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EXTENDING THE ORANI MODEL OF THE AUSTRALIAN ECONOMY : ADDING FOREIGN INVESTMENT TO A MINIATURE VERSION

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EXTENDING THE ORANI MODEL OF THE AUSTRALIAN ECONOMY :

ADDING FOREIGN INVESTMENT TO A MINIATURE VERSION

by

Peter B. Dixon, B.R. Parmenter and Russell J. Rimmer

1. Introduction

ORANI is a large multisectoral model of the Australian economy.[1] In standard applications it identifies 113 industries, 230 commodities (115 domestically produced and 115 imported), 9 types of labour, 7 types of agricultural land and 113 types of capital (one for each industry). It contains explicit modelling, at this disaggregated level, of many types of commodity and factor flows, e.g., inputs to current production, inputs to capital creation, household consumption, exports and margin services (retail, wholesale and transport). The multi-product characteristics of production in Australian agricultural industries are also explicitly modelled, and a facility is included for disaggregating economy-wide results to the regional (State) level.

The reason for including so much detail in ORANI is to facilitate its use by a variety of Government agencies with interests in different spheres of economic policy. Among the agencies which have used the model are the Industries Assistance Commission, the Bureau of Agricultural Economics, the Bureau of Industry Economics and the Premier's Department of South Australia. Applications of ORANI made by these agencies and other groups include simulations of the effects on industries, occupations and regions of changes in tariffs, the exploitation of mineral resources, changes in world commodity prices, changes in the exchange rate, the adoption of import parity pricing for oil products, subsidies to ailing industries, the move towards equal pay for women, changes in real wages and the adoption of Keynesian demand stimulation policies. Each of these applications draws upon different aspects of the model's detail.

In handling such a large model, three features of our procedures have been helpful. The first is that the theoretical structure has been kept simple. Lags are not explicitly included and similar behavioral assumptions and functional forms are employed for each industry. Consequently, results from ORANI can be explained intuitively in terms of the model's theory. Second, we solve the model by linear approximations. This enables us to obtain solutions at modest cost, and has the additional advantages of allowing flexibility in the selection of exogenous variables and of facilitating changes to the structure of the model. Finally, we have devised miniature versions of the model for experimenting with new developments prior to their implementation in the main model.

In section 2 of the paper we outline the theory of our computing procedure including a method for overcoming the problem of linearization errors. In section 3 we introduce the current miniature version of ORANI (MO81). With MO81 we are experimenting with ways of accounting explicitly for capital inflow in long-run simulations. Foreign investment is important to the long-run development of the Australian economy but is not a crucial consideration in the short-run problems which have been the subjects of most ORANI applications to date. Results of our experiments are reported in section 4. This section also includes an illustration of our method for eliminating linearization errors. Section 5 contains conclusions.

2. Computing ORANI Solutions : Theoretical Overview

The ORANI model can be represented as a system of m equations in n variables, with $n > m$:

$$F(X) = 0 , \tag{2.1}$$

where X is an $n \times 1$ vector of variables (demands, prices, employment levels, etc.) and F is an m-component vector of differentiable functions. In each application of ORANI, the equations are interpreted as refering to one point of time (the solution year), although the point of time chosen varies with applications. For example, in a study of the long-run effects of a change in protection, the solution year might be 1990. Then (2.1) would be interpreted as saying that in 1990, demand will equal supply, prices will equal costs, etc.. For a study of short-run effects, the solution year might be 1983. As we will see, the choice of the solution year has implications for (a) the choice of values for various parameters, (b) the setting of the initial solution and (c) the allocation of variables between the endogenous and exogenous categories.

While the selection of the $(n - m)$ variables to be treated as exogenous varies across applications of the model, in any particular application we can denote the endogenous variables by the $m \times 1$ vector X_1 and the exogenous variables by the $(n - m) \times 1$ vector X_2. Then (2.1) can be rewritten as

$$F(X_1, X_2) = 0 , \tag{2.2}$$

and a solution for the model can be written as a system of m equations of the form

$$X_1 = G(X_2) \quad , \tag{2.3}$$

where

$$F\big(G(X_2), X_2\big) = 0 \quad , \tag{2.4}$$

for all X_2 in some subset of R^{n-m}. In computations with ORANI, we are concerned with the effects on the endogenous variables of given changes to the exogenous variables. Thus the computational problem is to evaluate ΔX_1, where

$$\Delta X_1 = G(X_2^F) - G(X_2^I) \; , \tag{2.5}$$

and where X_2^I and X_2^F are the initial and final values for the exogenous variables.

The numbers of equations and variables in ORANI is extremely large and the F vector includes a variety of functional forms. However, in all ORANI computations we have two important pieces of information. First, we know an initial solution, (X_1^I, X_2^I). That is, we have a set of values for demands, prices, employment levels, etc., such that

$$F(X_1^I \; , X_2^I) = 0 \quad . \tag{2.6}$$

This initial solution is provided by the demands, prices, employment levels, etc., revealed in the initial-situation input-output data. These data are the source of parameters relating to cost shares, sales shares, etc., in the system (2.1). If the solution year chosen for an application of ORANI is 1990, then the initial-situation input-output data are a balanced set of input-output tables for 1990. These tables can be constructed by simply applying a uniform expansion factor to all flows in the most recently published input-output accounts. Alternatively, their construction could be based on detailed forecasts. In most of our work with ORANI, we have implicitly adopted the simple mechanical approach.[2] We have, therefore, reported results from the model in the following way : if we increase tariffs by 25 per cent, then in 1990 the output of cars will be x per cent higher than it would have been without the change in tariffs. The value assumed, $X^I(\text{cars})$, for what output would have been is part of our mechanically projected input-output data. If our aim were to produce a projection of the form

$$X^F(\text{cars}) = X^I(\text{cars})\ (1 + \frac{x}{100}) ,$$

for the output of cars in 1990 under an increase in tariffs, then our assumed value for $X^I(\text{cars})$ would be vital. However, in applications of ORANI, we have concentrated on the percentage change results, x. These are comparatively insensitive to plausible variations in X^I.

The second piece of information we have is how to evaluate any selected components of an $m \times n$ matrix $A(X)$, which has the property that

$$A(X) = F_X(X)\hat{X} , \qquad (2.7)$$

for all X satisfying (2.1), where $F_X(X)$ is the $m \times n$ matrix of first-order partial derivatives of F with respect to X and $\hat{}$ denotes a diagonal matrix.

The derivation of an $A(X)$ matrix is straight-forward for ORANI and we believe this is true for most members of the current generation of computable general equilibrium models. The task is to translate the system

(2.1) into a percentage-change or elasticities form. The system becomes

(2.8) $$A(X)x = 0 ,$$

where, for X satisfying (2.1), the ij^{th} element of $A(X)$ is given by

$$A_{ij}(X) = \frac{\partial F^i}{\partial X(j)} X(j) ,$$

and where the j^{th} component of x (denoted by $x(j)$) is the percentage change in the j^{th} variable (denoted by $X(j)$),[3] that is

(2.9) $$x(j) = 100\left(\frac{d(X(j))}{X(j)}\right) .$$

The A_{ij} often turn out to be cost shares or sales shares. Assume for example, that the i^{th} equation in (2.1) is

(2.10) $$F^i(X) = \frac{Q_1 + Q_2}{Q} - 1 = 0 ,$$

where Q_1 and Q_2 are demands by agents 1 and 2 and Q is total sales. Then

(2.11) $$\left(\frac{\partial F^i}{\partial Q_r}\right)Q_r = \frac{Q_r}{Q} , \qquad r=1,2 ,$$

and

(2.12) $$\left(\frac{\partial F}{\partial Q}\right)Q = -\frac{Q_1 + Q_2}{Q^2} Q .$$

Letting Q_1, Q_2 and Q be the first three variables, we can write the i^{th} row of $A(X)$ as

(2.13) $$A_{i.}(X) = (S_1, S_2, -1, 0, \ldots, 0) ,$$

where

(2.14) $$S_r = Q_r/Q , \qquad r=1,2.$$

Thus, $A_{i1}(X)$ and $A_{i2}(X)$ are the shares of agents 1 and 2 in total sales.

It is worth noting that in deriving (2.13), we have simplified (2.12). This explains the comment following (2.7). The $A(X)$ which we compute for ORANI necessarily satisfies (2.7) only for X satisfying (2.1). In our example, the third element in the i^{th} row of $A(X)$ is

$$A_{i3}(X) = -1 \quad .$$

This coincides with the value for $(\partial F/\partial Q)Q$ given in (2.12) only if Q_1, Q_2 and Q satisfy (2.10).

To make use of the $A(X)$ matrix in solving the computational problem (2.5), we reformulate the problem as one in numerical integration. We start by differentiating (2.4) to obtain

$$(2.15) \qquad F_1(X_1, X_2)\, G_2(X_2) + F_2(X_1, X_2) = 0 \quad ,$$

where X_2 is in the relevant subset of R^{n-m} and X_1 and X_2 jointly satisfy (2.2). F_1 and F_2 are $m \times m$ and $m \times (n - m)$ submatrices of F_X. F_1 contains the derivatives with respect to the endogenous variables, X_1, and F_2 contains the derivatives with respect to the exogenous variables, X_2. G_2 is the $m \times (n - m)$ matrix of first-order partial derivatives of G.

Next, we assume that

$$(2.16) \qquad \text{determinant} \left\{F_1\left(X_1^I, X_2^I\right)\right\} \neq 0 \quad .$$

This is a sufficient condition to ensure the existence in a neighbourhood of (X_1^I, X_2^I) of a unique vector of G functions (functions satisfying (2.4)) passing through (X_1^I, X_2^I) (see the implicit functions theorem, Apostol

(1957, pp.146-148)). It also allows us to rearrange (2.15) as

$$G_2(X_2) = -F_1^{-1}(X_1, X_2)\ F_2(X_1, X_2) \quad , \tag{2.17}$$

where it is understood that (X_1, X_2) is in a neighbourhood of (X_1^I, X_2^I) and that X_1 and X_2 jointly satisfy (2.2).

Finally, we substitute from (2.7) into (2.17). This gives

$$G_2(X_2) = -\hat{X}_1\ A_1^{-1}(X)\ A_2(X)\hat{X}_2^{-1} \quad , \tag{2.18}$$

for all (X_1, X_2) satisfying (2.2) and lying in a neighbourhood of (X_1^I, X_2^I). Thus, for any value of (X_1, X_2), we can evaluate a matrix $B(X_1, X_2)$ which has the property that

$$G_2(X_2) = B(X_1, X_2) \quad , \tag{2.19}$$

for all X_1 and X_2 satisfying (2.2) and lying in a neighbourhood of (X_1^I, X_2^I). The formula for B is the right hand side of (2.18).

The computation of ΔX_1 in (2.5) can now be handled as a standard problem in numerical integration. We have an initial point (X_1^I, X_2^I) satisfying

$$X_1^I = G(X_2^I) , \tag{2.20}$$

and we have a formula, (2.18), for evaluating the derivatives of G. We can proceed by, for example, Euler's method. We divide the change in the exogenous variables, $\Delta X_2 (= X_2^F - X_2^I)$, into n equal parts. Then we approximate $G(X_2^F) - G(X_2^I)$ by

$$(\Delta X_1)^n = \sum_{q=0}^{n-1} (\Delta X_1)_q^n \quad , \tag{2.21}$$

where the $(\Delta X_1)_q^n$ are given by

$$(2.22) \qquad (\Delta X_1)_0^n = \frac{1}{n} B\left(X_1^I, X_2^I\right)\Delta X_2 ,$$

and

$$(2.23) \qquad (\Delta X_1)_q^n = \frac{1}{n} B\left(X_1^I + \sum_{r=0}^{q-1} (\Delta X_1)_r^n , \quad X_2^I + \frac{q}{n}\Delta X_2\right)\Delta X_2 ,$$

for $q=1,\ldots,n-1$.

