IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

A joint study by the Australian Bureau of Statistics, the Department of Employment and Industrial Relations, the Department of Environment, Housing and Community Development, the Department of Industry and Commerce and the Industries Assistance Commission

DURABLES IN THE CONSUMPTION FUNCTION

by

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NON-PARAMETIC SIGN TEST FOR TESTING AUTOCORRELATION

Let y = number of changes of sign of residuals. Assuming positive and negative values are equally likely to occur, the probability of sign change is 0.5 .

Thus y is Binomial with parameters $n - 1$ and 0.5 .

For n large ,

$$\frac{y - 0.5(n-1)}{\sqrt{0.25(n-1)}} \sim N(0,1) \quad .$$

The null hypothesis assumes independence of the residuals. The test is two-tailed.

The second term on the right side of (A1) is a/c. As T approaches t*, this term approaches the slope at t* of the planning schedule made at T, i.e.,

$$\frac{a}{c} \rightarrow \left.\frac{\partial k_{Tj}(t)}{\partial t}\right]_{t = t^*} ,$$

or equivalently ,

$$\frac{a}{c} \rightarrow \left.\frac{\partial k_{t^*j}(t)}{\partial t}\right]_{t = t^*} ,$$

where the derivative is understood to be a right derivative. (Note that $k_{t^*j}(t)$ is not defined for $t < t^*$).

We conclude that

$$\dot{k}^*_{tj} = \left.\frac{\partial k_{tj}(t^*)}{\partial t}\right]_{t = t^*} + \left.\frac{\partial k_{t^*j}(t)}{\partial t}\right]_{t = t^*} ,$$

where the first derivative is defined from the left, while the second is defined from the right.

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1. INTRODUCTION

In the classical literature of linear expenditure systems (LES) savings was taken as predetermined and no attempt was made to analyse the inter-temporal consumption - savings choice.[1] To add this dimension to the consumer's allocation process it is necessary to model explicitly the trade off between current and future consumption. The initial *operational* theory to include this concept within the framework of the modern systems approach was Lluch's[2] extended linear expenditure system (ELES).

ELES is a system of commodity expenditue equations with endogenous total consumption. Prices, income and price and income expectations are the predetermined variables in the system. With a framework of continuous planning time, the analysis is centred on inter-temporal utility maximization under certainty in the tradition of Fisher [12] and Tintner [37][38]. The solution represents the optimal

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* This paper is exploratory in nature, being the first attempt to estimate a complete system of demand equations from Australian data in a way which explicitly recognizes durability. It is based on recent theoretical work by Dixon and Lluch [10]. Results reported here are taken from my M.Ec. thesis at Monash University. I am grateful for comments and guidance to Peter B. Dixon and Alan A. Powell, and to Michael McAleer for pointing out some notational errors in the first printing of this draft. All remaining errors are my own.

1. See Stone [32], Parkes [23], Pollak and Wales [29] and Goldberger and Gamaletsos [13].
2. See Lluch [17].

allocation of expenditures at the beginning of a consumer plan when the instantaneous utility functions are Klein-Rubin [16]. It also provides answers to questions about savings' responsiveness to relative price changes; an estimate of marginal propensity to consume (mpc); and a measure of the bias due to the treatment of aggregate consumption as an exogenous variable in LES.[1, 2]

In applied demand studies, the empirical approach, whether it be a single equation estimation or the estimation of a complete expenditure system, is to divide total expenditure into various commodity groups. The category of durables and its specific inter-temporal links with current and past expenditure has resulted in many unresolved difficulties. Part of the problem centres on the need for a distinction between the demand for a stock of a durable good, the purchase of a durable and the demand for the services it renders. A common approach to this problem is to construct models which allow the flow of services (estimated as a constant proportion of the stock of durables held in the period) to affect current expenditure decisions; these current decisions in turn affect the future levels of stocks and thus set up a dynamic relationship between consumption and its previous values. In the single equation approach, researchers such as Stone and Rowe [33], Houthakker and Taylor [15] and Williams [40] have attempted to capture what appears to be the essence of the durable goods' expenditure process. In the systems approach, however, it is often true that no theoretical distinction is made in explaining expenditures on non-durables and durables.[3] Nevertheless there have been studies which have

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1. Since ELES contains LES and the aggregate consumption function, the bias due to the exogenous treatment of aggregate consumption in LES can be measured since the two components can be estimated separately under a specification (viz., ELES) which involves only predetermined variables.
2. For a more comprehensive treatment of ELES, see Powell [31], Ch. 6.
3. Recent surveys of demand literature include Powell [31], Theil [36], Phlips [26], and Brown and Deaton [7].

is constructed by taking points at which the planning schedules cross the t^* abscissa. The fifth schedule, shown as joining the dots, is the path of the actual holdings of durable j. Actual holdings are assumed to equal currently planned holdings and therefore the "actual" schedule links the initial points on each of the planned schedules.

The change in actual holdings over the period t to t^* can be written as

$$k_{t^*j} - k_{Tj} = \left[k_{t^*j} - k_{Tj}(t^*)\right] + \left[k_{Tj}(t^*) - k_{Tj}\right] .$$

Then replacing k_{t^*j} and k_{Tj} by $k_{t^*j}(t^*)$ and $k_{Tj}(T)$, and dividing through by $t^* - T$, we obtain

$$\text{(A1)} \qquad \frac{k_{t^*j} - k_{Tj}}{t^* - T} = \frac{\left[k_{t^*j}(t^*) - k_{Tj}(t^*)\right]}{t^* - T} + \frac{\left[k_{Tj}(t^*) - k_{Tj}(T)\right]}{t^* - T}$$

In terms of the diagram, the expression on the left of the equality is $(a + b)/c$, and as T approaches t^*, this becomes the slope of the actual schedule at t^*, i.e., $\dot{k}_{t^*j}$.

The first term on the right of (A1) is represented by b/c. For T close to t^*, this term approximates the slope of the $k_{tj}(t^*)$ schedule for any t in the interval $[T, t^*]$. In the limit as T approaches t^* b/c becomes the slope (defined only to the left since $k_{tj}(t^*)$ does not exist for $t > t^*$) of the $k_{tj}(t^*)$ schedule at t^* , i.e.,

$$\frac{b}{c} \rightarrow \left.\frac{\partial k_{tj}(t^*)}{\partial t}\right]_{t = t^*}$$

where the derivative is understood to be a left derivative.

