IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

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ESTIMATION OF HOUSEHOLD DEMAND RESPONSES USING CROSS-SECTION DATA : A SYSTEMS INTERPRETATION OF PODDER'S RESULT

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1. Introduction

This paper has two aims : to set out at the theoretical level the manner in which linear demand systems may be used to extract maximum information from household budget study data and to apply these theoretical results to Podder's (1971) empirical findings to investigate income, price and family size effects on Australian consumption patterns.

The demand systems considered are based on maximization of the Klein-Rubin (1947-48) utility function. Atemporal maximization of this function yields Stone's (1954) Linear Expenditure System, LES, in which expenditures on commodities are expressed as a function of price and total consumption expenditure. Intertemporal maximization à la Lluch (1973) yields the Extended Linear Expenditure System, ELES, in which income replaces total consumption expenditure as an explanatory variable. The singular advantage of using ELES in a cross-section context is that it permits estimation of price elasticities when data on prices are not available. This is not possible using more conventional demand systems in which total expenditure is the (exogenous) explanatory variable : it is the endogenizing of saving in ELES which provides the extra information to permit estimation of price responses. Furthermore, ELES enables measurement of the responsiveness of saving to changes in relative prices.

Section 2 of the paper explores cross-section estimation of LES and ELES. Specific attention is paid to inter-relationships between the parameter values obtained using alternative estimating techniques. In section 3 Podder's results are given a systems interpretation, which permits estimation of price responses. The effect of family size on expenditure patterns is explored in some detail. Finally, some concluding remarks are made in section 4.

2. Estimation of the Parameters of the Klein-Rubin Utility Function Using Cross-section Data

2.1 Income versus total expenditure as an explanatory variable.

The Klein-Rubin (additive) utility function is given by

$$U = \sum_{j=1}^{n} \beta_j \ \ell n \ (q_j - \gamma_j) \qquad (1)$$

where U is an index of utility of a consumer or household, and q_j, $j=1, \ldots, n$, represents quantities of goods purchased. The γ_i parameters may be thought of as "subsistence quantities", utility being defined only when $q_j > \gamma_j$, for all j. The β_i parameters are shown below to represent marginal budget shares.

Lluch's (1973) intertemporal maximization of (1) subject to a wealth constraint yields an expenditure system, ELES, of the following form :

$$v_{ih} = p_{ih} \gamma_i + \beta_i^* (y_h - \sum_j p_{jh} \gamma_j) + u_{ih}, \qquad \begin{matrix}(i=1,\ldots n)\\(h=1,\ldots H)\end{matrix} \qquad (2)$$

where

$$\beta_i^* = \mu\beta_i \ , \ 0 < \beta_i < 1 \ , \qquad \sum_{j=1}^{n} \beta_j = 1$$

and where v_{ih} is expenditure on the i^{th} good by the h^{th} household, p_{ih} is the price of the i^{th} good paid by the h^{th} household, y_h is income of the h^{th} household[(1)] and u_{ih} is an error term. The parameter β_i^* represents the marginal propensity to consume the i^{th} good and $\sum_j \beta_j^* = \mu \sum_j \beta_j = \mu$ is the aggregate marginal propensity to consume.

Typically, cross-section data contain no price information and it is necessary to assume all households face the same set of prices[(2)]. Thus in (2), $p_{ih}\,\gamma_i$ must be replaced by γ_i^*, say, where γ_i^* measures "subsistence expenditure" on the i^{th} good at prices prevailing during the budget survey. Making this substitution, equation (2) may be rewritten as[(3)]

$$v_{ih} = \alpha_i^* + \beta_i^* y_h + u_{ih} \qquad (3)$$

$$\alpha_i^* = \gamma_i^* - \beta_i^* \sum_{j=1}^{n} \gamma_j^* \qquad (4)$$

$$\beta_i^* = \mu\,\beta_i \quad . \qquad (5)$$

Summing (3) over commodities yeilds a Keynesian aggregate consumption function :

$$v_h = \alpha^* + \mu y_h + u_h \qquad (6)$$

where

$$\alpha^* = \sum_j \alpha_j^* = (1-\mu) \sum_j \gamma_j^* \qquad (7)$$

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(1) In Lluch's (1973) general formulation the income term is a permanent income measure. In this paper we will consider only current income, which in Lluch's formulation is equivalent to assuming that the present value of expected changes in labour income is zero.

(2) Korean results reported in Williams (1975) suggest that if cross-section price variations are of the order of 5 to 10 per cent then assuming them away does not affect parameter estimates.