If we set $n=1$, our approximation is

$$(2.24) \qquad G(X_2^F) - G(X_2^I) = B(X_1^I, X_2^I)\Delta X_2 \quad .$$

This is Johansen's (1960) approach. If we set $n=2$, then we obtain

$$(2.25) \qquad G(X_2^F) - G(X_2^I) = \frac{1}{2} B\left(X_1^I, X_2^I\right)\Delta X_2$$

$$+ \frac{1}{2} B\left(X_1^I + (\Delta X_1)_0^1 , \quad X_2^I + \frac{1}{2}\Delta X_2\right)\Delta X_2 \quad ,$$

where

$$(2.26) \qquad (\Delta X_1)_0^1 = \frac{1}{2} B\left(X_1^I, X_2^I\right)\Delta X_2 \quad .$$

With the first term on the right hand side of (2.25) we approximate the effect on X_1 of moving X_2 from X_2^I to $X_2^I + \frac{1}{2}\Delta X_2$. In the second term, B is re-evaluated to give an estimate of $G(X_2^I + \frac{1}{2}\Delta X_2)$. Then this estimate is used in approximating the effect on X_1 of moving X_2 from $X_2^I + \frac{1}{2}\Delta X_2$ to X_2^F.

Provided that the derivatives of B satisfy various boundedness conditions (these are detailed in DPSV, section 35), it can be shown that

$$(2.27) \qquad \lim_{n\to\infty}(\Delta X_1)^n = G(X_2^F) - G(X_2^I) \quad .$$

Our experience (see section 5) with applications of ORANI suggests that $G(X_2^F) - G(X_2^I)$ can usually be approximated to a high level of accuracy using a low value of n. $n=2$ is normally adequate.

3. A Miniature Version of ORANI, MO81

3.1 Introduction

Tables 1 - 4 set out the miniature version of ORANI, MO81. Table 1 lists the equations in the percentage change form, (2.8). Tables 2 and 4 define the variables and coefficients appearing in Table 1. Three possible selections for the exogenous variables are given in Table 3. In Table 4 we have also given examples of how the coefficients are evaluated for our miniature model using hypothetical input-output data. These data are discussed in section 4 of the paper. We have chosen to locate Table 4 together with the data tables (Tables 5 and 6) in subsection 4.1.

The equations in ORANI can be classified under five headings : I final demands, II industry inputs and outputs, III zero pure profit conditions, IV market clearing and V miscellaneous. This last group includes definitions of various aggregates such as the balance of trade and the consumer price index. In MO81 we have added a category VI, macroeconomic closure. This category includes an aggregate consumption function and equations describing the foreign share in capitalist income. In the current ORANI model the macro aggregates are usually treated as being exogenous and no explicit consideration is given to the foreign share in domestic capital. At present we are using MO81 to experiment with various equations which will allow a more complete treatment of these variables. Results from these experiments are discussed in section 4.

In describing MO81, we will first work through the equations in categories I - V. In subsection 3.3 we will consider the closure options which are available in a model restricted to these categories. This

Table 1 - The MO81 Equations : A Linear System in Percentage Changes[(a)]

Identifier	Equation	Subscript Range[(b)]	Number	Description
I. FINAL DEMANDS				
(T1)	$x^{(3)}_{(is)} = c^R - \left(p^{(3)}_{(is)} - \sum_{r=1}^{2} S^{(3)}_{(ir)} p^{(3)}_{(ir)}\right)$	i=1,...,g, s=1,2.	2g	Household demands for commodities.
(T2)	$x^{(2)}_{(is)j} = y_j - \left(p^{(2)}_{(is)j} - \sum_{r=1}^{2} S^{(2)}_{(ir)j} p^{(2)}_{(ir)j}\right)$	i=1,...,g, s=1,2, j=1,...,h.	2gh	Demands for inputs to capital creation.
(T3)	$p^{*}_{(i1)} = -\gamma_i x^{(4)}_{(i1)} + f^{(4)}_{(i1)}$	i=1,...,g.	g	Export demand functions.
II. INDUSTRY INPUTS AND OUTPUTS				
(T4)	$x^{(1)}_{(is)j} = z_j - \left(p^{(1)}_{(is)j} - \sum_{r=1}^{2} S^{(1)}_{(ir)j} p^{(1)}_{(ir)j}\right)$	i=1,...,g+1, j=1,...,h, s=1,2.	2(g+1)h	Demands for intermediate and primary factor inputs.
(T5)	$x^{(0)}_{(i1)j} = z_j + \left(p^{(0)}_{(i1)} - \sum_{q=1}^{g} H^{(0)}_{(q1)j} p^{(0)}_{(q1)}\right)$	i=1,...,g, j=1,...,h.	gh	Commodity supplies by industry.
III. ZERO PURE PROFIT CONDITIONS				
(T6)	$\sum_{q=1}^{g} H^{(0)}_{(q1)j} p^{(0)}_{(q1)} = \sum_{i=1}^{g+1} \sum_{s=1}^{2} H^{(1)}_{(is)j} p^{(1)}_{(is)j}$	j=1,...,h.	h	Zero pure profits in production.
(T7)	$p^{*}_{(i2)} + t_i + \phi = p^{(0)}_{(i2)}$	i=1,...,g.	g	Zero pure profits in importing.

(a) The variables and coefficients are defined in Tables 2 and 4.

(b) The number of domestically-produced goods is g, the number of imported goods is g and the number of industries is h. In standard applications of ORANI, g=115 and h=113. Some ORANI industries produce several goods and some goods are produced by several industries. In MO81, g=2 and h=2.

... *continued*

Table 1 continued ...

Identifier	Equation	Subscript Range	Number	Description
(T8)	$p^{*}_{(i1)} + v_i + \phi = p^{(4)}_{(i1)}$	i=1,...,g.	g	Zero pure profits in exporting.
(T9)	$\pi_j = \sum_{i=1}^{g} \sum_{s=1}^{2} H^{(2)}_{(is)j}\, p^{(2)}_{(is)j}$	j=1,...,h.	h	Zero pure profits in capital creation.
(T10)	$p^{(k)}_{(is)} = p^{(0)}_{(is)}$	i=1,...,g, s=1,2, k=3,4.	4g	Zero pure profits in the distribution of goods.
(T11)	$p^{(k)}_{(is)j} = p^{(0)}_{(is)}$	i=1,...,g, s=1,2, j=1,...,h, k=1,2.	4gh	
IV. MARKET CLEARING				
(T12)	$\sum_{j=1}^{h} x^{(0)}_{(q1)j}\, W^{(0)}_{(q1)j} = \sum_{k=1}^{2} \sum_{j=1}^{h} x^{(k)}_{(q1)j}\, W^{(k)}_{(q1)j} + \sum_{k=3}^{4} x^{(k)}_{(q1)}\, W^{(k)}_{(q1)}$	q=1,...,g.	g	Demand equals supply for domestically produced commodities.
(T13)	$\sum_{j=1}^{h} x^{(1)}_{(g+1,1)j}\, W^{(1)}_{(g+1,1)j} = \ell$		1	Demand for labor equals employment of labor.
(T14)	$x^{(1)}_{(g+1,2)j} = k_j$	j=1,...,h.	h	Demand equals employment of capital in each industry.

... continued

Table 1 continued ...

Identifier	Equation	Subscript Range	Number	Description
V. MISCELLANEOUS EQUATIONS				
(T15)	$x^{(0)}_{(q2)} = \sum_{k=1}^{2} \sum_{j=1}^{h} x^{(k)}_{(q2)j} \; W^{(k)}_{(q2)j} + x^{(3)}_{(q2)} \; W^{(3)}_{(q2)}$	q=1,...,g.	g	Import volumes.
(T16)	$m = \sum_{q=1}^{g} M_{(q2)} \left(p^{*}_{(q2)} + x^{(0)}_{(q2)} \right)$		1	Foreign currency value of imports.
(T17)	$e = \sum_{q=1}^{g} E_{(q1)} \left(p^{*}_{(q1)} + x^{(4)}_{(q1)} \right)$		1	Foreign currency value of exports.
(T18)	$\Delta B = (Ee - Mm)/100$		1	Balance of trade.
(T19)	$\xi^{(3)} = \sum_{i=1}^{g} \sum_{s=1}^{2} H^{(3)}_{(is)} \; p^{(3)}_{(is)}$		1	Consumer price index.
(T20)	$c^{R} = c - \xi^{(3)}$		1	Real aggregate consumption.
(T21)	$y^{R} = \sum_{j=1}^{h} W^{y}_{j} \; y_{j}$		1	Aggregate real investment.
(T22)	$\pi = \sum_{j=1}^{h} W^{y}_{j} \; \pi_{j}$		1	Investment-goods price index.
(T23)	$f_{R} = c^{R} - y^{R}$		1	Ratio of real aggregates.

... *continued*

Table 1 continued ...

Identifier	Equation	Subscript Range	Number	Description
(T24)	$r_j = \left(p^{(1)}_{(g+1,2)j} - \pi_j\right)Q_j$	j=1,...,h.	h	Rates of return on capital in each industry.
(T25)	$y_j = I^{(1)}_j k_j + I^{(2)}_j B_j (r_j - \omega) + f^{(2)}_j$	j=1,...,h.	h	Investment in each industry.
(T26)	$k = \sum_{j=1}^{h} W^k_j \; k_j$		1	Aggregate capital stock.
(T27)	$t = \sum_{i=1}^{g} \left(\zeta^t_i t_i + p^*_{(i2)} + x^{(0)}_{(i2)} + \phi\right) T^t_i$		1	Aggregate tariff revenue.
(T28)	$v = \sum_{i=1}^{g} \left(\zeta^v_i v_i + p^*_{(i1)} + x^{(4)}_{(i1)} + \phi\right) T^v_i$		1	Aggregate export subsidies.
(T29)	$p^{(1)}_{(g+1,1)j} = I_{(g+1,1)j} \; \xi^{(3)} + f_{(g+1,1)j} + f_{(g+1,1)}$	j=1,...,h.	h	Wage indexation.
(T30)	$r_j = r + f^r_j$	j=1,...,h.	h	Relative rates of return.
	Total number of equations in categories I - V =		9gh+11g+9h+12	

... continued

Table 1 continued ...

Identifier	Equation	Subscript Range	Number	Description
VI.	MACROECONOMIC CLOSURE			
(T31)	$c = f_c + \psi_1 \left(\sum_{j=1}^{h} \left(p^{(1)}_{(g+1,1)j} + x^{(1)}_{(g+1,1)j} \right) N_{(g+1,1)j} \right) + \psi_2 t - \psi_3 v + \psi_4 \left(q + \sum_{j=1}^{h} \left(p^{(1)}_{(g+1,2)j} + x^{(1)}_{(g+1,2)j} \right) N_{(g+1,2)j} \right)$		1	Consumption function.
(T32)	$u = (s - \pi)\left(\frac{U+1}{\tau U}\right)$		1	Determination of the domestic ownership share in the capital stock. (T32–T34)
(T33)	$q + k = u\Gamma$		1	
(T34)	$s = c - f_c/(1 - F_c)$		1	
	Total number of equations in category VI	=	4	
	Total number of equations in the complete model	=	9gh+11g+9h+16	

Table 2 - The MO81 Variables

Variable	Subscript Range[(a)]	Number	Description
Variables appearing in categories I - V			
$x^{(3)}_{(is)}$	i=1,...,g, s=1,2.	2g	Household demands for commodities
$x^{(k)}_{(is)j}$	i=1,...,g, s=1,2, j=1,...,h, k=1,2.	4gh	Demands for inputs of commodities for current production (k=1) and capital creation (k=2)
$x^{(1)}_{(g+1,s)j}$	j=1,...,h, s=1,2.	2h	Demands for labor (s=1) and capital (s=2) by industry
$x^{(4)}_{(i1)}$	i=1,...,g.	g	Export volumes
$x^{(0)}_{(q2)}$	q=1,...,g.	g	Import volumes
$x^{(0)}_{(i1)j}$	i=1,...,g, j=1,...,h.	gh	Commodity supplies by industry
z_j	j=1,...,h.	h	Industry activity levels
c		1	Aggregate consumption
c^R		1	Real aggregate consumption
y^R		1	Real aggregate investment
y_j	j=1,...,h.	h	Capital creation for each industry
ℓ		1	Aggregate employment
k_j	j=1,...,h.	h	Employment of capital in each industry
k		1	Aggregate capital employed
m		1	Foreign currency value of imports
e		1	Foreign currency value of exports

(a) The number of domestically-produced goods is g, the number of imported goods is g and the number of industries is h. In standard applications of ORANI, g=115 and h=113. Some ORANI industries produce several goods and some goods are produced by several industries. In MO81, g=2 and h=2.