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subjective discount rate ; ρ is the market rate of interest;[1] p_{ti} , P_{tj} are respective prices at which the consumer, at time t, expects non-durable i and durable j to be sold throughout the planning period (i.e., constant price expectations);[2] $q_{ti}(\tau)$, $R_{tj}(\tau)$ are the respective time paths of planned purchases of non-durable i and durable j ; $k_{tj}(\tau)$ is the consumer's planned holdings of stock of durables, at time t ; σ_j is the decay rate for durable j ; z_t/ρ is the present value of the consumer's expected life-time income, at time t, from his financial assets and sales of labour ; and τ is planning time.

It should be noted that :

(a) The utility function is assumed to be inter-temporally separable.

(b) The assumption of a constant subjective discount rate, δ , implies that the consumer's behaviour is Strotz-consistent.[3]

(c) The problem of maintaining stock equilibrium by discontinuous jumps has been overcome by assuming that at each point of time t , the consumer adjusts his stocks to their currently optimal levels and that the consumer's expectations about prices and income are changing smoothly, i.e., p_{ti} , P_{tj} and z_t are continuous functions of real time t .

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1. For simplicity ρ is treated as time invariant.

2. In the event that the representative consumer expects prices to change over the planning period, each of the p_{ti} and P_{tj} would be functions of planning time τ . Explicit assumptions on growth laws governing these time paths would then have to be introduced.

3. If the consumer's discount rate is constant and if his price and income expectations are realized and remain unchanged, his plans for time τ , made at time t , will be the same as his plans for time τ made at time t + s. See Strotz [34], Pollak [28] and Blackorby et al. [5].

The planning problem (1) - (3) can be solved using Lagrange/Euler maximization methods[1] to obtain an expression for the consumer's planned expenditure path for non-durable i and his planned investment in durable j , i.e.,

(4) $$p_{ti}q_{ti}(\tau) = p_{ti}\gamma_i + \mu\beta_i^* z_t^* \exp(\rho-\delta)(\tau-t) \quad ,$$

and

(5) $$p_{tj}k_{tj}(\tau) = p_{tj}\theta_j + \mu\eta_j^* z_t^* \exp(\rho-\delta)(\tau-t) \quad ,$$

where

(6.1) $\mu = {}^{\delta}/_{\rho}$, the marginal propensity to consume out of supernumerary permanent income ;

(6.2) $z_t^* = z_t - \sum_i p_{ti}\gamma_i - \sum_j \sigma_j\theta_j P_{tj}$, is supernumerary permanent income, i.e., income over that required to purchase, at time t , the basic bundle of consumables together with the cost of meeting depreciation on the basic bundle of durables ;

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1. See sections 3.8 - 3.14 [14].

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TABLE 5 (continued)

Estimates of :	$\sigma_1 = \sigma_2 = 10$	$\sigma_1 = \sigma_2 = 1000$	ELES
Shift term parameter for Motor Vehicles	0.0043 (0.002)	0.00002 (0.002)	#
Shift term parameter for Household Durables	0.005 (0.002)	0.00003 (0.003)	#
Market discount rate	- 1.712 (18.018)	26.689 (170967)	#
Time preference discount rate	- 1.330 (13.994)	12.455 (79790)	#
Marginal propensity to consume	0.776 (0.011)	0.467 (0.108)	0.780 (0.010)

\# Not applicable

(1) The estimates of the discount rates, ρ and δ, continued their erratic behaviour ($\hat{\rho}$ = - 173% to 2668% ; $\hat{\delta}$ = - 133% to 1245%) as σ_j changed from 10 to 1000. This behaviour was previously noted. Nevertheless, a surprising feature was that $\hat{\mu}$, their ratio, experienced a considerable change (0.777 to 0.490) for the first time. This counter intuitive result may have been caused by accumulated rounding errors and possible number underflow or overflow when a decay value of 1000 was used.

(2) There were noticeable fluctuations in gamma estimates, e.g., travel and other non-durables both moved from negative values (- 0.813 and - 37.712 respectively) to positive values (14.190 and 41.649 respectively) as σ_j changed from 10 to 1000.

TABLE 1 : UNCONSTRAINED ESTIMATES OF DELES CONSUMPTION FUNCTION AUSTRALIA, 1959-74

Commodity Group	Estimates of Stone-Geary Parameters†		Estimates of Marginal Budget Shares		Z Statistic for Independence of Residuals[1]
Nondurables	$\hat{\gamma}_i$		$\hat{\beta}_i$	$\hat{\beta}^*_i$	
Food	24.340 (4.892)		0.141 (381.216)	0.151 (0.009)	-5.92
Tobacco Cigarettes and Alcoholic Drinks	19.141 (2.163)		0.067 (182.668)	0.073 (0.005)	-1.89
Clothing	-8.001 (6.556)		0.116 (315.683)	0.126 (0.011)	-1.64
Rent	24.562 (2.015)		0.132 (357.985)	0.143 (0.006)	-0.88
Travel and Communication	13.792 (1.329)		0.032 (85.921)	0.034 (0.003)	-1.13
Other Nondurables	45.918 (7.951)		0.366 (992.976)	0.395 (0.012)	-1.39
Durables	$\hat{\theta}_j$	$\widehat{\theta_j \sigma_j}$	$\hat{\eta}_j$	$\hat{\eta}^{**}_j$	
Motor Vehicles	7.953 (139564.787)	2.926 (4.697)	0.067 (943.924)	0.040 (0.007)	-0.63
Household Durables	302.538 (9933434.89)	59.450 (8.280)	0.078 (1372.526)	0.038 (0.010)	-3.65

Estimates of:		
Shift Term parameter for Motor Vehicles	$\widehat{\mu \eta}^*_1$	0.053 (0.007)
Shift Term parameter for Household Durables	$\widehat{\mu \eta}^*_2$	0.074 (0.008)
Market Discount rate	$\hat{\rho}$	0.627 (23117.586)
Time preference discount rate	$\hat{\delta}$	0.452 (1660.248)
Marginal propensity to consume	$\hat{\mu}$	0.721 (0.015)

† Units are dollars per capita in 1966-7 prices.
* Asymptotic standard errors are in parentheses.

1. Test is two tailed and the null hypothesis assumes independence of the residuals. See Appendix B.

consumption was exceeded by the 'subsistence minima' values. This problem was also encountered by Powell [30], Pollak and Wales [29], and Goldberger and Gamaletsos [13].

(v) The estimates of utility function parameters, $\hat{\beta}_i$ and $\hat{\eta}_j$, were insignificant. By contrast, the marginal budget shares, $\hat{\beta}_i^*$ and $\hat{\eta}_j^{**}$, as defined in (6.3) and (8.1) had significant estimates. Although it is difficult to compare these results with other empirical studies due to the particular split up of personal consumption expenditure, it does appear obvious that the estimates of marginal budget shares for durables (0.040 for motor vehicles and 0.038 for household durables) are too low.