(3) Estimation of ELES from cross-section data is discussed in more detail in Belandria (1971), Betancourt (1973), Howe (1974), and Powell (1973, 1974).

and $v_h = \sum_j v_{jh}$, i.e. total consumption expenditure, and $u_h = \sum_j u_{jh}$.

Under the conventional atemporal maximization of the Klein-Rubin utility function (1) subject to a budget constraint total expenditure is assumed exogenous. The resulting expenditure equations, LES, are for the h^{th} household

$$v_{ih} = p_{ih}\,\gamma_i + \beta_i\,(v_h - \sum_{j=1}^{n} p_{jh}\,\gamma_j) + \varepsilon_{ih} \qquad \begin{matrix}(i=1,\ldots,n)\\(h=1,\ldots,H)\end{matrix} \qquad (8)$$

where the only new notation is the error term ε_{ih}. It is now apparent that β_i represents the marginal budget share for the i^{th} good.

Again assuming no price variation across households we obtain :

$$v_{ih} = \alpha_i + \beta_i\,v_h + \varepsilon_{ih} \qquad (9)$$

where

$$\alpha_i = \gamma_i^* - \beta_i \sum_{j=1}^{n} \gamma_j^* \quad . \qquad (10)$$

Notice, however, that equations (9) - (10) may also be obtained by eliminating the income term, y_h, from the ELES equations (3) - (7). Under this derivation, however, the error term on (9) becomes

$$\varepsilon_{ih} = u_{ih} - \beta_i\,u_h \quad . \qquad (11)$$

It follows that if ELES is the maintained hypothesis then ordinary least squares (OLS) estimation of (9) would yield inconsistent parameter estimates since the error term, ε_{ih}, is then correlated with the explanatory variable, v_h,

i.e. $\text{plim}\,(\sum_h v_h\,\varepsilon_{ih})/H \neq 0$, (for all i).

The above result is formally equivalent to that obtained by Summers (1959) working outside a demand systems framework. He was concerned solely with the biases involved in using total expenditure rather than income in estimating Engel curves. Liviatan (1961) extended Summers' work by arguing that even if income theoretically is the correct explanatory variable it is inappropriate to use it as an explanatory variable in Engel curve analysis (assuming estimation is by OLS) as it is likely to contain measurement error. Liviatan's solution was to use total expenditure as the explanatory variable but estimate by instrumental variables (IV), using measured income as the instrument. In the context of LES/ELES, this amounts to estimating equation (9) by IV, using y_h as instrument, and thus obtaining consistent estimates of α_i, β_i. Exactly the same estimates of β_i would however be obtained from estimating ELES (3) by OLS and calculating β_i from (1)

$$\beta_i = \beta_i^* / \sum_j \beta_j^* = \beta_i^*/\mu \quad .$$

The proof is simple :

$$\tilde{\beta}_i = \frac{\sum_h v_{ih}\, y_h}{\sum_h v_h\, y_h} = \frac{\sum_h v_{ih}\, y_h}{\sum_h y_h^2} \Bigg/ \frac{\sum_h v_h y_h}{\sum_h y_h^2} = \hat{\beta}_i^* / \hat{\mu} \qquad (12)$$

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(1) OLS estimation of the consumption function (6) yields an identical value of μ to that obtained by summing the OLS estimates of β_i^* in (3).

where "~" denotes an IV estimate and "^" an OLS estimate[1], $\hat{\mu}$ is the OLS estimate from (7) and for convenience variables are expressed as deviations from sample means. A similar result was obtained by Liviatan outside a demand systems framework.

The rule for calculating "income" responses of demand now seems clear : estimate the equation in ELES (3) by OLS and unscramble the estimates of β_i ; these estimates are consistent even in the presence of measurement error in income; the OLS estimates of the components of β_i, namely β_i^* and μ, are, however, consistent only in the absence of measurement error in income[2].

Turning to price responses, if estimates of γ_i^*, $i=1,\ldots,n$, can be obtained then all price elasticities can be calculated using the systems approach. If η_{ij} is the (uncompensated) elasticity of demand for the i^{th} good with respect to the j^{th} price, then

$$\eta_{ij} = \begin{cases} [\gamma_i^*(1-\beta_i)/v_i]-1 & , \quad i=j \\ -\beta_i\,\gamma_j^*/v_i & , \quad i\neq j \end{cases} \qquad (13)$$

Putting aside for one moment the statistical properties of any estimators, estimates of the intercept term, α_i, in LES (9) are not sufficient to permit the unscrambling of γ_i^* values. Essentially this is because the parameter estimates are such as to ensure that the budget constraint holds globally, i.e. $\sum_j \hat{v}_{jh} = v_h$ for all households. In particular $\sum_j \hat{\alpha}_j = 0$ and there are only n-1 independent linear expressions (10) in the n different γ_i^*. Notice, however, that one additional piece of information -- either

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(1) This convention is retained throughout the paper.