... *continued*

Table 2 continued ...

Variable	Subscript Range	Number	Description
Variables appearing in categories I - V *(continued)*			
ΔB		1	Balance of trade
$p^{(k)}_{(is)}$	i=1,...,g, s=1,2, k=3,4.	4g	Prices paid by households and exporters for commodities
$p^{(k)}_{(is)j}$	i=1,...,g, j=1,...,h, s=1,2, k=1,2.	4gh	Prices paid by industries for commodity inputs to production and capital creation
$p^{(0)}_{(is)}$	i=1,...,g, s=1,2.	2g	Basic prices of domestic and imported commodities
$p^{(1)}_{(g+1,s)j}$	j=1,...,h, s=1,2.	2h	Prices paid by industries for use of primary factors (s=1 for labor and s=2 for capital)
$p^{*}_{(i1)}$	i=1,...,g.	g	Foreign currency prices of exports
$p^{*}_{(i2)}$	i=1,...,g.	g	Foreign currency prices of imports
π_j	j=1,...,h.	h	Prices of units of capital
π		1	Investment-goods price index
$\xi^{(3)}$		1	Consumer price index
r_j	j=1,...,h.	h	Rates of return on capital in each industry.
r		1	Useful variables for exogenizing relative rates of return while endogenizing absolute rates of return
f^r_j		h	(as above)
$f^{(4)}_{(i1)}$	i=1,...,g.	g	Shifts in foreign export demands
f_R		1	Ratio of real aggregate consumption to real aggregate investment
$f^{(2)}_j$	j=1,...,h.	h	Shift variable in industry investment equations

... continued

Table 2 continued ...

Variable	Subscript Range	Number	Description
Variables appearing in categories I - V *(continued)*			
$f_{(g+1,1)j}$	j=1,...,h.	h	Variable used to allow variations across industries in wage rate movements
$f_{(g+1,1)}$		1	Normally interpreted as the real wage rate
ϕ		1	Exchange rate ($A/$US)
v_i	i=1,...,g.	g	One plus ad valorem export subsidies
t_i	i=1,...,g.	g	One plus ad valorem tariff rates
v		1	Aggregate export subsidy
t		1	Aggregate tariff revenue
ω		1	Expected rate of return in all industries, adjusted for risk
Total number of variables in categories I - V =	9gh + 15g+12h+17		
Additional variables introduced in category VI			
f_c		1	Shift in the average propensity to consume
q		1	Domestic share in the ownership of capital
u		1	Rate of growth in real domestic saving over the period 0 to τ
s		1	Domestic saving
Number of additional variables =		4	
Total number of variables in categories I - VI = 9gh+15g+12h+21			

Table 3 - Typical Lists of Exogenous Variables

(1) Short-run, restricted model	(2) Long-run, restricted model	(3) Long-run, complete model	No. of Variables
$p^*_{(i2)}$	$p^*_{(i2)}$	$p^*_{(i2)}$	g
$f^{(4)}_{(i1)}$	$f^{(4)}_{(i1)}$	$f^{(4)}_{(i1)}$	g
t_i	t_i	t_i	g
$v_q \forall q \epsilon G$	$v_q \forall q \epsilon G$	$v_q \forall q \epsilon G$	g
$x^{(4)}_{(q1)} \forall q \notin G$	$x^{(4)}_{(q1)} \forall q \notin G$	$x^{(4)}_{(q1)} \forall q \notin G$	
k_j	r_j	r_j	h
$f_{(g+1,1)j}$	$f_{(g+1,1)j}$	$f_{(g+1,1)j}$	h
$f_{(g+1,1)}$	ℓ	ℓ	1
$f^{(2)}_j$	$f^{(2)}_j$	$f^{(2)}_j$	h
c^R	f_R	f_c	1
y^R	ΔB	ω	1
r	r	r	1
ϕ	ϕ	ϕ	1
		Total number of exogenous variables =	4g+3h+5

will indicate the possibilities available in the current version of ORANI. Then in subsection 3.4 we will consider the equations in category VI. The additional closure options which are available in the complete model are discussed in subsection 3.5.

3.2 The equations of MO81, categories I - V

I Final demands

ORANI includes equations describing final demands by households, capital creators, foreigners (export demands) and the government. Only the first three types appear in our miniature version.

Underlying the estimation of the ORANI household demand functions is the assumption of constrained utility maximization. It is assumed that $X^{(3)}_{(is)}$, $i=1,\ldots,g$, $s=1,2$, maximize

$$U\left(X^{(3)}_{(11)},\ldots,X^{(3)}_{(g1)}\ ;\ X^{(3)}_{(12)},\ldots,X^{(3)}_{(g2)}\right), \tag{3.1}$$

subject to

$$\sum_{i=1}^{g}\sum_{s=1}^{2} P^{(3)}_{(is)}\ X^{(3)}_{(is)} = C , \tag{3.2}$$

where U is a utility function describing household preferences. C is aggregate household expenditure. $X^{(3)}_{(is)}$ and $P^{(3)}_{(is)}$ are household consumption and the price to households of good i from source s. The superscript (3) is used to denote households. (Later we will be using superscript (1) to denote intermediate usage, superscript (2) to denote demands for inputs to capital creation and superscript (4) to denote export demands.) Source 1 (i.e., $s=1$) is domestic supplies while $s=2$ indicates foreign supplies, i.e., imports. Thus domestic and imported commodities are treated as distinct. For example, domestically produced cars are treated as different products from imported cars although the two are modelled as being good substitutes. Much of the econometric effort associated with ORANI has been concerned with estimating elasticities of substitution between imported and domestic products belonging to the same input-output categories. (Details are in DPSV, section 29(a).) By handling imported and domestic products as imperfect substitutes, we avoid the unsatisfactory consequences of either of the two extremes frequently found in modelling exercises. On the one hand, the assumption of perfect substitutability, which is implied when imported cars and domestic cars are treated as the same product, is inconsistent with the empirical observation that prices of imported and domestic commodities can move independently of each other without causing the exclusion of either product from the market. At the other extreme, the quantity share of imports in each market is assumed to be fixed, i.e., imports are treated as non-competitive. The implied assumption, that the elasticity of substitution between imported and domestic cars is zero, is inconsistent with the observation that the quantity share of imported cars in the domestic market responds to changes in the relative prices of imported and domestic products.

On solving problem (3.1) - (3.2), we obtain demand functions of the form

$$(3.3) \qquad X^{(3)}_{(is)} = f^{(3)}_{(is)}\left(P^{(3)}_{(11)},\ldots,P^{(3)}_{(g1)} \ ; \quad P^{(3)}_{(12)},\ldots,P^{(3)}_{(g2)} \ ; \quad C\right) ,$$

$$i=1,\ldots,g, \quad s=1,2.$$

In our miniature model, we assume that the utility function (3.1) has the form

$$U = \min_{i=1,\ldots,g} \left\{ \frac{X^{(3)}_{(i\cdot)}}{A^{(3)}_{(i\cdot)}} \right\}, \tag{3.4}$$

where the $A^{(3)}_{(i\cdot)}$ are positive parameters and the $X^{(3)}_{(i\cdot)}$ are defined by

$$X^{(3)}_{(i\cdot)} = \text{Cobb-Douglas}\left\{X^{(3)}_{(i1)}, X^{(3)}_{(i2)}\right\}, \quad \text{for } i=1,\ldots,g. \tag{3.5}$$

This specification implies that consumers derive utility from "effective" units of goods 1,2,...,g, where an effective unit of good i is an aggregation of units of domestically produced and imported good i. The aggregation is defined by a Cobb-Douglas function. Of course, both (3.4) and (3.5) could be replaced by more general forms. In ORANI, the equations corresponding to these have, respectively, additive and CES forms (see DPSV, section 14). Under (3.4) and (3.5), however, the percentage-change form of (3.3) is very simple. One representation is that given in equation (T1) of Table 1. There we have used c^R (the percentage change in <u>real</u> household expenditure) rather than c. c^R is defined in the table by (T20) and (T19).

Without going through the derivation, it is easy to justify (T1). First, we notice that it implies that the expenditure elasticity of demand for each good is 1. This follows from the homotheticity of the utility function (3.4) - (3.5). Second, at any fixed level of real expenditure (c^R), (T1) implies that the demand for good i from either source is independent of the price of good j from either source for all

$j \neq i$. This follows from the adoption of the Leontief form in (3.4). Finally, (T1) implies that the elasticity of substitution $\left(\sigma_i^{(3)}\right)$ between alternative sources of good i is unity, i.e.,

$$(3.6) \qquad \sigma_i^{(3)} \equiv \frac{\eta_{(is)(it)}}{S_{(it)}^{(3)}} = 1 \ , \qquad \text{for } s \neq t,$$

where $S_{(it)}^{(3)}$ is the share of good i from source t in household expenditure on good i, and $\eta_{(is)(it)}$ is the compensated (fixed real expenditure) elasticity of demand for good i from source s with respect to changes in the price of good i from source t. A unitary substitution elasticity would be expected in view of the Cobb-Douglas assumption, (3.5).

The second set of final-demand equations in MO81 describe inputs to capital creation (buildings, plant and equipment). We assume that for each industry j, $j=1,\ldots,h$, these inputs

$$X_{(is)j}^{(2)} \ , \qquad i=1,\ldots,g, \quad s=1,2,$$

are chosen to minimize

$$(3.7) \qquad \sum_{i=1}^{g} \sum_{s=1}^{2} P_{(is)j}^{(2)} \, X_{(is)j}^{(2)} \ ,$$

subject to

$$(3.8) \qquad Y_j = \min_{i=1,\ldots,g} \left\{ \frac{X_{(i\cdot)j}^{(2)}}{A_{(i\cdot)j}^{(2)}} \right\} \ ,$$

and

$$(3.9) \qquad X_{(i\cdot)j}^{(2)} = \text{Cobb-Douglas}\left\{ X_{(i1)j}^{(2)} \ , \ X_{(i2)j}^{(2)} \right\} \ , \qquad \text{for } i=1,\ldots,g,$$

where $A^{(2)}_{(i\cdot)j}$ is a positive parameter, $X^{(2)}_{(is)j}$ is the quantity of good i from source s used in creating capital for industry j, $P^{(2)}_{(is)j}$ is the price to industry j of its inputs of (is) used for capital creation, $X^{(2)}_{(i\cdot)j}$ is the "effective" input of good i to industry j for capital creation and Y_j is the number of units of capital created for industry j. Thus, whatever the value of Y_j, industry j is assumed to choose its inputs to minimize the costs of creating Y_j units of capital. The technology for creating capital is described by (3.8) and (3.9). Primary factors do not appear in either of these equations. The use of primary factors in creating buildings, etc., is accounted for via the inputs from the construction industry. Equations (3.8) and (3.9) are close to those adopted in ORANI. However, in ORANI, (3.9) is generalized to the CES form.

On solving problem (3.7) - (3.9), we obtain demand functions of the form

$$(3.10) \qquad X^{(2)}_{(is)j} = f^{(2)}_{(is)j}\left(P^{(2)}_{(11)j},\ldots,P^{(2)}_{(g1)j}\ ;\ P^{(2)}_{(12)j},\ldots,P^{(2)}_{(g2)j}\ ;\ Y_j\right),$$

$$\text{for } i=1,\ldots,g,\ \ s=1,2 \text{ and } j=1,\ldots,h.$$

Under the specification (3.8) - (3.9), the percentage-change form of (3.10) reduces to (T2).