(vi) The estimates of ρ and δ were 62.7% and 45.2% respectively. Depreciation rates per quarter for motor vehicles and household durables were calculated at 30.8% and 17.8% respectively. As with (iii) the asymptotic standard errors of these ML estimates were so large that they need not be taken seriously.

(vii) The estimated m.p.c. of 0.721 was highly significant. This value was consistent with the work of Powell [30] and Lluch and Williams [19], who reported a similar magnitude for U.S. data.

4.2 Conditional Estimates

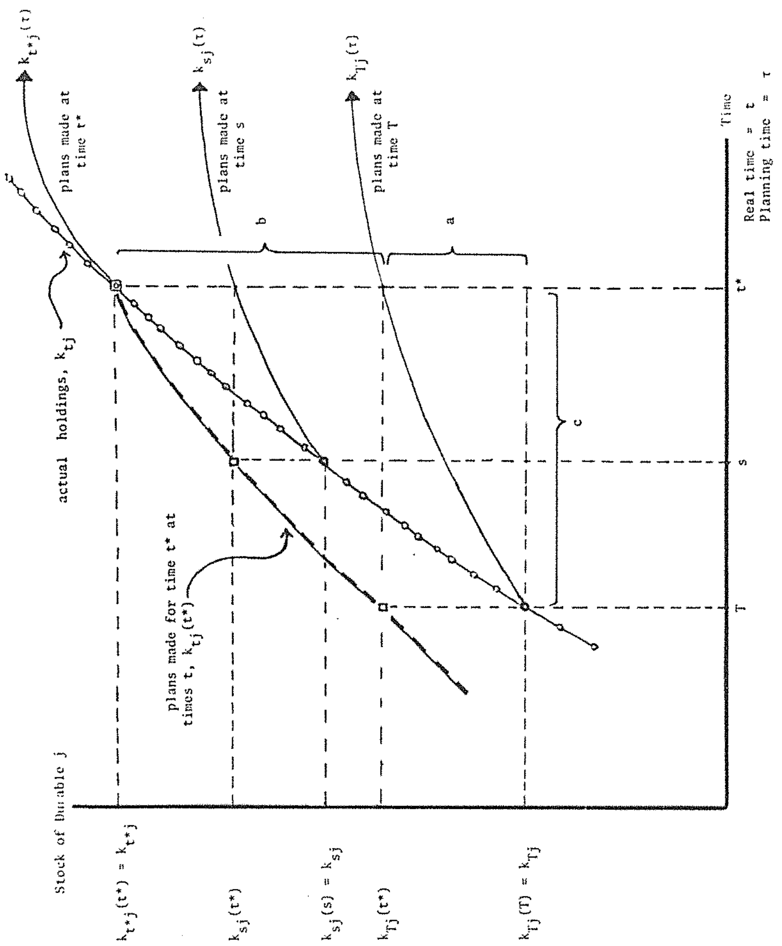
The estimation of the DELES consumption function was attempted using more plausible a priori values for depreciation rates.[1, 2] Using

1. See Filmer [11].
2. See Broadbent [6].

TABLE 5 : ESTIMATES OF DELES AND ELES CONSUMPTION FUNCTION: AUSTRALIA 1959 - 74*

Commodity Group	Estimates of Stone-Geary Parameters†			Estimates of Marginal Budget Shares				
	$\sigma_1=\sigma_2=10$	$\sigma_1=\sigma_2=1000$	ELES	$\sigma_1=\sigma_2=10$		$\sigma_1=1000$		ELES
Nondurables	$\hat{\gamma}_i$	$\hat{\gamma}_i$	$\hat{\gamma}_i$	$\hat{\beta}_i$	$\hat{\beta}_i^*$	$\hat{\beta}_i$	$\hat{\beta}_i^*$	$\hat{\beta}_i$
Food	28.324 (4.145)	40.686 (4.603)	28.180 (4.192)	0.108 (0.021)	0.106 (0.007)	0.105 (0.973)	0.105 (0.015)	0.105 (0.007)
Tobacco, Cigarettes and Alcoholic Drinks	28.284 (1.875)	25.681 (1.973)	26.697 (1.974)	0.038 (0.008)	0.038 (0.004)	0.056 (0.516)	0.056 (0.008)	0.040 (0.004)
Clothing	19.401 (6.571)	3.668 (6.132)	20.083 (6.504)	0.053 (0.013)	0.052 (0.010)	0.124 (1.154)	0.124 (0.018)	(0.050) (0.010)
Rent	11.672 (2.463)	27.558 (2.276)	9.562 (2.550)	0.146 (0.027)	0.144 (0.005)	0.130 (1.204)	0.130 (0.009)	0.151 (0.005)
Travel and Communication	-0.813 (1.553)	14.190 (1.338)	-1.153 (1.529)	0.053 (0.010)	0.052 (0.002)	0.042 (0.394)	0.042 (0.005)	0.052 (0.002)
Other Nondurables	-37.712 (9.609)	41.649 (9.423)	-36.608 (9.394)	0.500 (0.090)	0.492 (0.011)	0.422 (3.922)	0.422 (0.022)	(0.485) (0.011)
Durables	$\theta\hat{\sigma}_j$	$\theta\hat{\sigma}_j$	$\theta\hat{\sigma}_j$	$\hat{\eta}_j$	$\hat{\eta}_j^{**}$	$\hat{\eta}_j$	$\hat{\eta}_j^{**}$	$\hat{\eta}_j$
Motor Vehicles	-4.560 (2.760)	5.180 (3.122)	-4.874 (2.759)	0.047 (0.075)	0.054 (0.003)	0.054 (3.633)	0.054 (0.008)	0.054 (0.003)
Household Durables	34.597 (3.072)	45.338 (3.930)	34.986 (3.219)	0.054 (0.086)	0.062 (0.004)	0.067 (4.531)	0.067 (0.009)	0.062 (0.004)

† Units are dollars per capita in 1966-7 prices. * Asymptotic standard errors are in parentheses.
# Not applicable.



attempted to construct models for durable goods within a systems framework. Parks [25], Diewert [9], Weber [39], Motley [21], and Dixon and Lluch [10] are recent contributions to this cause.

The purpose of this paper is to present the Dixon/Lluch model and the empirical results obtained when the model (DELES) was applied to Australian data. The paper is structured as follows. In sections 2 and 3 respectively, the main features of the model and the data base are discussed. Section 4 briefly outlines the steps taken to make the model operational and discusses the main results. Section 5 contains a summary and suggestions for further work.