(2) Since within LES and within ELES the same explanatory variables appear in each equation of the respective system, estimation by OLS on a commodity-by-commodity basis is equivalent to systems maximum likelihood estimation. The outlined estimation method also ensures that $\sum_j \tilde{\beta}_j = 1$ as required.

an estimate of <u>one</u> γ_i^* or of $\sum_j \gamma_j^*$ -- would permit unscrambling. With ELES, on the other hand, the n estimates α_i^*, i=1,...,n, in (3) do not satisfy any constraint and the values of γ_i^*, i=1,...,n, may be unscrambled from estimates of α_i^* and β_i^* using

$$\gamma_i^* = \alpha_i^* + [\beta_i^*(1-\sum_j\beta_j^*)^{-1} \sum_j \alpha_i^*] \qquad (i=1,\ldots,n) \quad . \qquad (14)$$

The question of obtaining <u>consistent</u> estimates of γ_i^* must now be faced. If measurement errors in income are thought to be unimportant then consistent estimates of (β_i^*, α_i^*) and therefore of γ_i^*, may be obtained from OLS estimation of (3)(1). If measurement error is present, these results will be inconsistent. One way around this problem would be to fall back on IV estimates of α_i in LES, i.e.,

$$\tilde{\alpha}_i = \overline{v}_i - \tilde{\beta}_i \overline{v} \qquad (i=1,\ldots,n) \qquad (15)$$

where a bar denotes sample mean values, and to use an independent estimate of the "subsistence sum", $\Sigma \gamma_j$, to unscramble the γ_i^* estimates. Now in LES, $\Sigma \gamma_j^*$ is related to the expenditure elasticity of the marginal utility of total expenditure, ω, by

$$\omega = -v/(v-\sum_j \gamma_j^*) \quad . \qquad (16)$$

Since estimates of ω are plentiful(2) and are reasonably similar for countries at a given level of development, this suggests using them to obtain estimates of $\Sigma \gamma_j^*$. Denoting such an estimate as $\Sigma \hat{\gamma}_j^*$, it

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(1) Note that OLS estimation of the consumption function (6) would yield identical estimates of μ $(= \sum_j\beta_j^*)$ and α^* $(= \sum_j\alpha_j^*)$ to those obtained from OLS estimation of (3).

(2) See Brown and Deaton (1972), de Janvry, Bieri and Nuñez (1972), Lluch and Powell (1975), and Lluch and Williams (1974, 1975).

follows that "IV" estimates of γ_i^*, i=1,...,n, may be obtained from (10) as

$$\tilde{\gamma}_i^* = \tilde{\alpha}_i + \tilde{\beta}_i \sum_j \hat{\gamma}_j^* \qquad (17)$$

and price elasticities calculated using (13).(1)

2.2 Introduction of family size

In cross-section analysis variables other than income or total expenditure are likely to be important causes of variations in expenditure patterns across households. Family size effects, in particular, are likely to be substantial. In estimating Engel curves outside a demand systems context it is a simple matter to include additional explanatory variables in the demand equations. Corresponding estimating equations may be developed within the framework of LES/ELES by assuming that the "subsistence parameters" are a function of relevant variables such as family size.(2) Thus if "subsistence" expenditure for the h^{th} household, γ_{ih}^*, is assumed to depend linearly on family size(3), f, i.e.

$$\gamma_{ih}^* = \gamma_{0i}^* + \gamma_{1i}^* f_h \qquad (i=1,\ldots,n) \qquad (18)$$

(1) One alternative approach to the problem of measurement error in income might be to replace current measured income with a permanent income measure, perhaps by specifying permanent income to be a function of socio-economic variables as in Zellner (1970), Goldberger (1972), Powell (1974) and Musgrove (1975). Direct IV estimation of (3) is another possibility.

(2) If β_i or β_i^*, i=1,...,n, are also thought to be functions of family size etc., then it would seem preferable to subdivide the sample and fit the models in simple form (i.e. as set out in section 2.1) to the separate subgroups.