The third set of final demand equations in Table 1 describe demands for exports. Export demand functions in MO81 (and ORANI) take the form

$$(3.11) \qquad P^{*}_{(i1)} = \left(X^{(4)}_{(i1)}\right)^{-\gamma_i} F^{(4)}_{(i1)}, \qquad i=1,\ldots,g,$$

where $P^{*}_{(i1)}$ is the foreign currency receipt per unit of export of good i,

$X^{(4)}_{(i1)}$ is the volume of exports of good i and $F^{(4)}_{(i1)}$ is an export-demand shift variable. $F^{(4)}_{(i1)}$ will increase if there is an increase in foreign demand. The non-negative parameter γ_i is the reciprocal of the foreign elasticity of demand for good (i1). The percentage-change form of (3.11) is (T3).

No satisfactory econometric estimates are available for export demand elasticities $(1/\gamma_i)$ for Australian products. In most ORANI computations, numbers ranging from 1.3 to 20 have been used with the particular number chosen for each commodity depending on Australia's share in the relevant world market and on views about demand elasticities in importing countries. (Details are in DPSV, section 29(f).)

Export demand functions for all g domestic commodities are specified in MO81 (and ORANI). For commodities that are not exported, $X^{(4)}_{(i1)}$ can be fixed exogenously at an appropriately small level. This procedure is computationally more convenient than including export demand equations for some commodities and not for others.

II Industry inputs and outputs

Each industry in MO81 is assumed to choose its inputs to minimize the cost of achieving its activity or overall output level. The cost minimization problems produce functions explaining demands for intermediate and primary factor inputs in terms of industry activity levels and input prices. Equation (T4) arises from assuming that industry j chooses its inputs of domestically produced and imported materials, $X^{(1)}_{(is)j}$, i=1,...,g, s=1,2, its input of labor, $X^{(1)}_{(g+1,1)j}$, and its input of capital, $X^{(1)}_{(g+1,2)j}$, to minimize

$$(3.12) \qquad \sum_{i=1}^{g+1} \sum_{s=1}^{2} P^{(1)}_{(is)j} \; X^{(1)}_{(is)j} \quad ,$$

subject to

$$(3.13) \qquad Z_j = \min_{i=1,\ldots,g+1} \left\{ \frac{X^{(1)}_{(i\cdot)j}}{A^{(1)}_{(i\cdot)j}} \right\} ,$$

and

$$(3.14) \qquad X^{(1)}_{(i\cdot)j} = \text{Cobb-Douglas}\left\{ X^{(1)}_{(i1)j} \; , \; X^{(1)}_{(i2)j} \right\} \quad , \qquad i=1,\ldots,g+1,$$

where the $A^{(1)}_{(i\cdot)j}$ are positive parameters; $P^{(1)}_{(is)j}$, for i=1,...,g, s=1,2, is the price to industry j of good i from source s to be used as an intermediate input; $P^{(1)}_{(g+1,s)j}$ is the price to industry j of primary factor (denoted by subscript g+1) of type s (s=1 for labor and s=2 for capital) and Z_j is the activity level in industry j.

To explain the commodity composition of each indutry's output, we solve a revenue maximization problem. Equation (T5) is derived via the problem of choosing

$$X^{(0)}_{(i1)j} \; , \qquad i=1,\ldots,g \qquad \text{(j's commodity outputs)},$$

to maximize revenue,

$$(3.15) \qquad \sum_{i=1}^{g} P^{(0)}_{(i1)} \; X^{(0)}_{(i1)j} \quad ,$$

subject to the transformation constraint

$$(3.16) \qquad Z_j = \left[\sum_{i=1}^{g} \left\{ X^{(0)}_{(i1)j} \right\}^2 \; A^{(0)}_{(i1)j} \right]^{1/2} \quad ,$$

where the $A^{(0)}_{(i1)j}$ are positive parameters and $P^{(0)}_{(i1)}$ is the basic price for domestically produced good i. (This price carries no j subscript because it is assumed in MO81 (and ORANI) that all producers of good (i1) receive the same price.)

Thus, each industry is modelled as if it makes two separate sets of decisions. First, it chooses a vector of inputs to minimize the cost of giving itself a particular production possibilities set. Then, from that set, it chooses the revenue maximizing commodity output combination. (The situation is illustrated in Figure 1 for the two-input two-output case.) The separation of the input and output problems implies that inputs are not specific to products. This simplification is suitable where the principal inputs are of a general nature, e.g. labor, tractors and fertilizer in an agricultural industry.

In ORANI, the cost minimizing problems are more elaborate than (3.12) - (3.14). Agricultural land is included as a primary factor and labor is subdivided into 9 skill categories. The CES function and Hanoch's (1971) CRESH function appear instead of the Cobb-Douglas functions in (3.14). However, the Leontief form is retained in the ORANI equations corresponding to (3.13). Efforts were made to estimate substitution elasticities between capital and labor in general and between labor skill categories. These efforts are reported in DPSV, sections 29(b) and 29(c). While some success was achieved in estimating averages over all industries for the various substitution elasticities, no satisfactory results were obtained for individual industries.

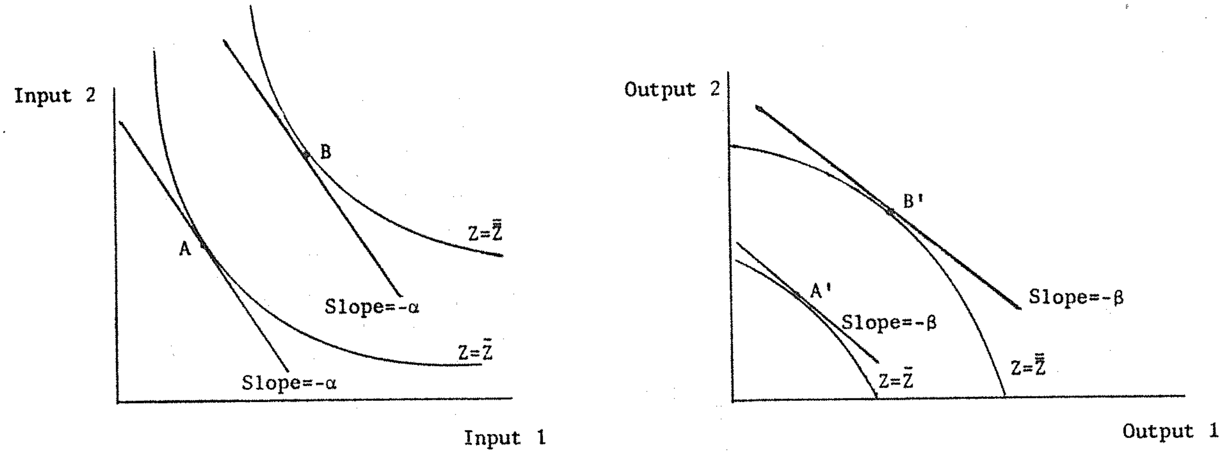

Figure 1 : Isoquants and Transformation Frontiers

If the ratios of input and output prices are α and β respectively and $Z = \bar{Z}$, then the input combination will be at A and the output combination will be at A'. If Z is set at the higher value, $\bar{\bar{Z}}$, then the input and output combinations move to B and B'.

Revenue maximization problems are specified in ORANI only for the agricultural industries. Non-agricultural commodities are modelled as if they are produced by single product industries. The generalization to multiproduct production functions was considered worthwhile in modelling the agricultural sector because most Australian farms are multi-product enterprises and Australian farmers frequently shift their output mixes between wheat, wool, cattle and other products in response to changes in product prices. Estimates of various pairwise product transformation elasticities are available for three Australian agricultural industries defined on a regional basis : high-rainfall zone farming, pastoral zone farming and wheat-sheep zone farming. Details of these estimates and of the CRETH functional form used to specify the transformation frontiers are in DPSV, sections 12 and 29(d), and Vincent, Dixon and Powell (1980).

III Zero pure profit conditions

The zero pure profit conditions, (T6), in our miniature version of ORANI equate the percentage changes in revenue per unit of activity in each industry to the percentage change in costs per unit of activity. Because the production functions, (3.13) - (3.14) and (3.16), exhibit constant returns to scale, no quantity variables appear in (T6). Revenue and costs per unit of activity are functions of output and input prices alone. Thus, percentage changes in revenue and costs per unit of activity are appropriately weighted averages of percentage changes in output and input prices.

Equations (T7) and (T8) are derived from

$$P^*_{(i2)}\ T_i\ \Phi = P^{(0)}_{(i2)}\ , \qquad i=1,\ldots,g, \tag{3.17}$$

and

$$P^*_{(i1)}\ V_i\ \Phi = P^{(4)}_{(i1)}\ , \qquad i=1,\ldots,g. \tag{3.18}$$

Equation (3.17) relates the basic price of imported product i $\left(P^{(0)}_{(i2)}\right)$ to the costs of importing, i.e., the foreign currency cost $\left(P^*_{(i2)}\right)$ converted by the exchange rate (Φ) and inflated by the power of the tariff (T_i).[4] Similarily, equation (3.18) relates the costs of exporting domestic commodity i (its at port price $\left(P^{(4)}_{(i1)}\right)$) to the receipt from exporting, i.e., the foreign currency price $\left(P^*_{(i1)}\right)$ converted by the exchange rate and inflated by the power of the export subsidy (V_i). In the case of non-exportables, the export subsidy can be treated as an endogenous variable. Then the relevant component of (3.18) simply defines the value of the export subsidy, a variable which may be meaningless but which is also harmless. Thus (3.18) has no unfortunate implications for the way in which the model determines domestic prices of non-exportables.

The fourth zero-pure-profit condition in Table 1, (T9), equates the percentage change in the price of a unit of capital for industry j with the percentage change in the cost of constructing such a unit. The technology for producing capital for industry j exhibits constant returns to scale (see (3.8) - (3.9)). Consequently, the percentage change in the cost of a unit of capital for industry j is simply a weighted average of the percentage changes in the relevant input prices where the weights are cost shares.

The final two zero-pure-profit conditions, (T10) and (T11), provide the link between purchasers' prices and basic prices. In MO81 there is no allowance for transport, wholesale, retail or other margins costs. Thus, in MO81 the absence of pure profits in the distribution of goods implies that purchasers' prices are equal to basic prices. In ORANI there is a detailed treatment of margins. Changes in purchasers' prices are modelled as reflecting not only changes in basic prices but also changes in the prices of margin services and changes in sales taxes.

VI Market clearing

Condition (T12) equates percentage changes in demands and supplies for domestically produced goods. The percentage change in the supply of domestically produced good q is a weighted average of the percentage changes in the supplies of this good by each industry. Imports are not added to domestic production in determining total supplies. This is because commodities from foreign and domestic sources are treated as distinct. The percentage change in the demand for domestically produced good q is a weighted average of the percentage changes in demands by intermediate users, capital creators, households and foreigners.

Equations (T13) and (T14) are the percentage-change forms of the equations

$$\sum_{j=1}^{h} X^{(1)}_{(g+1,1)j} = L \quad , \tag{3.19}$$

and

$$X_{(g+1,2)j} = K_j \quad , \qquad j=1,\dots,h. \tag{3.20}$$

The left hand sides of (3.19) and (3.20) are the economy-wide demand for labor and the demand for capital by industry j. Capital is assumed to be industry specific. Thus demands for capital are not aggregated across industries. On the right hand sides of (3.19) and (3.20) are the employment level for labor and the employment of capital of type j. Thus we are assuming that demands for factors are satisfied. This does not mean that we are necessarily assuming full employment. Although we could set L and K_j exogenously at full-employment levels, an obvious alternative is to set some factor prices exogenously and to let the model determine the corresponding factor employment levels. Under this latter specification our assumption would be that some factor markets are slack, i.e., supply constraints play no role in determining employment levels for some factors.