2. THE MODEL : DELES

Dixon and Lluch model durability explicitly by allowing the flow of services from stocks of durables, not the current purchases of durables, to appear as arguments in the consumer's utility function. The consumer is assumed to receive utility from his consumption of non-durables and the flow of services of durables. He plans his current and future holdings of goods so as to maximize the present value of a utility functional (1) subject to a budget constraint (2) and a stock flow identity (3) which assumes that durables decay exponentially, i.e.,

Maximize

(1) $$\int_t^\infty \exp(-(\tau-t)\delta) \left[\sum_i \beta_i \log(q_{ti}(\tau)-\gamma_i) + \sum_j \eta_j \log(k_{tj}(\tau)-\theta_j) \right] d\tau ,$$

subject to

(2) $$z_t/\rho - \int_t^\infty \exp(-(\tau-t)\rho) \left[\sum_i P_{ti} q_{ti}(\tau) + \sum_j P_{tj} R_{tj}(\tau) \right] d\tau = 0 ,$$

and

(3) $$\dot{k}_{tj}(\tau) - R_{tj}(\tau) + \sigma_j k_{tj}(\tau) = 0 ,$$

where

γ_i, β_i, θ_j, η_j are parameters in the instantaneous Klein-Rubin utility function [16] such that γ_i and θ_j are the "basic needs" of non-durable i and durable j respectively and where $\sum_i \beta_i + \sum_j \eta_j = 1$; δ is the consumer's

APPENDIX A

STOCK HOLDING UNDER CONTINUOUS STOCK EQUILIBRIUM

by

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A critical point in the derivation of DELES is the partitioning of the change in holdings into two parts - the movement along a plan and the movement of the plan, see equation (7). We have found the following diagrammatic analysis to be helpful in seeing what is happening, and John Iacono has agreed to our appending it in case some other readers also find it to be useful.

In the figure, we have shown five schedules. The three marked with arrow heads are consumer plans. They show how the consumer, at each of the points in real time, T, s, and t^*, plans his future holdings of durable j. For our particular illustration, changes in the consumer's expectations over the period T to t^* have been causing upward revisions - the later planning schedules lie above the earlier ones. The upward revisions are also illustrated by the fourth schedule (shown as joining the □'s) . For each point of real time, T, s, t^* etc., this schedule shows the planned holding of durable for time t^*, i.e., it shows the points $k_{tj}(t^*)$ for all $t \leq t^*$. The positive slope implies that changes in the consumer's expectations were causing him to increase his holdings of j, planned for time t^*, as time approached t^*. The reader will see that the $k_{tj}(t^*)$ schedule

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$$\beta_i^* = \frac{\beta_i}{1-\delta \sum_j \frac{\eta_j}{\sigma_j+\rho}} \quad ; \tag{6.3}$$

$$\eta_j^* = \frac{\eta_j}{\sigma_j+\rho-\delta(\sigma_j+\rho) \sum_j \frac{\eta_j}{\sigma_j+\rho}} \quad . \tag{6.4}$$

Using the assumption that the consumer's currently planned consumption of non-durables and his currently planned holding of durables are equal to his actual consumption and holding, the change in the consumer's holding of durables, at time t , is given by totally differentiating (5) with respect to τ and evaluating the result at $\tau = t$, i.e.,[1]

$$\dot{k}_{tj} = \left[\frac{\partial k_{tj}(\tau)}{\partial \tau}\right]_{\tau=t} + \left[\frac{\partial k_{tj}(\tau)}{\partial P_{tj}}\dot{P}_{tj} + \frac{\partial k_{tj}(\tau)}{\partial z_t^*}\dot{z}_t^*\right]_{\tau=t} \quad . \tag{7}$$

The above result, together with (3) and (5), is used to obtain the actual expenditure equation for durables, at time t, i.e.,

$$P_{tj}R_{tj} = \sigma_j\Theta_jP_{tj} + \mu\eta_j^{**}z_t^* - \mu\eta_j^*\left[\frac{\dot{P}_{tj}}{P_{tj}} - \frac{\dot{z}_t^*}{z_t^*}\right]z_t^* \quad , \tag{8}$$

---

1. A diagrammatic explanation of the partition of $\dot{k}_{tj}$ on the right of (7) is given in Appendix A.

where

$$(8.1) \qquad \eta_j^{**} = \eta_j^{*}(\sigma_j + \rho - \delta) \ .$$

For non-durables the actual expenditure equation, at time t, is obtained from (4), i.e.,

$$(9) \qquad p_{ti}q_{ti} = p_{ti}\gamma_i + \mu\beta_i^{*}z_t^{*} \ .$$

The third term on the right hand side of (8) is the Dixon/Lluch shift term. It has been hypothesised that failure to recognize durability will lead to a misspecified system via the left out shift term; i.e., in ELES estimation estimates of η_j^{**} and β_i^{*} may be incorrectly interpreted as estimates of utility function parameters η_j and β_i .

(1) utilizing the alternative option within DELES of specifying the demand equation for new purchases of durables by adjusting observed incoming stocks to a new desired level ;

(2) a reformulation of the model treating all n commodity groups as potentially durable (the first anti-model in section 4.3) ;

(3) taking explicit account of interest rates that vary in actual time (so that non-exponential inflation can be accounted for) ;

(4) utilizing a more appropriate definition of permanent income;[1] and

(5) by the introduction of money, either as another "consumer durable," or within an extended theory of portfolio choice.

1. See Powell [30].

term (Table 5) but also produced an anomalous estimate for the mpc and unexpected variations in gamma values and in estimates of marginal budget shares. The suggestion that these fluctuations are caused by large rounding errors may be valid but does need further investigation.

Limitations of the DELES framework itself should also be noted. Many assumptions were made about the structure of relevant relationships. Planning under certainty, perfect capital and second-hand markets[1] are restrictive assumptions of the model. The concept of continuous replanning (where people review decisions on what stock to hold at least daily) is unreal. Also price and income expectations are not allowed to undergo discontinuous shifts in historical time; i.e., there is no allowance for abrupt changes in expectations. These problems plus the abasence of discontinuities in purchases (so that durable good invisibilities are ruled out) may make DELES best suited for applications to aggregate data [10].

The obvious conclusion, apart from a provisional acceptance of the Dixon/Lluch basic concept, is that it may be expecting too much for a model to obtain meaningful estimates of interest reates, mpc, utility parameters, marginal budget shares and depreciation rates with only price, expenditure and savings as input data. Hopefully the gap between complete demand systems for non-durables and demand for durables could be futher closed by :

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1. See page 9, [10].