(3) This has been done previously by Betancourt (1973) and Howe (1974).

then the estimating equations for ELES (3) and LES (9) become, respectively,

ELES
$$v_{ih} = \alpha_i^* + \beta_i^* y_h + \delta_i^* f_h + u_{ih} \tag{19}$$

where

$$\alpha_i^* = (\gamma_{0i}^* - \beta_i^* \sum_j \gamma_{0j}^*) \tag{20}$$

$$\delta_i^* = (\gamma_{1i}^* - \beta_i^* \sum_j \gamma_{1j}^*) \tag{21}$$

and

LES
$$v_{ih} = \alpha_i + \beta_i v_h + \delta_i f_h + \varepsilon_{ih} \tag{22}$$

where

$$\alpha_i = (\gamma_{0i}^* - \beta_i \sum_j \gamma_{0j}^*) \tag{23}$$

$$\delta_i = (\gamma_{1i}^* - \beta_i \sum_j \gamma_{1j}^*) \quad . \tag{24}$$

The ELES aggregate consumption function becomes

$$v_h = \alpha^* + \mu y_h + \delta^* f_h + u_h \tag{25}$$

where

$$\alpha^* = \sum_j \alpha_j^* = (1-\mu)\sum_j \gamma_{0j}^* \tag{26}$$

and

$$\delta^* = \sum_j \delta_j^* = (1-\mu)\sum_j \gamma_{1j}^* \quad . \tag{27}$$

If it is thought appropriate to estimate ELES by OLS then estimates of μ, β_i, γ_{0i}^* and γ_{1i}^* may be obtained from estimates of α_i^*, β_i^* and δ_i^*. As was the case with one explanatory variable, however, LES estimates of (α_i, δ_i) do not permit an unscrambling of the underlying parameters $(\gamma_{0i}^*, \gamma_{1i}^*)$: to satisfy the budget constraint, the coefficients of <u>all</u>

variables other than total expenditure must sum to zero over equations (commodities). Two pieces of additional information are now required for identification -- $\sum_j \gamma^*_{0j}$ and $\sum_j \gamma^*_{1j}$ are the obvious candidates.

As before, there is a relationship between the OLS estimates of ELES and the IV estimates of LES. If measured income is used as the instrument for total expenditure, and family size acts as its own instrument, then

$$\hat{\beta}_i^* = \hat{\mu}\,\tilde{\beta}_i$$

$$\hat{\alpha}_i^* = \hat{\alpha}^*\,\tilde{\beta}_i + \tilde{\alpha}_i$$

$$\hat{\delta}_i^* = \hat{\delta}^*\,\tilde{\beta}_i + \tilde{\delta}_i \quad .$$

The proof of these results is given for the general case of m explanatory variables in an appendix. If price elasticities are to be calculated from LES estimates only (equation (22)), then one additional piece of outside information on the "subsistence parameters" is required for each additional explanatory variable introduced.

3. A Systems Interpretation of Podder's Results

In his Economic Record (1971) article "Patterns of Household Consumption Expenditures in Australia", Nripesh Podder fitted Engel curves to 1966-68 Australian household survey data collected by Macquarie University(1). He first regressed expenditure for nine categories of goods (which was not an exhaustive classification) on total expenditure

(1) The sample comprised about 5,500 families living in urban areas.

using OLS but experimenting with alternative functional forms. The double-log specification was preferred and two extensions to the analysis were carried out : (i) introduction of family size and (ii) estimation by IV.

Podder used IV in an attempt to overcome the simultaneity problems discussed in the previous section. Since he used measured household income as the instrument for total expenditure his IV results may be given a systems interpretation in the manner outlined above.

As Podder's results refer to elasticities it is convenient to commence by interpreting his total expenditure elasticities within an additive systems framework and thereby deriving price elasticities. After this is done, estimates of the parameters of the Klein-Rubin utility function will be obtained.

3.1 Price Elasticities

For any system of demand equations derived from an additive utility function all price elasticities, η_{ij}, may be expressed as a function of the expenditure elasticities, η_i, the Frisch parameter, ω, and average budget shares, w_i. Thus

$$\eta_{ii} = \omega^{-1}\eta_i - \eta_i w_i (1 + \omega^{-1}\eta_i) \qquad (28)$$

$$\eta_{ij} = -\eta_i w_j (1+\omega^{-1} \eta_j) \qquad i \neq j \quad . \qquad (29)$$

Estimates of ω may be obtained from other Australian empirical studies[(1)]. They tend to show values lying between -3 and -2; for simplicity we will

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(1) See Lluch and Powell (1975), Lluch and Williams (1974, 1975), Tran Van Hoa (1969), Powell (1966).

take $\hat{\omega} = -2.5$. Podder's IV estimates of η_i are given in table 1, and estimates of w_i are similarly reproduced. Substituting all these values into (28) - (29) yields the own-price elasticities and cross-elasticities with respect to the price of food given in table 1[1]. Since under a LES interpretation ω is a function of $\Sigma \gamma_j^*$ (see (16)), which in turn is a function of family size under our specification of section 2.2, the price elasticities in table 1 must be interpreted as pertaining to a family of average size. The estimates show that the own-price elasticities of demand for transport, recreation and durables are close to unity in absolute values. Fuel and food are the least responsive to price changes.