V <u>Miscellaneous equations</u>

Equations (T15) - (T18), (T27) and (T28) are concerned with trade aggregates. (T15) gives the percentage change in the imports of each commodity as a weighted average of the percentage changes in demands by intermediate users, capital creators and households. (T16) and (T17) define the percentage changes in the foreign currency values of aggregate imports and exports. (T18) defines the change in the balance of trade. The change is used rather than the percentage change because the balance of trade is a variable whose value can have either sign. There are obvious difficulties in using percentage changes for variables whose values can pass through zero. Equations (T27) and (T28) are derived from the equations

$$(3.21) \qquad T = \sum_{i=1}^{g} (T_i - 1) P^*_{(i2)} \Phi X^{(0)}_{(i2)} ,$$

and

$$(3.22) \qquad V = \sum_{i=1}^{g} (V_i - 1) P^*_{(i1)} \Phi X^{(4)}_{(i1)} ,$$

where T is the aggregate collection of tariff revenue and V is the aggregate payment of export subsidies.

Equations (T19) - (T23) and (T26) define various macro indices. In (T19) and (T22), the percentage changes in the consumer price index and the investment-goods price index are defined as weighted averages of percentage changes in commodity prices and the prices of units of capital. (T20) and (T21) define the percentage changes in real consumption expenditure and real investment expenditure and (T23) defines the percentage change in the ratio of these two variables. (T26) provides an index of the size of the aggregate capital stock. It defines the percentage change in the economy's capital stock as a weighted average of the percentage changes in the capital stocks of the industries.

Equations (T24) and (T25) tie down the percentage changes in investment by industry, y_j, $j=1,\ldots,h$. (T24), which gives the percentage changes in the rates of return on capital in each industry, is derived from the equation

$$(3.23) \qquad R_j = \frac{P^{(1)}_{(g+1,2)j}}{\Pi_j} - D_j , \qquad j=1,\ldots,h,$$

where R_j is the rate of return on capital in industry j, $P^{(1)}_{(g+1,2)j}/\Pi_j$ is the ratio of the rental price to the purchase price of a unit of capital in industry j and D_j is the rate of depreciation. The D_j's are assumed to be technologically fixed.

There are a variety of theories about investment which are consistent with (T25). One such theory starts with the assumption that investors are cautious. They behave as if they expect that expansions in the capital stock of any industry will lower the rate of return on the industry's capital. More particularly, investors in industry j behave as if their expectations in the solution year concerning the response of their rate of return in the following year (R_j^+) to changes in the size of the industry's capital stock are described by the equation

$$R_j^+ = R_j\left(\frac{K_j^+}{K_j}\right)^{-\beta_j} , \tag{3.24}$$

where R_j is the rate of return in the solution year, K_j is the capital stock in the solution year and K_j^+ is the "planned" capital stock, i.e., the level planned in the solution year for the following year. β_j is a positive parameter. If the planned capital stock is the same as the current capital stock, then (3.24) implies that the rate of return expected for the future is the current rate of return. If, however, investment plans are set so that K_j^+/K_j will reach A (see Figure 2), then investors will behave as if they expect the rate of return to fall to B.

Next, we assume that investment plans are set as if they are intended to equate expected rates of return across industries, i.e., there exists a number Ω such that

$$R_j\left(\frac{K_j^+}{K_j}\right)^{-\beta_j} = \Omega , \quad j=1,\dots,h. \tag{3.25}$$

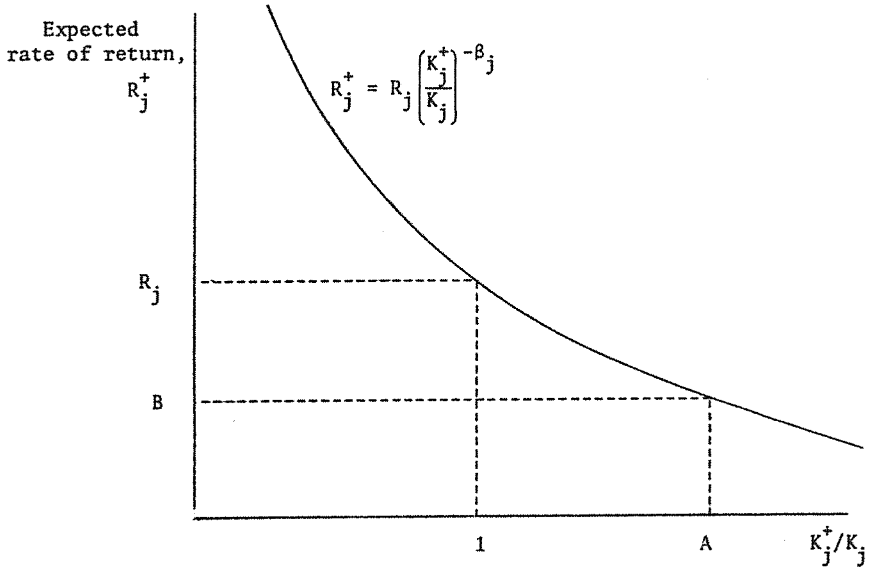

Figure 2 : Expected Rate-of-Return Schedule for Industry j

Finally, we assume that

$$K_j^+ = K_j(1 - D_j) + Y_j \;, \qquad j=1,\ldots,h. \tag{3.26}$$

That is, planned capital stock is the sum of current capital stock (appropriately depreciated) and current investment.

In percentage-change form, (3.25) and (3.26) become

$$r_j - \beta_j(k_j^+ - k_j) = \omega \;, \qquad j=1,\ldots,h \;, \tag{3.27}$$

and

(3.28) $$k_j^+ = k_j(1 - \Delta_j) + y_j \Delta_j \ , \qquad j=1,\ldots,h,$$

where

$$\Delta_j = Y_j/K_j^+ \ , \qquad j=1,\ldots,h.$$

On using (3.28) to eliminate k_j^+ from (3.27) we obtain

(3.29) $$y_j = k_j + B_j(r_j - \omega) \ , \qquad j=1,\ldots,h \ ,$$

where [5]

(3.30) $$B_j = \frac{1}{\beta_j \Delta_j} \ , \qquad j=1,\ldots,h.$$

In (T25) we have shown (3.29) with two extra coefficients, $I_j^{(1)}$ and $I_j^{(2)}$, and an extra variable $f_j^{(2)}$. If we set all the $I_j^{(1)}$'s and $I_j^{(2)}$'s at one and the $f_j^{(2)}$'s at zero, then (T25) reduces to (3.29). However, (T25) allows other possibilities. For some industries, the investment theory set out in equations (3.24) - (3.26) might not be considered appropriate, e.g.,in government-dominated industries investment might be considered to be independent of changes in rates of return. In such cases, $I_j^{(1)}$ and $I_j^{(2)}$ can be set at zero and the percentage change in investment in industry j can be determined by the exogenously chosen value for $f_j^{(2)}$. Another possibility is to set just $I_j^{(2)}$ at zero. For example, we might wish to specify investment by industry j as

(3.31) $$Y_j = (H_j + D_j)K_j \ , \qquad j=1,\ldots,h,$$

where H_j, the rate of growth of capital in industry j in the solution year, is treated as a constant. This specification is popular in long-run planning models. With H_j and D_j held constant, (3.31) reduces to

(3.32) $$y_j = k_j \quad , \qquad j=1,\ldots,h.$$

(3.32) can be accommodated in (T25) by setting $I_j^{(1)}$ at one, and $I_j^{(2)}$ and the variable $f_j^{(2)}$ at zero.

Rather than treating the H_j's as constants, we could assume that

(3.33) $$K_j = K_j(0)(1 + H_j)^{\tau} \quad ,$$

that is,

(3.34) $$H_j = \left(\frac{K_j}{K_j(0)}\right)^{1/\tau} - 1 \quad ,$$

where $K_j(0)$ is the capital stock in industry j in the base year, the base year being τ years before the solution year. Under (3.31) and (3.34) we are assuming that growth rates in the solution year are the same as the average growth rates established between the base year and the solution year. With the H_j's treated as variables, the percentage-change form of (3.31) is

(3.35) $$y_j = h_j \frac{H_j}{H_j + D_j} + k_j \quad , \qquad j=1,\ldots,h.$$

Equation (3.34) implies that

(3.36) $$h_j = \frac{1}{\tau} \frac{H_j + 1}{H_j} k_j \quad , \qquad j=1,\ldots,h.$$

Substitution of (3.36) into (3.35) gives

(3.37) $$y_j = \left(\frac{H_j + 1}{\tau(H_j + D_j)} + 1\right) k_j \quad , \qquad j=1,\ldots,h.$$

To accommodate (3.37) by (T25), we set

$$(3.38) \qquad I_j^{(1)} = \left(\frac{H_j + 1}{\tau(H_j + D_j)} + 1\right) , \qquad j=1,\ldots,h.$$

Again, $I_j^{(2)}$ and $f_j^{(2)}$ are set at zero for all j.

The final pair of equations for discussion in this section is (T29) and (T30). Equation (T29) allows flexibility in the treatment of wages. If we fix the coefficient $I_{(g+1,1)j}$ at one for all j and the variable $f_{(g+1,1)j}$ at zero, then $f_{(g+1,1)}$ is the percentage change in real wages in each industry. Variations in relative wages across industries can be introduced by using non-uniform values for the $f_{(g+1,1)j}$, $j=1,\ldots,h$. The effects of policy changes and other economic shocks in an environment of partial wage indexation can be studied by setting the $I_{(g+1,1)j}$ at values different from one. Equation (T30) is useful if we wish to assume that all rates of return move by the same endogenously determined percentage. We simply set f_j^r exogenously at zero for all j and treat r as an exogenous variable. Exogenously determined changes in the relative rates of return can be handled by assigning different values to the f_j^r for different j. If the f_j^r are treated as endogenous variables, then (T30) has no influence on results from the model apart from the values for the f_j^r's themselves.

3.3 Closure options in MO81 restricted to categories I - V

In this subsection we consider the closure options available in the restricted MO81 model formed by equations (T1) - (T30), i.e., categories I - V in Table 1. This restricted model has 9gh+11g+9h+12 equations and 9gh+15g+12h+17 variables (see Tables 1 and 2). It can be closed by treating 4g+3h+5 variables as exogenous. Columns 1 and 2 of Table 3 give two lists of exogenous variables which could be used in the restricted model. The first list is suitable for studies of the short-run effects of tariff changes, etc., and the second is suitable for long-run studies. Other choices of exogenous variables are available for both short- and long-run simulations.

The first two variables, $p^*_{(i2)}$ and $f^{(4)}_{(i1)}$, in columns 1 and 2 of Table 3 are the percentage changes in the foreign currency prices of imports and the percentage shifts in the export demand curves. MO81, in common with ORANI, contains no equations describing either the conditions of supply of foreign products or the positions of foreign demand curves for domestic products. It is difficult, therefore, to imagine a plausible MO81 experiment in which either $p^*_{(i2)}$ or $f^{(4)}_{(i1)}$ is treated as endogenous. By placing $p^*_{(i2)}$ in the exogenous category we are adopting the small country assumption on the import side, i.e., the prices of imports are independent of local demands. Because Australia is a major supplier of wool, wheat, sugar and various mineral products, the small country assumption is not appropriate on the export side. Consequently, it is only the positions of the foreign demand curves which are exogenized. With $p^{*}_{(i2)}$ and $f^{(4)}_{(i1)}$ exogenous, we are allowing for the computation of answers to questions of the form : what were (or will be) the effects of past (or projected) changes in foreign import supply prices and foreign export demands.

The third group of exogenous variables in columns 1 and 2 of Table 3 are the tariffs. In most ORANI computations, the tariffs have been exogenous. It is possible, however, to conduct experiments in which some or all the tariffs are endogenous. For example, we might wish to compute the level of protection which would be required to maintain current employment levels in footwear, say, in the face of exogenously given movements in foreign prices, domestic wages and the exchange rate. For such a computation, footwear employment would replace the footwear tariff on the exogenous list.