3. THE DATA

DELES was fitted using quarterly Australian personal consumption expenditure for the period 1959-74 [1]. Consumption expenditure and disposable income were expressed in nominal dollars on a per head of population basis, the population figures coming from [2], [3].

The implicit price deflators for the non-durable categories were those estimated by the Institute of Applied Economic and Social Research [8], which were in part derived from information obtained from the Australian Bureau of Statistics.

The implicit price deflators for household durables were obtained in the following manner. The annual series was obtained by taking the ratio of current expenditure on household durables to expenditure at average 1966-67 prices [1]. A series was then sought as a base for interpolating the annual series. For the period December, 1959, to June, 1967, the implied deflator series for personal consumption expenditure on durables developed in [6] was used. For June, 1967, to December, 1974, the CPI series [4] for Household Appliances and Furniture and Floor Coverings was used.

The implicit price deflators for motor vehicles used the series "sales in base year service units" developed in [11] and personal expenditure on purchase of motor vehicles [1]. The implicit price deflators were calculated by dividing personal expenditure by sales in base year service units.

4. THE RESULTS

The model was used to incorporate two durable items - purchases of motor vehicles and purchases of household durables - into a system where their parameters were estimated simultaneously with those of the following non-durable items :

(i) Food
(ii) Tobacco, Cigarettes and Alcoholic Drinks
(iii) Clothing
(iv) Rent
(v) Travel and Communication
(vi) Other Non-durables .

Under the assumption of DELES, the per capita expenditure equations for non-durable i , by the representative consumer, at time t , is obtained from (4), i.e.,

$$p_{ti}q_{ti} = p_{ti}\gamma_i + \mu\beta_i^* z_t^* \quad . \tag{10}$$

Using the relationship :

$$\dot{z}_t^* = \left[\frac{\partial z_t^*}{\partial z_t}\dot{z}_t + \sum_i \frac{\partial z_t^*}{\partial p_{ti}}\dot{p}_{ti} + \sum_j \frac{\partial z_t^*}{\partial P_{tj}}\dot{P}_{tj} \right] \tag{11}$$

$$= \dot{z}_t - \sum_{i=1}^{6} \gamma_i \dot{p}_{ti} - \sum_{j=1}^{2} \sigma_j \Theta_j \dot{P}_{tj} \quad ,$$

rate resulted in implausibly low values of the marginal budget shares of durables, by contrast insistence only on plausible depreciation rates (Table 2) resulted in marginal budget shares $(\hat{\beta}_i^*$ and $\hat{\eta}_j^{**})$ for both durables and non-durables virtually the same as ELES estimates and of plausible magnitude. The estimates of the Stone-Geary parameters and marginal propensity to consume were likewise almost identical to the values obtained in ELES. Although the difference between the estimates of the utility function parameters, β_i and η_j (which also occur in ELES), and those for β_i^* and η_j^{**} (which are specific to DELES) could represent a measure of the bias due to not explicitly recognising durability [10], this difference was almost totally insensitive to changes in the depreciation rate. The reasons for this behaviour (whether economic or statistical) remain to be investigated.

Apart from the significance of the shift term, the most striking feature is the robustness of the estimated mpc, $\hat{\mu}$, under the specification changes represented by Tables 1 - 4. These estimates ranged from 0.777 to 0.682 with t values ranging from 73.23 to 54.31. By contrast the estimates of ρ and δ are wildly erratic. This feature was noted by Powell [30] in the context of the ELES consumption function. However, unlike Powell [30], the asymptotic standard errors obtained were not always large enough (Tables 2 and 3) to dismiss the estimates as unlikely. This difficulty may simply reflect the inability of these data and this methodology to give reliable individual interpretation to these estimates.

The enforced elimination of durability within DELES (by making the decay rates so large that durables become effectively non-durables) obtained the expected results for the size and significance of the shift

5. SUMMARY

DELES is a system of expenditure equations in which durability is explicitly recognised. Basic to the model is the introduction of continuous replanning where infinitesimal adjustment to stocks is possible. The adoption of a specific instantaneous utility function and an exponential decay rate for durables led to Dixon and Lluch's derivation of durable expenditure equations which included a "shift term."

The expenditure equations were made operational and applied to quarterly Australian data. The shift term was tested for significance under various changes in model specifications. The results were promising and clearly indicated a solid foundation for the acceptance of the Dixon/Lluch model as a significant contribution to complete systems estimations. Nevertheless many problems were left unresolved.

In all estimations the presence and possible consequences of autocorrelation have simply been noted; transformation along the lines explored in [19] may be an appropriate next step.

In the unconstrained estimation (Table 1) the estimates of the market rate of interest, ρ , of the consumer's subjective discount rate, δ , and of the depreciation rates for both categories of durables were far in excess of any plausible values. When computations were made with more realistic rates of interest, ρ , and depreciation rates (Table 4), the estimates of marginal budget shares for durables, η_j , were improbably low, and subsistence values for durables, $\sigma_j\theta_j$, were far in excess of sample purchases (in quantum terms). Whilst the enforcement of plausible values for both the market rate of interest and the depreciation

the equation for actual expenditure on durable j , at time t , becomes

(12) $$P_{tj}R_{tj} = \sigma_j\theta_j P_{tj} + \mu\eta_j^{**} z_t^* - \mu\eta_j^* \left[\frac{\dot{P}_{tj}}{P_{tj}} - \frac{\dot{z}_t}{z_t^*} + \frac{\sum_{i=1}^{6} \gamma_i \dot{P}_{ti}}{z_t^*} + \frac{\sum_{j=1}^{2} \sigma_j\theta_j \dot{P}_{tj}}{z_t^*} \right] z_t^* .$$

Equations (10) and (12) have been derived on the assumption that variables are in continuous time. For estimation purposes it was necessary to adopt a discrete approximation for an observed period t . This was done using a finite time analogue of differentials of logarithms involving operators A, B and C where

$$Ax_t = (x_t - x_{t-1})/\tfrac{1}{2}(x_t + x_{t-1}) \text{ replaces } d\,\ell n\, x_t/dt ;$$

$$Bx_t = (1 - L)x_t = x_t - x_{t-1} \text{ replaces } \dot{x}_t ;$$

$$Cx_t = \tfrac{1}{2}(1 + L)x_t = \tfrac{1}{2}(x_t + x_{t-1}) \doteq x_{t-\frac{1}{2}} \text{ replaces } x_t .$$

This approach is similar to that adopted by the "Rotterdam School" of demand theorists [35].