3.2 Estimates of the Klein-Rubin Parameters

Podder's IV estimates of total expenditure elasticities may be combined with average budget shares to give IV estimates of marginal budget shares, β_i: from the definition of total expenditure elasticities under LES/ELES,

$$\beta_i = \eta_i / w_i \qquad (i=1,\dots,9) \quad .$$

These estimates are given in the second last column of table 1. Note that they sum to 1.12 for the nine commodities. Failure to employ a systems approach has yielded estimates of marginal budget shares (and expenditure elasticities) which overstate true values[2]. They should be thought of as upper bounds.

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(1) Other cross-price effects are much less important. This is essentially so because the average budget shares for goods other than food are relatively small and the income effect is correspondingly small.

(2) Unless the remaining goods are thought to be collectively inferior -- in which case they do not satisfy the theoretical requirements of the Klein-Rubin utility function.

Table 1 : Estimates of Price Elasticities and Klein-Rubin Parameters, Australian Cross-Section Data, 1966-68[1]

Commodity[2]	Mean Exp. v_i	Average Budget Share w_i	Elasticities: Exp. η_i	Elasticities: Own Price η_{ii}	Elasticities: Food Price η_{i1}	Marg. Budget Share β_i	Sub. Exp. $\bar{\gamma}_i^*$
1. Food	1176	.3004	0.467	- .301		.140	956
2. Clothing	312	.0797	1.614	- .691	- .394	.129	111
3. Vice	194	.0496	1.868	- .771	- .456	.093	49
4. Fuel	139	.0355	0.455	- .195	- .111	.016	114
5. Transport	375	.0958	2.428	- .978	- .593	.233	11
6. Recreation	230	.0587	1.533	- .648	- .375	.090	89
7. Medical	158	.0404	2.301	- .928	- .562	.093	13
8. Housing	383	.0978	0.785	- .367	- .168	.077	263
9. Durables	427	.1091	2.297	- .939	- .561	.251	35
Residual	521	.1330				- .121	710
Total	3915	1.0000				1.000	2349

1. Table is based on Podder (1971), Tables III and VI.

2. The full definitions given in Podder on p. 386 are :
   1. Food, including food consumed while absent from home.
   2. Clothing, manchester goods, and footwear.
   3. Cigarettes, tobacco and liquor.
   4. Fuel, gas, electricity and telephone.
   5. Fares and motor vehicle expenses (other than initial purchase cost)
   6. Medical, dental, chemist needs, and funeral.
   7. Recreation and entertainment.
   8. House maintenance and overhead, furnishing and rent [no imputations are included].
   9. Consumer durables, including travel and hobby goods.

Estimates of the "subsistence" parameters at mean family size, $\bar{\gamma}_i^*$, may now be obtained in two <u>equivalent</u> ways : from rearranging the LES estimating equations (9) - (10),[1] i.e.,

$$\bar{\gamma}_i^* = \bar{v}_i - \beta_i(\bar{v} - \Sigma\gamma_j^*) ,$$

or from the own-price elasticity formula (13), i.e.,

$$\bar{\gamma}_i^* = \bar{v}_i(1 + \eta_{ii})/(1 - \beta_i) .$$

The resultant estimates of $\bar{\gamma}_i^*$ are given in the last column of table 1. The "subsistence-sum" for the nine commodities is $1639 per household (in 1967-68 prices). Food accounts for about 40 per cent of total subsistence expenditure; the next largest estimates occur for Housing, Fuel and Clothing, in that order. The "subsistence" expenditures are always less than actual mean expenditures. If the β-values for Podder's nine commodities were scaled to sum to unity, the effect would be to raise the $\bar{\gamma}^*$- estimates for these goods by an average of 12 per cent and lower the implicit $\bar{\gamma}^*$ - estimate for the remaining commodities by about one-quarter.

