-.1013	.0294	-.0734	.4056	-.5348	.1679	-.0392	.0469	.0989
	(.2078)	(.2045)	(.1504)	(.2443)	(.4010)	(.1314)	(.1635)	(.2461)	(.1765)
3 children									
α_i	-167.9	336.9	-520.9	457.7	-646.8	-426.6	-194.8	464.4	698.1
	(700.3)	(381.6)	(418.3)	(681.1)	(1211.4)	(524.5)	(520.3)	(480.2)	(557.5)
β_i	.4192	-.0487	.2210	.0148	.2775	.1819	.1763	-.0924	-.1496
	(.2027)	(.1105)	(.1211)	(.1972)	(.3507)	(.1518)	(.1506)	(.1390)	(.1614)
ρ_i	-.4030	.0894	-.1524	.2155	-.1277	-.0719	-.1334	.2799	.3036
	(.2570)	(.1401)	(.1535)	(.2500)	(.4446)	(.1925)	(.1910)	(.1763)	(.2046)
4 children									
α_i	1216.5	-225.0	11.7	126.6	-425.9	-547.3	-723.1	756.3	-190.0
	(821.2)	(386.9)	(573.3)	(609.2)	(1502.4)	(495.5)	(664.4)	(764.0)	(425.8)
β_i	.0209	.1092	.0792	.1400	.1887	.2315	.3054	-.1809	.1060
	(.2385)	(.1124)	(.1665)	(.1769)	(.4364)	(.1439)	(.1930)	(.2219)	(.1237)
ρ_i	.1963	-.1137	-.0082	-.0593	-.0020	-.1608	-.2002	.3836	-.0397
	(.3090)	(.1456)	(.2157)	(.2292)	(.5654)	(.1865)	(.2500)	(.2875)	(.1602)

1. Standard errors are given in parentheses.

The number of children in the household seems to influence the β_i-estimates less than does the age of the household head. Nevertheless, some patterns emerge. The marginal budget share for food shows some tendency to increase with family size : it is estimated to be around 0.10 for both young and old families with three children. Among young households the β_i-estimates for transport increase with family size, whereas those for recreation fall with family size. This could, however, just be an income effect as household income is inversely related to family size for young households (see table 1). An income effect is observed for these β_i-estimates among old households : the marginal budget shares for transport and recreation tend to be negatively and positively associated, respectively, with household income. There is some tendency for the marginal budget share for medical goods to increase with family size. Overall, however, it would be a reasonable approximation to assume constant marginal budget shares across family sizes <u>within a given age group</u>.[1]

Within each of the nine consumer groups the linearity assumption of the model was tested using spline functions as outlined in the previous section. Experimentation with more than one join point proved unsuccessful in that the linear segments tended to move erratically and coefficients were badly determined. Even with only two segments, that is, one join point, the same problems were encountered with data for young households. Presented results are therefore confined to old households and are given in table 4. The single join point used in the reported results is $4000, although some (unsuccessful) experimentation was conducted with other values. By using a value of $4000 the samples, which in several cases are rather small, are divided roughly in half.

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1. As is done, for example, by Betancourt (1977) and Musgrove and Howe (1977).

Instrumental variable estimates of the Engel curve coefficients, α_i and β_i in equation (8), are given in tables 2 and 3, together with estimates of their asymptotic standard errors. Negative values of the intercept, α_i , imply that the average budget share increases with total expenditure. This phenomenon is observed for clothing, durables, recreation and "other" among both young and old households. The effect is most pronounced for expenditure on durables by young households.

All of the estimates of marginal budget shares reported in table 3 lie in the expected interval of 0 to 1. About 80% of the 45 β_i-estimates for old households are significant at the 5 per cent level, whereas only a little over half of the β_i-estimates are significant for young households. The marginal budget share for housing is always significant at the 5 per cent level for old households, but it is never significant for young households. Only one of the nine β_i-estimates for cigarettes and alcohol is significant. Expenditure on these goods appears to be unrelated to "income." In contrast, expenditure on clothing is estimated to be significantly influenced by "income," within a given family type, in all but one case.

Comparing young and old families of a given size, the marginal budget share for durables is always markedly higher for young families, whereas the β_i-values for housing and cigarettes and alcohol are higher for old families. The marginal budget share for recreation is markedly higher for old households if we confine comparison to households with children.