The estimating equations used were the behavioural equations (10) and (12) in discrete form with error terms added. The final form of the estimating equations were estimated using Wymer's RESIMUL package [41].

The eight DELES equations were first estimated for all parameters. They were then estimated with extraneous values of depreciation rates and market interest rates. Finally, DELES with large depreciation rates was compared to estimates of ELES.

4.1 Unconstrained Estimates

Principal results for the estimation of all parameters are listed in Table 1. The dominant features of the results presented are :

(i) The residuals on food and household durables indicate that the error specification of no serial correlation does not held for these commodities.[1] (Note, if serial correlation is present, standard errors of sample estimates may be seriously understated.)

(ii) The estimates of the shift term parameters for both categories of non-durables were highly significant. Respective t values for motor vehicles and household durables were 7.03 and 8.77.

(iii) All estimates of Stone-Geary parameters for non-durables were significant except for clothing. This estimate was also negative suggesting that the demand for clothing is elastic with respect to its own price. However, the relatively large asymptotic standard error indicates that the result need not be taken seriously.

(iv) The estimate of the purchases of durables ($\sigma_j \theta_j$) required to maintain the subsistence stock was insignificant for motor vehicles. In the case of household durables the actual quantum

1. See Appendix B.

(3) There were marked variations between the estimates of the marginal budget shares for the DELES model ($\sigma_1 = \sigma_2 = 1000$) and the ELES model; e.g., the estimate of the marginal budget share for clothing varied from 0.124 (DELES) to 0.050 (ELES).

It is quite possible that the erratic behaviour in the estimates of both marginal budget shares and gamma values may also be attributed to accumulated rounding errors which could occur with a large value of the decay rate.

become insignificant; the denominators of $\hat{\beta}_i^*$ and $\hat{\eta}_j^{**}$ would approach one, i.e., $\hat{\beta}_i^* \approx \hat{\beta}_i$ and $\hat{\eta}_j^{**} \approx \hat{\eta}_j$; and estimates of marginal budget shares and gamma values should approximate the estimates obtained by a systems estimation which ignored durability, viz., ELES. The latter approach was used in this study.

Decay rates of 10 and 1000 for both durable categories were imposed on the DELES model.[1] Table 5 lists the main results together with the estimates obtained using ELES.

For $\sigma_1 = \sigma_2 = 10$, the estimates of the shift term were still significant at the 5% level, i.e., the model continued to detect durability when only the slightest elements were allowed to be present. The main difference between the estimates obtained earlier (Tables 2 and 3) and those obtained for this decay value were that $\hat{\beta}_i$ and $\hat{\eta}_j$ now approached β_i^* and η_j^{**} respectively. All other estimates remained virtually unchanged and were almost identical to those obtained using ELES.

When $\sigma_1 = \sigma_2 = 1000$ was used, the size of the shift terms became very small (0.00002 for Motor Vehicles; 0.00003 for Household Durables) and were statistically insignificant. Although the desired equality between $\hat{\beta}_i$'s and $\hat{\beta}_i^*$'s (and $\hat{\eta}_j$'s and $\hat{\eta}_j^{**}$'s) remained there emerged some apparent model inconsistencies in other estimates :

1. $\sigma_1 = \sigma_2 = 10$ is an approximate depreciation rate of 99.996% per quarter.

the values 0.0645 for motor vehicles[1] and 0.078 for household durables[2] the model was rerun. The principal results are listed in Table 2. Notable differences and similarities between this estimation and the unconstrained estimation were :

(i) Tobacco, clothing and rent were the commodity groups which could not reject the hypothesis of no serial correlation.

(ii) The estimates of shift term parameters were still significant at the 5% level.

(iii) Estimates of Stone-Geary parameters for travel, other non-durables and motor vehicles were all negative although the category, other non-durables, was the only on of the trio that was significantly different from zero. Rather than accept the economic implication that the demand for Other Non-durables is elastic with respect to its own price, it is perhaps more likely that this is another example of the well known erratic behaviour of gamma estimates.

(iv) All estimates of β_i, η_j, β_i^* and η_j^{**} were significant. The η_j estimates of 0.172 and 0.197 for motor vehicles and household durables respectively appear to be more plausible values than those obtained in the unconstrained estimates. The large gaps between η_j^{**} and $\hat{\eta}_j$ may indicate the bias associated

1. $\sigma_1 = 0.0645$ is a depreciation rate of approximately 6.25% per quarter.

2. $\sigma_2 = 0.078$ is a depreciation rate of approximately 7.5% per quarter.

TABLE 2 : ESTIMATES OF DELES CONSUMPTION FUNCTION USING A PRIORI DEPRECIATION RATES,# AUSTRALIA, 1959-74*

Commodity Group	Estimates of Stone-Geary Parameters†		Estimates of Marginal Budget Shares		Z Statistic for Independence of Residuals[1]
Nondurables	$\hat{\gamma}_i$		$\hat{\beta}_i$	$\hat{\beta}_i^*$	
Food	28.321 (4.145)		0.076 (0.005)	0.107 (0.007)	-4.91
Tobacco Cigarettes and Alcoholic Drinks	28.280 (1.875)		0.027 (0.003)	0.038 (0.004)	-0.63
Clothing	19.393 (6.570)		0.037 (0.007	0.052 (0.010)	-1.13
Rent	11.673 (2.463)		0.103 (0.004)	0.144 (0.005)	-0.88
Travel	-0.815 (1.553)		0.037 (0.002)	0.052 (0.002)	-2.90
Other Nondurables	-37.699 (9.607)		0.351 (0.011)	0.492 (0.011)	-2.90
Durables	$\hat{\theta}_j$	$\widehat{\theta_j \sigma_j}$	$\hat{\eta}_j$	$\hat{\eta}_j^{**}$	
Motor Vehicles	-72.972 (44.158)	-4.561 (2.759)	0.172 (0.008)	0.054 (0.003)	-4.66
Household Durables	461.252 (40.973)	34.593 (3.072)	0.197 (0.008)	0.062 (0.004)	-3.15

Estimates of:		
Shift term parameter for Motor Vehicles	$\widehat{\mu\eta_1}^*$	0.0044 (0.0018)
Shift term parameter for Household Durables	$\widehat{\mu\eta_2}^*$	0.0050 (0.002)
Market Discount rate	$\hat{\rho}$	42.744 (18.136)
Time preference discount rate	$\hat{\delta}$	33.199 (14.137)
Marginal propensity to consume	$\hat{\mu}$	0.777 (0.011)

$\sigma_1 = 0.0645$, $\sigma_2 = 0.078$ [Equivalent to annual depreciation rates of 25% and 30% respectively.]
† Units are dollars per capita in 1966-7 prices.
* Asymptotic standard errors are in parentheses.