3.3 Family-size Effects

No use has yet been made of the information contained in the coefficients of family size in Podder's work. Just as it was possible to obtain estimates of β_i from corresponding elasticity estimates, it is possible to convert family-size elasticities into marginal responses. Denoting the

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(1) Recall that an estimate of $\Sigma\bar{\gamma}^*$ is implicit in the assumed value of ω ; $\omega = -2.5$ implies $\Sigma\bar{\gamma}_j^* = 2349$.

elasticity of expenditure on the i^{th} commodity with respect to family size, f, by ζ_i, then

$$\delta_i \quad = \quad \zeta_i \bar{v}_i / \bar{f} \quad ,$$

where δ_i is the coefficient of family size in the LES estimating equation (22). These IV estimates of δ_i are given in column (2) of table 2. For given total expenditure, expenditure on Recreation, Housing, Transport and Vice decreases with family size, for the five remaining goods it increases. The (negative) coefficient for the remaining goods is obtained from the requirement that the δ_i-estimates sum to zero over commodities.

IV estimates of the intercept term in equations (22) may be obtained from

$$\tilde{\alpha}_i \quad = \quad \bar{v}_i - \tilde{\beta}_i \bar{v} - \tilde{\delta}_i \bar{f} \ .$$

These are given in column (3) of table 2.

Thus, IV estimates have now been obtained of all the coefficients in the LES estimating equations (22) which allow for family size effects. However, it was pointed out in section 2 that two pieces of additional information are required in order to unscramble estimates of the "subsistence" parameters γ^*_{0i} and γ^*_{1i} in equation (18). The estimate of ω of - 2.5 provides one piece of additional information. This implies that $\Sigma\bar{\gamma}^*_j \ = \ 2349$, and from (18)

$$2349 \quad = \quad \Sigma\gamma^*_{0j} + \bar{f}\ \Sigma\gamma^*_{1j} \ . \qquad (30)$$

Table 2 : Parameter Estimates of LES Incorporating Family Size Effects, Australian Cross-Section Data, 1966-68

Commodity	δ_i	α_i	γ^*_{0i}		γ^*_{1i}	
			$\Sigma\gamma^*_{1j}=0$	$\gamma^*_{1j}\geq 0$	$\Sigma\gamma_{1j}=0$	$\gamma^*_{1j}\geq 0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. Food	134.3	159.9	489.5	290.9	134.3	191.4
2. Clothing	9.8	- 225.5	76.7	- 105.4	9.8	62.2
3. Vice	- 1.6	- 163.5	54.3	- 77.0	- 1.6	36.2
4. Fuel	8.7	45.3	83.3	60.4	8.7	15.3
5. Transport	- 6.2	- 514.2	32.3	- 297.1	- 6.2	88.6
6. Medical	3.5	- 134.6	76.9	- 50.5	3.5	40.2
7. Recreation	- 31.5	- 96.4	122.1	- 9.6	- 31.5	6.3
8. Housing	- 31.3	191.1	371.5	262.8	- 31.3	0.0
9. Durables	3.6	- 566.5	22.4	- 332.5	3.6	105.6
Residual	- 89.3	1304.5	1020.6	1191.7	- 89.3	- 138.5
Total	0.0	0.0	2349.0	933.6	0.0	407.3

A number of options suggest themselves for the second piece of outside information:

(i) assume that the average propensity to save is independent of family size, i.e., using (27), $\Sigma\gamma^*_{1j} = 0$;

(ii) assume that "subsistence" expenditure on vice (cigarettes, tobacco and alcohol) is independent of family size, i.e., $\gamma^*_{13} = 0$;

(iii) assume for each of the nine commodities considered by Podder that household "subsistence" expenditure never declines as family size increases, i.e., $\gamma^*_{1i} \geqslant 0$ for all i ;

(iv) estimate the ELES aggregate consumption function (25) - (27) by OLS and obtain a direct estimate of $\Sigma\gamma^*_{1j}$. This will be a consistent estimate only if income is measured without error. (Since Podder uses gross household income as his IV this is the appropriate explanatory variable to use in estimating (25).[1])

Assumptions (i) and (ii) give similar results and only the former is considered, i.e., $\Sigma\gamma^*_{1j} = 0$. Under (iii), $\Sigma\gamma^*_{1j} = 407$ and under (iv) $\Sigma\gamma^*_{1j} = 317$. For simplicity, we present detailed results only for the two extreme values of $\Sigma\gamma^*_{1j}$, i.e., assumptions (i) and (iii). The two sets of estimates of γ^*_{0i} are given in columns (4) and (5) of table 2, the estimates for γ^*_{1i} in columns (6) and (7). Under either assumption "subsistence" expenditure on food increases markedly with family size; "subsistence" expenditure

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(1) If one could place high confidence in the estimates of (25), then it would not even be necessary to assume an outside value for ω -- one could obtain direct ELES estimates of the Klein-Rubin parameters.

on housing and recreation on the other hand does not seem to show much variation.