The fourth set of exogenous variables is a selection of export subsidies and export levels. In most ORANI computations, the set G contains the labels of Australia's major export commodities. For these commodities, the model is allowed to explain export levels and foreign currency export prices are allowed to play a dominant role in determining domestic prices. Notice that if v_i is exogenous, then $p^{(4)}_{(i1)}$ will fully reflect movements in $p^{*}_{(i1)}$ via (T8). For commodities not included in G, the export volume is exogenous and the foreign currency export price does not influence the domestic price. As we noted earlier, if v_i is endogenous, then the only role of the i^{th} component of (T8) is in the determination of the value of v_i. Non-exportables and minor export commodities are usually excluded from G.

The fifth set of variables in columns 1 and 2 of Table 3 are capital stocks (column 1) and rates of return (column 2). For short-run analysis, where the solution year is only one or two years ahead, the effects of tariff changes, etc., on capital stocks are ignored. It is assumed that the shock (e.g., tariff change) under investigation can affect the allocation of investment across industries but that insufficient time passes for capital stocks to reach levels noticeably different from those they would have reached in the absence of the shock. For long-run analysis, where the solution year is say ten years ahead, it assumed that rates of return are unaffected by the shock under investigation. The assumption is that in the long run, industries which are favoured by the shock will grow relative to other industries until rates of return are restored to their historic norms. Thus, we are assuming that tariff changes, etc., affect rates of return in the short run and capital stocks in the long run.

The sixth and seventh sets of exogenous variables concern wages and aggregate employment. We will assume that the coefficient $I_{(g+1,1)j}$ appearing in (T29) is unity for all j. Thus, the percentage change in real wages payable in industry j is $\left(f_{(g+1,1)j} + f_{(g+1,1)}\right)$. In the short run, movements in real wages in Australia tend to be dominated by political pressures operating through and around a central wage-setting tribunal (The Conciliation and Arbitration Commission). Thus, in analysing the short-run effects of economic changes, it is realistic to treat $f_{(g+1,1)j}$ and $f_{(g+1,1)}$ as exogenous variables. In the long run, however, real wages have adjusted in a way which is consistent with the maintenance of approximately full employment. Where a long-run focus is adopted, it is appropriate to assume that changes in the aggregate level of employment are determined by demographic factors and are independent of tariff changes, etc.. That is, it is appropriate to treat ℓ as an exogenous variable and to allow the overall level of real wages to adjust to shocks via the endogenous treatment of $f_{(g+1,1)}$. Even in long-run simulations, the $f_{(g+1,1)j}$ would usually be treated as exogenous in models such as MO81 and ORANI. These models have no equations explaining industry-specific wage rates. MO81 has just one type of labor which is free to move between industries. In standard applications, ORANI recognizes 9 types of labor. These labor types are distinguished on an occupational (not an industrial) basis.

The eighth set of variables in columns 1 and 2 of Table 3 are the shift terms $\left\{f_j^{(2)}\right\}$ which appear in the investment equations, (T25). Their role was explained following (3.30) in subsection 3.2.

The next two variables listed in column 1 of Table 3 are the percentage changes in real aggregate household expenditure and real aggregate investment, c^R and y^R.[6] By including these in column 1 we are setting an economic environment in which real aggregate demand is controllable independently of other variables appearing in the column. The underlying assumption is that policy makers have available macro instruments, not included in our model, by which they can influence c^R and y^R, at least in the short run. Another option is to set exogenously the change in the balance of trade, ΔB, and the percentage change in the ratio of consumption to investment f_R. Then our model would indicate the change in real domestic absorption which would need to accompany a tariff cut, say, in order to maintain a target level for the balance of trade.

Neither of these options (exogenizing c^R and y^R or ΔB and f_R) is satisfactory for long-run analysis. In the absence of a better alternative, the second option is shown in column 2 of Table 3 as the standard long-run macro closure for the restricted MO81 model. (This is also the standard closure used in long-run ORANI simulations.) The problem is that for long-run simulations it is desirable to treat all the variables c^R, y^R, ΔB and f_R as endogenous. For example, our model might indicate that a tariff reform in 1981 would cause a 10 per cent increase in the amount of capital in 1990 consistent with earning the exogenously given rates of return, R_j, $j=1,\ldots,h$. That is, we might obtain the result

$$k = 10 , \tag{3.39}$$

where k is the percentage change in the aggregate capital stock defined in equation (T26). The effect of the tariff reform on consumption, investment and the balance of trade would then depend on (among other things) the way in which the 10 per cent extra expansion in the capital stock over the period 1981-1990 was financed. If the extra capital is owned mainly by foreign investors, then we could expect the income accruing from it in 1990 to be reflected in an increased balance of trade surplus to finance additional profit repatriation. On the other hand, if the extra capital were domestically owned, then the income accruing from it in 1990 would increase domestic absorption (c^R and y^R). In ORANI and the restricted version of MO81, there is no explicit consideration of domestic and foreign ownerhsip. In subsection 3.4 we consider a way of rectifying this deficiency.

The second last variable in columns 1 and 2 of Table 3 is r. In Table 1, r appears only in equation (T30). As explained in our discussion of this equation in subsection 3.2, r will normally be set exogenously. The exception is when we wish to exogenize relative rates of return across industries without exogenizing absolute rates of return. Then we can exclude r and r_j, $j=1,\ldots,h$, from the list of exogenous variables and include k (the percentage change in the aggregate capital stock) and f_j^r, $j=1,\ldots,h$ (the shift terms in (T30)). When k and the f_j^r's replace r and the r_j's as exogenous variables, we are assuming that tariff changes and other shocks have no effect on long-run capital accumulation or on relative rates of return, but that they can affect the average rate of return across all industries. The exogenous treatment of k is attractive in ORANI and the restricted version of MO81 when we wish to draw conclusions about efficiency and gains and losses to the domestic economy. For example, if we were interested in the welfare effects of a tariff reform, then we might set

$$k = \ell = 0$$

and compute the effects of the tariff change on the level of production arising from a fixed level of resources. The difficulty with letting k

move is that ORANI and the restricted version of MO81 give no indication of capital ownership. Hence, they give no indication of how a changed level of capitalist income would be allocated between domestic residents and foreigners.

The last variable in columns 1 and 2 of Table 3 is the percentage change in the exchange rate, ϕ. It acts as a numeraire, i.e., it determines the percentage change in the absolute price level. Natural alternatives to ϕ as the numeraire include the average wage rate and the consumer price index.

3.4 Macro equations in MO81, category VI

For a small open economy such as Australia's, a reasonable assumption in long-run analysis is that rates of return on capital are exogenous. A tariff reform, for example, might be expected to affect rates of return in the short run. But for the long run, we can assume that adjustments in the level of foreign investment will force domestic rates of return into line with foreign rates of return.

There is no difficulty in exogenizing rates of return in ORANI and allowing capital stocks to adjust. However, as we have seen in the previous subsection, there is a problem in determining the implied levels of foreign ownership of domestic capital. Because ORANI, as presently constituted, includes no domestic savings function, the model provides no basis for projecting the share of capital accumulation which is financed from domestic sources. In this section we show how MO81 can be expanded to accommodate domestic savings, and domestic and foreign ownership. A similar expansion of ORANI is planned as part of our future research.

The first step is to add a consumption function. We assume that in the solution year (1990, say) consumption is related to domestic income by

$$(3.40) \quad C = F_c \left[\sum_{j=1}^{h} P^{(1)}_{(g+1,1)j} \; X^{(1)}_{(g+1,1)j} + Q\left(\sum_{j=1}^{h} P^{(1)}_{(g+1,2)j} \; X^{(1)}_{(g+1,2)j} \right) + T - V \right].$$

F_c is the average propensity to consume. The first term inside the square brackets is total labor income. Q is the domestic share in the ownership of capital. Thus, the second term in the square brackets is the capital income accruing to domestic residents. T and V are tariff revenue and export subsidies. In the context of MO81, the sum of the terms in the square brackets is the total income of domestic residents.

In percentage-change form, (3.40) becomes (T31), in Table 1 where the ψ_i's are the ratios of labor income, tariff revenue, export subsidies and domestic capitalist income to total domestic income. $N_{(g+1,1)j}$ and $N_{(g+1,2)j}$ are industry j's shares in total labor and capital income.

Equation (T31) introduces two variables not appearing in the earlier equations in Table 1 : f_c, the percentage change in the average propensity to consume and q, the percentage change in the domestic share of capitalist income. In long-run simulations, it is reasonable to set

f_c exogenously at zero. To tie down q, we require some additional equations.

We assume that

$$(3.41) \qquad Q(\tau)\left[\sum_{j=1}^{h} \Pi_j(\tau)\ K_j(\tau)\right] = Q(0)\left[\sum_{j=1}^{h} \Pi_j(\tau)\ K_j(0)(1 - D_j)^{\tau}\right] + S^* ,$$

where

$$(3.42) \qquad S^* = \sum_{t=0}^{\tau-1}\left[\frac{S(t)}{\sum_{j=1}^{h} \Pi_j(t)\ Y_j(t)}\right]\left[\sum_{j=1}^{h} \Pi_j(\tau)\ Y_j(t)(1 - D_j)^{\tau-t-1}\right] .$$

The first point to note about these two equations is that we have added explicit time arguments to the variables. Time τ is the solution year, 1990 say, and time 0 is the base year, 1981 say. The base year can usually be interpreted as the year in which the tariff reform or other shock under investigation is introduced. The τ arguments are not strictly necessary in (3.41) and (3.42). Our convention until now has been to represent variables for the solution year without time arguments. However, expressions containing variables from several different points of time are probably clearer if we include all the time arguments.

Equation (3.41) says that the value of the domestically owned capital stock in the solution year (year τ) equals the depreciated value of the domestically owned base year capital stock plus S^*, the depreciated value of domestically owned capital acquired between the base year and the solution year. In more detail, the left hand side of (3.41) is the value[7] of the capital stock in the solution year, $\Sigma_j\ \Pi_j(\tau)\ K_j(\tau)$, multiplied by the domestic share, $Q(\tau)$. The first term on the right hand side is the value in the solution year of what remains of the base period capital stock $\left(\Sigma_j \Pi_j(\tau) K_j(0)(1 - D_j)^{\tau}\right)$ multiplied by the share $\left(Q(0)\right)$ which is domestically owned. (D_j is the annual rate of depreciation of the capital used by industry j.) The second term on the right hand side of (3.41), S^*, is defined by (3.42). In (3.42) we assume that the amount of capital installed for industry j at time $t + 1$ is $Y_j(t)$, i.e., the investment at time t. Of this, the amount remaining by the solution year is $Y_j(t)(1 - D_j)^{\tau-t-1}$. Thus, the total value in the solution year of capital installed as a result of investment at time t is $\Sigma_j \Pi_j(\tau) Y_j(t)(1 - D_j)^{\tau-t-1}$. The share of this capital which is domestically owned is assumed to be the ratio for time t of domestic savings, $S(t)$, to investment, $\Sigma_j \Pi_j(t) Y_j(t)$. For Australia, it is realistic to suppose that the savings-investment ratio will remain below unity over at least the next 10 years. However, (3.42) is interpretable even when savings exceeds investment in some periods. For those periods, (3.42) implies that domestic investors buy some existing units of capital from foreign owners.