1. Test is two-tailed and the null hypothesis assumes independence of the residuals. See Appendix B.

(iv) The severe fluctuations in the estimates of the price parameters (γ_i) for clothing (38.052 for $\rho = 0.08$ to - 45.486 for $\rho = 0.12$) and other non-durables (- 35.367 for $\rho = 0.08$ to 52.949 for $\rho = 0.12$) were further evidence of their known erratic behaviour.

(v) The estimates of mpc varied indirectly with the increase in ρ .

4.3 Estimates of An Anti-model

The main concept of the DELES model, the significance of the shift term, is well supported by the empirical evidence of Tables 1 - 4. Two other possibilities remain to test further the model and its ramifications for systems estimation. The first possibility centres on the model's ability to detect durability for the appropriate commodity groups. The approach would be to remodel the estimating equations for n commodity groups in which σ_1 is the decay rate for each group i . The expected result for this type of anti-model[1] would be that the model would be able to discern between durables and non-durables, i.e., for each non-durable expenditure equation, the estimate of σ_i should be sufficiently large to indicate that the commodity was a perishable, and furthermore, the respective shift terms in such cases should approach zero. The other alternative would be to force durability out of DELES by making the decay rates so large that the durables are approximately non-durables. In this situation, if durables in fact behaved like consumables the shift term should approach zero and

1. The model is really mis-specified. We know that food, clothing, etc., are non-durables but we are treating them as possible durables.

TABLE 4 (continued)

Estimates of :	ρ = 0.08	ρ = 0.1	ρ = 0.12
Shift term parameter for Motor Vehicles	0.020 (0.007)	0.026 (0.007)	0.022 (0.007)
Shift term parameter for Household Durables	0.017 (0.005)	0.023 (0.006)	0.022 (0.005)
Time preference discount rate	0.050 (0.001)	0.071 (0.001)	0.818 (0.001)
Marginal propensity to consume	0.723 (0.013)	0.705 (0.013)	0.682 (0.013)

The most striking feature of these estimates was the continued stability of the mpc and the highly significant estimates of the shift term parameters as the model specification changed. Other noteworthy points were :

(i) The behaviour of some marginal budget shares were quite sensitive to changes in ρ ; e.g., the estimate for clothing varied from 0.021 to 0.106 to 0.228 for the stated changes in ρ .

(ii) The estimates of the marginal budget shares for durables (< 0.01) were not plausible.

(iii) The approximate equality of the values of $\hat{\beta}_i^*$'s and $\hat{\beta}_i$'s. This was due to the use of an extraneous market discount rate and decay rates. Their combined effect forced the denominator of $\hat{\beta}_i^*$ close to one.[1]

1. For example, given $\sigma_1 = \sigma_2 = 0.0625$, $\rho = 0.12$, then $\hat{\delta} \approx 0.085$ for plausible values of $\hat{\mu}$.

Hence,

$$1 - \delta \sum_j \frac{\eta_j}{\sigma_j+\rho} > 0.954 \text{ for } \eta_j < 0.05 .$$

with models which fail to recognize durability. (The $\hat{\beta}_i^*$ and $\hat{\eta}_j^{**}$ values are approximately the same as the estimates of ELES which appear in Table 5.)

(v) The estimates of ρ and δ were now 4194.3% and 3257.6%! This wildly erratic behaviour of the market discount rate and time preference discount rate has also been noted in work by Powell [30]. Despite the extreme variation in $\hat{\rho}$ and $\hat{\delta}$, the estimate of the mpc, 0.777, was quite stable.

The model was rerun using various decay rates ranging from 0.0625 to 1 for both durables. The results in Table 3 are restricted, for brevity, to decay rates of $\sigma_1 = \sigma_2 = 0.25$. Apart from estimates of theta values for motor vehicles and household durables, the model was insensitive to considerable change in decay rates.

Further estimatations were calculated by imposing a priori values of ρ together with fixed decay rates. Table 4 lists the main results when values of ρ = 0.08, 0.10 and 0.12 were used with rates of $\sigma_1 = \sigma_2 = 0.0625$. There were only two gamma values, clothing and other non-durables for the model ρ = 0.10, which failed to survive a test of significant difference from zero. In each estimation, the hypothesis of independent error terms was rejected for the categories of food, motor vehicles and household durables. For the sake of brevity, test statistics for independence have been omitted from this table.

TABLE 3 : ESTIMATES OF DELES CONSUMPTION FUNCTION USING A PRIORI DEPRECIATION RATES,# AUSTRALIA, 1959-74

Commodity Group	Estimates of Stone-Geary Parameters†		Estimates of Marginal Budget Shares		Z Statistic for Independence of Residuals[1]
Nondurables	$\hat{\gamma}_i$		$\hat{\beta}_i$	$\hat{\beta}^*_i$	
Food	28.322 (4.145)		0.076 (0.005)	0.106 (0.007)	-5.42
Tobacco, Cigarettes and Alcoholic Drinks	28.283 (1.875)		0.027 (0.003)	0.038 (0.004)	-0.38
Clothing	19.397 (6.571)		0.037 (0.007)	0.052 (0.010)	-1.13
Rent	11.673 (2.462)		0.103 (0.004)	0.144 (0.005)	-0.88
Travel	-0.813 (1.553)		0.037 (0.002)	0.052 (0.002)	-2.90
Other Nondurables	-37.703 (9.608)		0.353 (0.011)	0.492 (0.011)	-2.90
Durables	$\hat{\theta}_j$	$\widehat{\theta_j \sigma_j}$	$\hat{\eta}_j$	$\hat{\eta}^{**}_j$	
Motor Vehicles	-18.242 (11.039)	-4.561 (2.760)	0.170 (0.008)	0.054 (0.003)	-4.91
Household Durables	138.380 (12.291)	34.595 (3.073)	0.195 (0.008)	0.062 (0.004)	-3.15

Estimates of:		
Shift term parameter for Motor Vehicles	$\widehat{\mu \eta_1^*}$	0.0043 (0.0018)
Shift term parameter for Household Durables	$\widehat{\mu \eta_2^*}$	0.0050 (0.002)
Market Discount rate	$\hat{\rho}$	41.943 (18.082)
Time preference discount rate	$\hat{\delta}$	32.576 (14.093)
Marginal propensity to consume	$\hat{\mu}$	0.777 (0.011)

\# $\sigma_1 = \sigma_2 = 0.25$ [Equivalent to annual depreciation rate of 88.5%]

† Units are dollars per capita in 1966-7 prices.