Table 2

IV ESTIMATES OF INTERCEPT IN ENGEL CURVES, α_i,

AUSTRALIAN HOUSEHOLDS, 1966-8 [1]

Commodity	HOUSEHOLD TYPE [2]								
	< 35				≥ 35				
	0	1	2	3	0	1	2	3	4
Food	652.7 (212.6)	821.2 (135.2)	568.3 (196.0)	789.9 (160.8)	953.4 (146.9)	731.3 (101.3)	972.8 (140.4)	926.2 (125.9)	780.8 (247.7)
Cigs., Alcohol	121.9 (106.5)	131.8 (77.0)	101.8 (119.4)	84.0 (98.7)	-38.7 (97.9)	88.5 (126.3)	35.5 (137.4)	94.3 (71.6)	27.4 (121.4)
Clothing	-248.4 (175.7)	-80.9 (75.2)	-333.4 (133.0)	-32.0 (85.5)	-182.6 (88.5)	-203.2 (98.3)	-222.6 (99.1)	-107.2 (76.5)	29.8 (187.8)
Housing	269.4 (281.6)	454.3 (140.9)	491.3 (203.5)	494.6 (135.9)	61.8 (120.9)	428.3 (124.0)	-171.6 (153.8)	-127.5 (125.1)	258.3 (197.7)
Durables	-581.5 (524.4)	-628.8 (257.3)	-730.7 (335.2)	-1307.8 (326.5)	-409.4 (252.3)	-298.9 (214.0)	241.0 (266.3)	-300.2 (228.7)	-430.3 (492.3)
Medical	-42.7 (286.0)	155.8 (80.7)	2.6 (132.4)	-18.4 (143.8)	160.7 (93.0)	-73.8 (91.5)	1.2 (82.9)	-231.5 (98.9)	-190.3 (162.2)
Transport	283.9 (219.0)	138.1 (124.5)	122.9 (189.9)	-1.7 (148.9)	-251.6 (150.7)	119.4 (111.5)	196.8 (111.2)	167.3 (101.1)	-278.6 (227.2)
Recreation	-423.4 (279.0)	-184.9 (101.7)	-86.8 (279.4)	-77.0 (113.5)	-194.2 (161.1)	-696.3 (148.1)	-603.8 (166.7)	-295.5 (84.5)	-95.3 (200.9)
Other	-31.8 (91.0)	-806.4 (149.0)	-135.8 (118.4)	68.4 (76.1)	-99.5 (120.8)	-95.4 (81.5)	-449.3 (120.3)	-126.0 (100.7)	-101.8 (138.0)

1. Standard errors are given in parentheses.
2. < 35 and ≥ 35 relate to age of household head in years; 0, 1, 2, 3, 4 refer to number of children in household. Only households with working heads and non-working children are included.

Table 3

IV ESTIMATES OF MARGINAL BUDGET SHARES, β_i,

AUSTRALIAN HOUSEHOLDS, 1966-8 [1]

Commodity	HOUSEHOLD TYPE [2]								
	< 35				≥ 35				
	0	1	2	3	0	1	2	3	4
Food	.0765 (.0507)	.0518 (.0336)	.1464 (.0509)	.1091 (.0377)	.0089 (.0390)	.1013 (.0223)	.0792 (.0296)	.1011 (.0274)	.1558 (.0519)
Cigs., Alcohol	.0113 (.0254)	.0082 (.0191)	.0179 (.0310)	.0253 (.0231)	.0639 (.0260)	.0364 (.0278)	.0379 (.0289)	.0219 (.0156)	.0311 (.0254)
Clothing	.1304 (.0419)	.0810 (.0187)	.1536 (.0345)	.0714 (.0200)	.1144 (.0235)	.1263 (.0217)	.1207 (.0209)	.1007 (.0167)	.0763 (.0393)
Housing	.1041 (.0672)	.0589 (.0350)	.0493 (.0528)	.0409 (.0318)	.1617 (.0321)	.0588 (.0273)	.2009 (.0324)	.1849 (.0273)	.0993 (.0414)
Durables	.3267 (.1251)	.3177 (.0639)	.3128 (.0870)	.4709 (.0765)	.2190 (.0669)	.1735 (.0472)	.0581 (.0561)	.1767 (.0498)	.1901 (.1031)
Medical	.0768 (.0682)	.0435 (.0200)	.0775 (.0344)	.0867 (.0337)	.0390 (.0247)	.0978 (.0202)	.0772 (.0175)	.1252 (.0216)	.1209 (.0340)
Transport	.0550 (.0523)	.0891 (.0309)	.0820 (.0493)	.1063 (.0349)	.1968 (.0400)	.0940 (.0246)	.0639 (.0234)	.0710 (.0220)	.1677 (.0476)
Recreation	.1776 (.0666)	.1045 (.0253)	.0883 (.0725)	.0708 (.0266)	.1297 (.0427)	.2348 (.0326)	.2041 (.0351)	.1285 (.0184)	.0828 (.0421)
Other	.1416 (.0217)	.2454 (.0370)	.0722 (.0307)	.0186 (.0178)	.0667 (.0320)	.0772 (.0180)	.1581 (.0253)	.0900 (.0220)	.0787 (.0289)

1. Standard errors are given in parentheses.
2. < 35 and ≥ 35 relate to age of household head in years; 0, 1, 2, 3, 4 refer to number of children in household. Only households with working heads and non-working children are included.