For the purposes of our miniature model, it is legitimate to simplify (3.41) and (3.42) by assuming that

(3.43) $$\Pi_j(t) = \Pi(t) , \quad \text{for } j=1,\dots,h \text{ and } t=0,\dots,\tau ,$$

and

(3.44) $$D_j = D , \quad \text{for } j=1,\dots,h.$$

(3.43) and (3.44) imply that the costs of constructing units of capital and depreciation rates are identical across industries. These assumptions would be justified if the technologies for constructing capital, (3.8) - (3.9), were identical for all j. This is, in fact, assumed in our implementation of MO81, see p.63. Under (3.43) and (3.44), equations (3.41) and (3.42) may be combined and written as

$$(3.45)\qquad Q(\tau)\left[\Sigma_j\ K_j(\tau)\right] = Q(0)\left[\Sigma_j\ K_j(0)\right](1 - D)^{\tau} + \sum_{t=0}^{\tau-1}\left[S(t)/\Pi(t)\right](1 - D)^{\tau-t-1}\ .$$

To make use of (3.45), we must provide an explanation for the stream of real savings $\left[S(t)/\Pi(t),\quad t=1,\ldots,\tau-1\right]$ over the period between the base year and the solution year. We assume that real savings grow exponentially, i.e.,

$$(3.46)\qquad \frac{S(t)}{\Pi(t)} = \frac{S(0)}{\Pi(0)}(1 + U)^t\ ,\qquad t=1,\ldots,\tau-1,$$

where

$$(3.47)\qquad U = \left[\frac{S(\tau)/\Pi(\tau)}{S(0)/\Pi(0)}\right]^{1/\tau} - 1\ .$$

Then on substituting from (3.46) into (3.45) we obtain

$$(3.48)\ Q(\tau)\ K(\tau) = Q(0)\ K(0)(1 - D)^{\tau} + \frac{S(0)}{\Pi(0)}\left[\frac{(1 + U)^{\tau} - (1 - D)^{\tau}}{D + U}\right]\ ,$$

where we have denoted the economy's aggregate capital stocks at times τ and 0 by $K(\tau)$ and $K(0)$. We treat $K(0)$, $S(0)$, $\Pi(0)$, $Q(0)$ and D as constants. Thus, in linear percentage-change form, (3.47) and (3.48) reduce to (T32) and (T33) where the coefficient in (T33) is

$$(3.49)\quad \Gamma = \left[\left(\frac{U\tau}{1+U}\right)\left(\frac{(1+U)^{\tau}}{(1+U)^{\tau} - (1-D)^{\tau}}\right) - \frac{U}{D+U}\right]\left(\frac{Q(\tau)\ K(\tau) - Q(0)\ K(0)(1-D)^{\tau}}{Q(\tau)\ K(\tau)}\right).$$

Equation (T34) in Table 1 relates percentage changes in consumption to percentage changes in savings. It is the percentage-change form of

$$(3.50)\qquad S = \left(\frac{1 - F_c}{F_c}\right) C ,$$

where (3.50) follows from (3.40).

3.5 Closure of MO81 when category VI is included

Category VI in Table 1 adds four equations and four variables to the model set out in categories I - V. The four new variables are f_c, the percentage change in the average propensity to consume; u, the percentage change in the growth rate of real domestic savings over the period from the base year to the solution year, i.e., year 0 to year τ; s, the percentage change in savings in year τ; and q, the percentage change in the domestic share in the ownership of the capital stock. Because we have added equal numbers of equations and variables, the closure options discussed in subsection 3.3 are still available. Results from the restricted model can be regarded as coming from the complete model with f_c, u, s and q treated endogenously. If these variables are endogenous, then values for the remaining endogenous variables can be computed without reference to the category VI equations.

The main reason for adding the category VI equations is to allow consumption (c^R), investment (y^R), the balance of trade (ΔB) and the consumption-investment ratio (f_R) all to be endogenous in a long-run simulation. This can be done by selecting the exogenous variables shown in column 3 of Table 3. Compared with column 2, (the preferred long-run closure in the restricted model), in column 3 we have replaced f_R and ΔB with f_c and ω.

By exogenizing f_c, we are linking consumption to domestic income. This is possible in the expanded MO81 model because we have added the consumption function (T31) and sufficient equations to explain domestic income. In particular, we have added sufficient equations to explain the domestic share in capital income. By exogenizing ω, we are allowing (T25) to explain not only the allocation of investment across industries but also total investment.

4. Implementation of MO81

4.1 The data for MO81 and the initial evaluation of the coefficients in Table 1

Data for MO81 are set out in Tables 5 and 6. These data are purely hypothetical. They are illustrative of the input-output data base which will be required for implementing a version of ORANI extended to include the types of equations appearing in category VI of Table 1.

In Table 4, we have indicated how the information in Tables 5 and 6 is used to give initial values for the coefficients of the equations in Table 1. Data are required for two points of time, the base year (year 0) and the solution year (year τ). In implementing MO81 we have set τ equal to 10. The data for the solution year are used to evaluate the share coefficients (cost shares, sales shares and revenue shares). Both base year and solution year data are necessary for evaluating the growth-related coefficients : U in (T32), Γ in (T33) and the H_j's used in some variants of (T25). As explained earlier (see pp. 4-5), data for the solution year must be obtained by projection. In the present exercise, Table 6 was obtained from Table 5 by assuming a growth rate of 5 per cent per annum for 10 years in all commodity and primary factor flows and zero growth in all prices.

Two further points concerning the input-output data are worth mentioning. First, as we noted on p.4, these data provide an initial solution, X^I, to (2.1). To obtain X^I, a convention is required with respect to quantity units. The most convenient approach is to define

Table 4 - The MO81 Coefficients and Parameters Appearing in Table 1

Equation	Coefficient or Parameter	Description and Evaluation[a]
(T1)	$S^{(3)}_{(ir)}$	Share of good i from source r in household purchases of good i, e.g., initial value for $S^{(3)}_{(11)} = 19.55/(19.55 + 1.63)$.
(T2)	$S^{(2)}_{(ir)j}$	Share of good i from source r in industry j's purchases of good i to be used as an input to capital creation, e.g., initial value for $S^{(2)}_{(11)1} = 1$, initial value for $S^{(2)}_{(22)1} = 3.26/(9.78 + 3.26)$.
(T3)	γ_i	Reciprocal of the foreign (export) demand elasticity for good i. Values adopted were $\gamma_1 = 0.5$, $\gamma_2 = 0.05$.
(T4)	$S^{(1)}_{(ir)j}$	For i=1,2, this is the share of good i from source r in industry j's purchases of good i to be used as an intermediate input, e.g., initially $S^{(1)}_{(11)1} = 16.28/(16.28 + 1.63)$. For i=3, this is the share of labor (r=1) or the share of capital (r=2) in industry j's payments to primary factors, e.g., initially $S^{(1)}_{(31)1} = 32.57/(32.57 + 8.14 + 8.14)$.
(T5)	$H^{(0)}_{(q1)j}$	Share of total revenue in industry j accounted for by sales of good q, e.g., initially $H^{(0)}_{(11)1} = 73.30/(73.30 + 26.06)$.
(T6)	$H^{(0)}_{(q1)j}$	Covered under (T5).
	$H^{(1)}_{(is)j}$	Share of input (is) in the total costs of production in industry j, e.g., initially $H^{(1)}_{(11)1} = 16.28/99.36$, $H^{(1)}_{(32)1} = 16.28/99.36$, $H^{(1)}_{(31)2} = 32.57/71.67$.
(T7)	None	

... *continued*

Table 4 continued ...

Equation	Coefficient or Parameter	Description and Evaluation[a]
(T8)	None	
(T9)	$H^{(2)}_{(is)j}$	Share of input (is) in the total cost of capital creation for industry j, e.g., initially $H^{(2)}_{(11)1} = 3.26/16.29$, $H^{(2)}_{(12)1} = 0$, $H^{(2)}_{(21)2} = 4.89/8.14$.
(T10)	None	
(T11)	None	
(T12)	$W^{(0)}_{(q1)j}$	Share of the total output of (q1) which is accounted for by industry j, e.g., initially $W^{(0)}_{(11)1} = 73.30/87.96$, $W^{(0)}_{(11)2} = 14.66/87.96$.
	$W^{(k)}_{(q1)j}$	Share of the total purchases of (q1) which is accounted for by industry j for purpose k (k=1, intermediate demand; k=2, input to capital creation). For example, initially $W^{(1)}_{(11)1} = 16.28/87.96$, $W^{(2)}_{(11)2} = 1.63/87.96$, $W^{(2)}_{(21)2} = 4.89/83.07$.
	$W^{(k)}_{(q1)}$	Share of the total purchases of (q1) which is accounted for by final user k (k=3, households; k=4, exports). For example, initially $W^{(3)}_{(11)} = 19.55/87.96$, $W^{(4)}_{(11)} = 34.21/87.96$.
(T13)	$W^{(1)}_{(g+1,1)j}$	Share of the total demand for primary factor of type 1 (i.e., labor) which is accounted for by industry j. For example, the initial value of $W^{(1)}_{(g+1,1)2}$ is 32.57/65.16.
(T14)	None	

... continued

Table 4 continued ...

Equation	Coefficient or Parameter	Description and Evaluation[(a)]
(T15)	$W^{(k)}_{(q2)j}$	Share of total purchases of imported good q which is accounted for by industry j for purpose k, e.g., initially $W^{(1)}_{(12)1} = 1.63/16.29$, $W^{(2)}_{(22)2} = 1.63/27.69$. Note : the denominator in these shares is the basic value (including duty) of the sales of good (q2).
	$W^{(3)}_{(q2)}$	Share of total purchases of good (q2) which is accounted for by households. Initially $W^{(3)}_{(12)} = 1.63/16.29$, $W^{(3)}_{(22)} = 11.40/27.69$.
(T16)	$M_{(q2)}$	Share of the total foreign currency value of imports accounted for by imports of good (q2). Initially $M_{(12)} = 14.66/34.21$, $M_{(22)} = 19.55/34.21$.
(T17)	$E_{(q1)}$	Share of the total foreign currency value of exports accounted for by exports of good (q1). Initially $E_{(11)} = 1$, $E_{(21)} = 0$. Note : there are no export taxes or subsidies in the data.
(T18)	E	Foreign currency value of exports. Initially $E = 34.21$.
	M	Foreign currency value of imports. Initially $M = 34.21$.
(T19)	$H^{(3)}_{(is)}$	Share of household expenditure devoted to good (is), e.g., initially $H^{(3)}_{(11)} = 19.55/74.93$.
(T20)	None	

... *continued*

Table 4 continued ...

Equation	Coefficient or Parameter	Description and Evaluation[(a)]
(T21)	W_j^y	Share of total investment accounted for by industry j. Initially $W_1^y = 16.29/24.43$, $W_2^y = 8.14/24.43$.
(T22)	W_j^y	Covered under (T21).
(T23)	None	
(T24)	Q_j	$Q_j = (R_j + D_j)/R_j$, i.e., Q_j is the ratio of the gross to the net rate of return in industry j. From Table 6, $D_1 = 8.14/162.89$, $D_2 = 4.07/81.45$ (these are treated as constants). Initially $R_1 = 8.14/162.89$, $R_2 = 4.07/81.45$. Hence, initially $Q_j = 2$ for j=1,2.
(T25)	$I_j^{(1)}$, $I_j^{(2)}$	Users of the model set values according to how investment by industry is to be modelled (see pp.34-39). Where formulae such as (3.38) are used, the H_j are valued according to (3.34) using values for $K_j(0)$ and K_j from Tables 5 and 6. For example, where $\tau = 10$, the initial value for H_1 is $(162.89/100)^{.1} - 1$.
	B_j	$B_j = 1/\beta_j\Delta_j$ where β_j is the elasticity of the expected rate of return schedule for industry j and Δ_j is the ratio of investment in the solution year to capital stock in the following year. For j=1,2, we set $\beta_j = 30$. Initially $\Delta_1 = 16.29/(162.89(.95) + 16.29)$ and $\Delta_2 = 8.14/(81.45(.95) + 8.14)$. <u>Note</u> : none of the results reported in section 4 are affected by the value used for B_j.

... continued

Table 4 continued ...

Equation	Coefficient or Parameter	Description and Evaluation[a]
(T26)	W_j^k	Share of the total capital stock accounted for by industry j. Initially, $W_1^k = 162.89/244.34$, $W_2^k = 81.45/244.34$.
(T27)	ζ_i^t	$\zeta_i^t = T_i/(T_i-1)$, i.e., ζ_i^t is the ratio of the power of the tariff on good i to the ad valorem rate. Initially $\zeta_1^t = 16.30/1.63$, $\zeta_2^t = 27.69/8.14$.
	T_i^t	Share of total tariff revenue accounted for by tariffs on good i. Initially, $T_1^t = 1.63/9.77$, $T_2^t = 8.14/9.77$.
(T28)	ζ_i^v	$\zeta_i^v = V_i/(V_i - 1)$, i.e., ζ_i^v is the ratio of the power of the export subsidy on good i to the ad valorem rate.
	T_i^v	Share of total export subsidies accounted for by export subsidies on good i. Note : our data base shows no export subsidies. Hence, in our computations, (T28) and the variable v are deleted.
(T29)	$I_{(g+1,1)j}$	User determined wage-indexing parameter, (see p.39).
(T30)	None	

... continued

Table 4 continued ...