* Asymptotic standard errors are in parentheses.

1. Test is two tailed and the null hypothesis assumes independence of the residuals. See Appendix B.

TABLE 4 : ESTIMATES OF DELES CONSUMPTION FUNCTION USING A PRIORI DEPRECIATION RATE# AND MARKET RATES OF INTEREST: AUSTRALIA, 1959-74*

Commodity Group	Estimates of Stone-Geary Parameters†			Estimates of Marginal Budget Shares					
	ρ=0.08	ρ=0.10	ρ=0.12	ρ=0.08		ρ=0.10		ρ=0.12	
Durables	$\hat{\gamma}_i$	$\hat{\gamma}_i$	$\hat{\gamma}_i$	$\hat{\beta}_i$	$\hat{\beta}^*_i$	$\hat{\beta}_i$	$\hat{\beta}^*_i$	$\hat{\beta}_i$	$\hat{\beta}^*_i$
Food	39.508 (3.965)	35.345 (4.159)	38.914 (4.117)	0.098 (0.008)	0.099 (0.008)	0.118 (0.008)	0.118 (0.008)	0.123 (0.009)	0.124 (0.009)
Tobacco, Cigarettes and Alcoholic Drinks	23.684 (1.845)	26.421 (1.831)	32.452 (1.688)	0.051 (0.004)	0.051 (0.004)	0.049 (0.004)	0.050 (0.004)	0.040 (0.005)	0.041 (0.005)
Clothing	38.052 (5.887)	0.193 (6.072)	-45.486 (6.142)	0.021 (0.011)	0.022 (0.010)	0.106 (0.011)	0.107 (0.011)	0.228 (0.011)	0.229 (0.011)
Rent	9.430 (2.875)	19.441 (2.315)	28.044 (1.748)	0.173 (0.006)	0.174 (0.006)	0.154 (0.006)	0.155 (0.006)	0.138 (0.006)	0.139 (0.006)
Travel and Communication	1.771 (0.491)	6.653 (1.500)	14.374 (1.255)	0.058 (0.002)	0.058 (0.002)	0.051 (0.002)	0.051 (0.002)	0.037 (0.002)	0.037 (0.002)
Other Nondurables	-35.367 (10.676)	7.740 (9.104)	52.959 (6.615)	0.590 (0.010)	0.591 (0.011)	0.510 (0.012)	0.513 (0.012)	0.421 (0.012)	0.423 (0.012)
Durables	$\widehat{\theta_j \sigma_j}$	$\widehat{\theta_j \sigma_j}$	$\widehat{\theta_j \sigma_j}$	$\hat{\eta}_j$	$\hat{\eta}^{**}_j$	$\hat{\eta}_j$	$\hat{\eta}^{**}_j$	$\hat{\eta}_j$	$\hat{\eta}^{**}_j$
Motor Vechicles	35.711 (3.235)	36.290 (3.300)	38.366 (3.353)	0.004 (0.001)	0.002 (0.0008)	0.006 (0.002)	0.003 (0.0009)	0.006 (0.002)	0.003 (0.001)
Household Durables	85.007 (3.789)	86.290 (3.891)	88.837 (3.904)	0.003 (0.001)	0.002 (0.0006)	0.005 (0.001)	0.003 (0.0008)	0.006 (0.0014)	0.003 (0.001)

\# $\sigma_1 = \sigma_2 = 0.0625$ † Units are dollars per capita in 1966-7 prices

* Asymptotic standard errors are in parentheses.

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[40] Williams, R.A., "Demand for Consumer Durables : Stock Adjustment Models and Alternative Specifications of Stock Depletion, "Review of Economic Studies, Vol. 39, 1972, pp. 281-295.

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[42] Wymer, C.R., "Full Information Maximum Likelihood Estimation with Non-Linear Restrictions", London School of Economics (mimeograph).

subjective discount rate ; ρ is the market rate of interest;[1] p_{ti} , P_{tj} are respective prices at which the consumer, at time t, expects non-durable i and durable j to be sold throughout the planning period (i.e., constant price expectations);[2] $q_{ti}(\tau)$, $R_{tj}(\tau)$ are the respective time paths of planned purchases of non-durable i and durable j ; $k_{tj}(\tau)$ is the consumer's planned holdings of stock of durables, at time t ; σ_j is the decay rate for durable j ; z_t/ρ is the present value of the consumer's expected life-time income, at time t, from his financial assets and sales of labour ; and τ is planning time.

It should be noted that :

(a) The utility function is assumed to be inter-temporally separable.

(b) The assumption of a constant subjective discount rate, δ , implies that the consumer's behaviour is Strotz-consistent.[3]

(c) The problem of maintaining stock equilibrium by discontinuous jumps has been overcome by assuming that at each point of time t , the consumer adjusts his stocks to their currently optimal levels and that the consumer's expectations about prices and income are changing smoothly, i.e., p_{ti} , P_{tj} and z_t are continuous functions of real time t .

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1. For simplicity ρ is treated as time invariant.

2. In the event that the representative consumer expects prices to change over the planning period, each of the p_{ti} and P_{tj} would be functions of planning time τ . Explicit assumptions on growth laws governing these time paths would then have to be introduced.

3. If the consumer's discount rate is constant and if his price and income expectations are realized and remain unchanged, his plans for time τ , made at time t , will be the same as his plans for time τ made at time t + s. See Strotz [34], Pollak [28] and Blackorby et al. [5].

The planning problem (1) - (3) can be solved using Lagrange/Euler maximization methods[1] to obtain an expression for the consumer's planned expenditure path for non-durable i and his planned investment in durable j , i.e.,

(4) $$P_{ti}q_{ti}(\tau) = P_{ti}\gamma_i + \mu\beta_i^* z_t^* \exp(\rho-\delta)(\tau-t) ,$$

and

(5) $$P_{tj}k_{tj}(\tau) = P_{tj}\theta_j + \mu\eta_j^* z_t^* \exp(\rho-\delta)(\tau-t) ,$$

where

(6.1) $\mu = \delta/\rho$, the marginal propensity to consume out of supernumerary permanent income ;

(6.2) $z_t^* = z_t - \sum_i P_{ti}\gamma_i - \sum_j \sigma_j\theta_j P_{tj}$, is supernumerary permanent income, i.e., income over that required to purchase, at time t , the basic bundle of consumables together with the cost of meeting depreciation on the basic bundle of durables ;

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1. See sections 3.8 - 3.14 [14].

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