Using (18) and (13), estimates of γ^*_{0i}, γ^*_{1i}, and β_i may be combined with data to yield estimates of price elasticities. Since household "subsistence" expenditures now depend on family size, it follows that so do the estimates of price elasticities. Values of η_{ii} for family sizes of two and six, for both assumptions (i) and (iii), are presented in table 3. The last two columns show the marginal effect on own-price elasticities of an additional family member. The values of η_{ii} for food, fuel and, to a lesser extent, housing, are only moderately sensitive to the assumption about $\Sigma\gamma^*_{1j}$. The own-price elasticity for food falls from around - 0.5 for a family of two to around zero for a family of six; the corresponding figures for fuel are around - 0.3 to near zero. For many commodities, however, the values are quite sensitive to the assumptions used. Further work is required to tighten up these elasticity estimates.

4. Concluding Remarks

By interpreting the parameter estimates of Engel curves within the context of an additive utility function (the Klein-Rubin) we have been able to obtain estimates of all price responses. Own-price elasticities obtained in this manner from Podder's work with the 1966-68 Macquarie survey data are quite plausible when evaluated at mean family size -- the range is - 0.20 to - 0.98, with the highest values, in absolute terms, occuring for durables and transport. By assuming that the "subsistence" parameters in the Klein-Rubin utility function are linear functions of family size we have been able to go further and make estimates of how "subsistence" expenditure

Table 3 : Family Size Effects on Own-Price Elasticities, LES Estimates, Australian Cross-Section Data, 1966-68

Commodity	Own-Price Elasticities				Marginal Effects on η_{ii} of Additional Family Member	
	$\Sigma\gamma^*_{1j} = 0$		$\gamma^*_{1i} \geqslant 0$		$\Sigma\gamma^*_{1j} = 0$	$\gamma^*_{1i} \geqslant 0$
	f = 2	f = 6	f = 2	f = 6		
1. Food	-.45	-.05	-.51	.05	.098	.140
2. Clothing	-.73	-.62	-.95	-.25	.027	.174
3. Vice	-.76	-.79	-1.02	-.34	-.007	.169
4. Fuel	-.29	-.04	-.36	.08	.062	.108
5. Transport	-.96	-1.01	-1.25	-.52	-.013	.181
6. Medical	-.67	-.61	-.88	-.25	.014	.159
7. Recreation	-.66	-1.39	-.98	-.84	-.181	.036
8. Housing	-.26	-.56	-.37	-.37	-.075	.0
9. Durables	-.95	-.92	-1.21	-.47	.006	.185

and price elasticities vary with family size. The results here are less successful. For food and fuel "subsistence" expenditure rises and the absolute value of the own-price elasticity falls quite markedly as family size increases. For both commodities, however, the own-price elasticity becomes positive for family size greater than six. This suggests that a linear specification fitted to the whole sample is too rigid and/or that the assumption of expenditure elasticities being independent of family size is incorrect (so that the γ-estimates are picking up both "income" and price effects). Whilst for other commodities positive price elasticities were not a problem, the estimates for a given commodity were quite sensitive to the assumption made in order to identify all the "subsistence" parameters.

The next step in analysis would seem to be to partition the sample data by such household characteristics as socio-economic class and stage in the life cycle. In this way it would be possible to check for variations in the Klein-Rubin parameters across sets of relatively homogeneous consumers. By judicious choice of "representative consumers" it may be possible to get reasonably reliable estimates of the parameters of the ELES aggregate consumption function and thus obviate the need to obtain outside information on the "subsistence" parameters.

APPENDIX

Here we derive the general relationships between OLS estimates of ELES and IV estimates of LES, where in obtaining the IV estimates income is used as an instrument for total expenditure and all other variables act as their own instruments.

The estimating equations to be considered are :

ELES

$$v_{ih} = \beta_i^* \, y_h + \sum_{j=0}^{m} \delta_{ji}^* \, x_{jh} \tag{A1}$$

$$v_h = \mu y_h + \sum_{j=0}^{m} \delta_j^{**} \, x_{jh} \tag{A2}$$

LES

$$v_{ih} = \beta_i \, v_h + \sum_{j=0}^{m} \delta_{ji} \, x_{jh} \tag{A3}$$

where notation is as in the text except for x_{jh} which represents an observation for the h^{th} household (h=1,...,H) on variable x_j; it is assumed that x_{0h}=1 for all h.

Define the following data vectors and matrix :

$$v_i^* = \begin{bmatrix} v_{i1} \\ v_{i2} \\ \vdots \\ v_{iH} \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_h \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_h \end{bmatrix}, \quad X = \begin{bmatrix} x_{01} & x_{11} & \cdots & x_{m1} \\ x_{02} & x_{12} & \cdots & x_{m2} \\ \vdots & \vdots & & \vdots \\ x_{0H} & x_{1H} & \cdots & x_{mH} \end{bmatrix}$$

(i=1,...,n), and the following vectors of parameters :

$$\delta_i^* = \begin{bmatrix} \delta_{0i}^* \\ \delta_{1i}^* \\ \vdots \\ \delta_{mi}^* \end{bmatrix}, \qquad \delta^{**} = \begin{bmatrix} \delta_0^{**} \\ \delta_1^{**} \\ \vdots \\ \delta_m^{**} \end{bmatrix}, \quad \text{and} \quad \delta_i = \begin{bmatrix} \delta_{0i} \\ \delta_{1i} \\ \vdots \\ \delta_{mi} \end{bmatrix}$$

Finally, let W be an H x (m+2) data matrix defined as

$$W = [y \vdots X].$$

Now IV estimates of LES are given by

$$W'v_i^* = W' [v \vdots X] \begin{bmatrix} \tilde{\beta}_i \\ \cdots \\ \tilde{\delta}_i \end{bmatrix}$$

$$= (W'v)\ \tilde{\beta}_i + (W'X)\ \tilde{\delta}_i , \qquad \text{(A4)}$$

and OLS estimates of the ELES aggregate consumption function (A2) by

$$W'\ v = W' [y \vdots X] \begin{bmatrix} \hat{\mu} \\ \cdots \\ \hat{\delta}^{**} \end{bmatrix} . \qquad \text{(A5)}$$

Substituting (A5) into (A4) yields

$$W'\ v_i^* = W' [y\ \hat{\mu} + X\ \hat{\delta}^{**}]\ \tilde{\beta}_i + (W'X)\ \tilde{\delta}_i$$

$$= (\hat{\mu}\ \tilde{\beta}_i)\ W'y + W'X\ [\hat{\delta}^{**}\ \tilde{\beta}_i + \tilde{\delta}_i] \quad . \qquad \text{(A6)}$$

using (A11) we obtain :

$$\sum_i \hat{\delta}^*_{ji} = \hat{\delta}^{**}_j \, , \tag{A12}$$

as expected, i.e. the OLS estimate of the coefficient of x_j in the ELES aggregate consumption function is identical to the sum of the OLS estimates of the coefficients of x_j in each of the individual expenditure equations.

However OLS estimation of (A1) produces

$$W' v_i^* = W' [y \vdots X] \begin{bmatrix} \hat{\beta}_i^* \\ \cdots \\ \hat{\delta}_i^* \end{bmatrix}$$

$$= \hat{\beta}_i^* W'y + (W'X) \hat{\delta}_i^* \quad . \qquad (A7)$$

Equating coefficients in (A6) and (A7) we derive the key results :

$$\hat{\beta}_i^* = \hat{\mu} \tilde{\beta}_i \qquad (A8)$$

$$\hat{\delta}_i^* = \hat{\delta}^{**} \tilde{\beta}_i + \tilde{\delta}_i \quad . \qquad (A9)$$

For the typical element of (A9)

$$\hat{\delta}^*_{ji} = \hat{\delta}^{**}_j \tilde{\beta}_i + \tilde{\delta}_{ji} \qquad \begin{pmatrix} i=1,\ldots,n \\ j=1,\ldots,m \end{pmatrix} \quad . \qquad (A10)$$

If x_j is family size, then $\hat{\delta}_j^{**}$ is expected to be positive and thus the OLS estimate of the coefficient of family size in ELES will tend to exceed the corresponding LES IV estimate (for all goods).

Notice that if we add the (A4) expressions over commodities i.e. calculate $\sum_i W' v_i^* = W' v$, and equate coefficients on the left and right hand sides of the resulting equation we obtain :

$$\sum_i \tilde{\beta}_i = 1 \quad \text{and} \quad \sum_i \tilde{\delta}_{ji} = 0 \qquad \text{(for all j)} \quad . \qquad (A11)$$

These results show that IV estimates of LES ensure that the budget constraint holds. If we now sum (A10) over commodities and substitute

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