Equation	Coefficient or Parameter	Description and Evaluation[a]
(T31)	ψ_i	ψ_i, i=1,...,4 are the shares in domestic income accounted for by wage income, tariff revenue, export subsidies and capital income accruing to domestic capitalists. In Table 6, wage income is 65.16, tariff revenue is 9.77, export subsidies are 0 and domestic capitalist income is (24.44)(183.26/244.34) = 18.33. Thus, domestic income is 93.24 and initially we have $\psi_1 = .70$, $\psi_2 = .10$, $\psi_3 = 0$ and $\psi_4 = 0.20$.
	$N_{(g+1,1)j}$	Share of industry j in total wage payments. Initially $N_{(g+1,1)1} = 32.57/65.16$, $N_{(g+1,1)2} = 32.57/65.16$.
	$N_{(g+1,2)j}$	Share of industry j in total returns to capital. Initially $N_{(g+1,2)1} = 16.28/24.44$, $N_{(g+1,2)2} = 8.14/24.44$.
(T32)	$(U+1)/\tau U$	In the base year (see Table 5) domestic income is 40 (wages) plus 6 (tariff revenue) plus 15(112.5/150) (domestically accruing capital income), i.e. domestic income is 57.25. Consumption is 46 leaving savings $S(0) = 11.25$. In the solution year (see Table 6) domestic income is 93.24 (see the discussion of ψ_i earlier in this table). Consumption is 74.93 leaving $S(\tau) = 18.31$. Commodity prices are constant as we move from Table 5 to Table 6 (see p.53). In particular, initially, $\Pi(\tau)/\Pi(0) = 1$. Hence, the initial value for U is $(18.31/11.25)^{.1} - 1 = 0.05$. With $\tau = 10$, we find that the initial value for the coefficient in (T32) is 1.05/.50 = 2.10.

... continued

Table 4 continued ...

Equation	Coefficient or Parameter	Description and Evaluation [a]
(T33)	Γ	See formula (3.49). With $\tau = 10$, $D = 0.05$, $Q(0) = .75$, $K(0) = 150$ and U, $Q(\tau)$ and $K(\tau)$ having initial values 0.05, .75 and 244.34, Γ has an initial value of 0.160.
(T34)	$1/(1-F_c)$	Initially F_c equals (74.93/93.24) = .80, i.e. F_c is the ratio of consumption to domestic income. This gives an initial value for the coefficient in (T34) of 5.1.

(a) Except when otherwise indicated, the initial share values were computed from the solution-year input-output data, i.e., Table 6.

Table 5 - Input-output Data Base for MO81 for Year 0, the Base Year

			Intermediate inputs to industries 1 and 2		Gross fixed capital formation by industries 1 and 2		Household consumption	Exports	Negative of import duty	Row Totals
Domestic commodities		1	10	8	2	1	12	21		54
		2	15	1	6	3	26	0		51
Imported commodities		1	1	8	0	0	1		-1	9
		2	5	2	2	1	7		-5	12
Labor			20	20						40
Gross operating surplus	Depreciation		5	2.5						7.5
	Net profit		5	2.5						7.5
Total costs			61	44	10	5	46	21	-6	181
Domestic commodity outputs		1	45	9						54
		2	16	35						51
Domestically-owned capital stocks			75	37.5						112.5
Foreign-owned capital stocks			25	12.5						37.5
Total capital			100	50						150

Table 6 - Input-output Data Base for MO81 for Year τ, the Solution Year[a]

			Intermediate inputs to industries 1 and 2		Gross fixed capital formation by industries 1 and 2		Household consumption	Exports	Negative of import duty	Row Totals
Domestic commodities		1	16.28	13.03	3.26	1.63	19.55	34.21		87.96
		2	24.43	1.63	9.78	4.89	42.35	0		83.07
Imported commodities		1	1.63	13.03	0	0	1.63		-1.63	14.66
		2	8.14	3.26	3.26	1.63	11.40		-8.14	19.55
Labor			32.57	32.57						65.16
Gross operating surplus	Depreciation		8.14	4.07						12.22
	Net profit		8.14	4.07						12.22
Total costs			99.36	71.67	16.29	8.14	74.93	34.21	-9.77	294.84
Domestic commodity outputs		1	73.30	14.66						87.96
		2	26.06	57.01						83.07
Domestically-owned capital stocks			122.17	61.09						183.26
Foreign-owned capital stocks			40.72	20.36						61.08
Total capital			162.89	81.45						244.34

(a) τ=10. All flows shown here are obtained from the corresponding flows in Table 5 by multiplying by $(1.05)^{10}$.

the quantity units so that all initial prices for commodities and primary factors in the solution year are unity. Then we can simply read the value flows in Table 6 as quantity flows. The balancing properties of Table 6 ensure that the quantity flows satisfy the market clearing conditions and that the value of production in each industry equals the value of output. We may assume that the free parameters are set so that the other equations in MO81 are also satisfied. For example, the exponents in the Cobb-Douglas function (3.5) can be assumed to be set at 19.55/21.18 and 1.63/21.18 for $i=1$ and 42.35/53.75 and 11.40/53.75 for $i=2$.

Second, we note that the data in Table 6 are in accord with the simplifying assumptions (3.43) and (3.44). The inputs used to create a unit of capital for industry 1 are identical to those used for industry 2. The rate of depreciation of capital in both industries is 5 per cent.

4.2 Illustrative results : the effects of a 3.4 per cent increase in the ad valorem tariff rate on commodity 2

In Table 7 we report two simulations of the effects of a one per cent increase in the power of the tariff on good 2 (the main import-competing commodity). This is equivalent to a 3.4 per cent increase in the ad valorem rate. (Notice from Table 6 that the initial value of T_2 is 1.42 (= (8.14 + 19.55)/19.55).

In making the computations we relied on the Johansen approach, (2.24). That is we computed x_1 according to

$$x_1 = -A_1^{-1}(X^I)\ A_2(X^I)x_2 \ , \tag{4.1}$$

where x_1 and x_2 are the vectors of percentage changes in the endogenous

Table 7 - Effects of a 3.4 per cent Increase in the ad valorem Tariff Rate on Good 2 : Long-run Simulations[a]

Variable	(1) Standard long-run simulation available with restricted model	(2) Standard long-run simulation available with complete model
Gross domestic product[b]	-0.06	-0.06
Aggregate capital (k)	-0.40	-0.40
Aggregate employment (ℓ)	0[c]	0[c]
Real aggregate consumption (c^R)	-0.06	0.05
Real aggregate investment (y^R)	-0.06	-0.40
100ΔB/GDP	0[c]	0.00
Imports, foreign currency value (m)	-0.27	-0.28
Exports, foreign currency value (e)	-0.27	-0.27
Domestic ownership share (q)	0.63	0.40
Average propensity to consume (f_c)	-0.16	0[c]
Activity level, industry 1 (z_1)	-0.48	-0.47
Activity level, industry 2 (z_2)	0.33	0.33

(a) All projections are percentage changes except those for ΔB. Changes in the balance of trade are expressed as a per cent of GDP. Results should be interpreted as deviations from the values which the variables would have reached in the solution year in the absence of the tariff change.

(b) Calculated as the appropriately weighted sum of c^R, y^R and 100ΔB/GDP.

(c) Set exogenously.

and exogenous variables. The only non-zero component of x_2 in the simulations was t_2 which was set at 1. In simulation 1 (column 1 of Table 7), the list of exogenous variables was that given in column 2 of Table 3. In simulation 2, the exogenous variables were those listed in column 3 of Table 3. In both simulations, the wage-indexation parameters, $I_{(g+1,1)j}$, j=1,2, in (T29) were set at unity. In simulation 1, $I_j^{(1)}$ and $I_j^{(2)}$ appearing in (T25) were also set at unity, i.e., (3.29) applies. Because ω is endogenous in this simulation, the role of (T25) is confined to allocating investment across industries. ω adjusts so that the y_j's emerging from (T25) are consistent with aggregate investment which is assumed to move in line with aggregate consumption. The allocation of investment across industries is of little importance in our miniature model because the technology for building capital is identical for the two industries. Thus, shifts of investment expenditure between the two industries in the solution year have no effect on input demands. In simulation 2, we set $I_j^{(1)} = 1$ and $I_j^{(2)} = 0$ for all j, i.e., (3.32) applies. Experiments with (3.37) have also been conducted. However, for long-run simulations, (3.32) seems more reasonable. The argument is that after 10 years, growth rates in capital stocks will have settled back to reflect the underlying growth in productivity and the size of the labor force. Capital growth rates in year 10 are assumed to be independent of the adjustments caused by the tariff change to the growth rates for the period 0 to 10. Notice, finally, that with $I_j^{(2)} = 0$, the value chosen in simulation 2 for the exogenous variable ω has no bearing on the results.

Simulation 1 is illustrative of the current ORANI model in standard long-run mode. In particular, it is assumed that the change in the tariff on good 2 has no effect on the balance of trade or on the ratio of consumption to investment. Consumption and investment are assumed to move by the same percentage as GDP. In simulation 2 it is assumed that the change in the tariff does not affect the average propensity to consume, i.e., consumption changes by the same percentage as income accruing to domestic residents.

The most obvious feature of the results in Table 7 is their similarity in the two experiments for all variables except those relating to the composition of domestic absorption (c^R, y^R, f_c and q). In both simulations the tariff change increases activity by the main producer (industry 2) of good 2, and depresses the main exporter (industry 1). The resulting effects on GDP depend on the sizes of (a) the efficiency losses, (b) the favourable terms-of-trade effects and (c) the contractions in the capital stocks necessary to maintain rates of return. The sizes of these three effects of the tariff increase are not very sensitive to changes in the composition of domestic absorption.

In analysing the results we found it helpful to make a third simulation of the effects of the tariff increase. The same conditions were assumed as in simulation 2 except that we held the aggregate capital stock fixed and allowed the economy-wide average rate of return to be endogenous. (Compared with simulation 2, in this third simulation, k and the f_j^r's, set at zero, replaced r and the r_j's in the exogenous category, see p.45.) We found that with $k=0$, the tariff increase caused GDP to increase by 0.04 per cent. The terms-of-trade effect dominated the conventional efficiency losses. The tariff increase also reduced the real rentals on units of capital because the production of exports (good 1) is relatively capital intensive. Real capital income fell by 0.31 per cent and real non-capital income (wages and tariff revenue) rose by 0.15 per cent. We have assumed that domestic residents own all of the non-capital income (which accounts for 80 per cent of their total income in the solution year) and 75 per cent of the capital income. Thus, in the fixed-capital experiment, the percentage change in the real income of domestic residents is

$$x_d^R = 0.8(0.15) + 0.2(-0.31) = 0.06 \quad .$$

This is also the percentage change in real consumption (c^R).

To arrive at the results in simulation 2, we must now allow for the reduction in the size of the capital stock necessary to restore rates of return (and thus real rentals) to their original levels. The necessary change is $k = -0.40$. The same is true for simulation 1 (see Table 7). This yields a reduction in GDP given, to a good approximation, by

$$gdp = S_K k = 0.25(-0.40) = -0.10 \, ,$$

where $S_K = .25$ is the capital share in value added shown in our data base. The net effect on GDP of the tariff increase in simulation 2 (and in simulation 1) is the sum of the effect with capital fixed and the effect of the reduction in the capital stock, i.e.,

$$gdp = 0.04 - 0.10 = -0.06.$$

Despite this reduction in GDP, the result for consumption ($c^R = 0.05$) in column 2 of Table 7 remains close to that in the fixed-capital case where c^R is 0.06, i.e., the reduction in the size of the capital stock has a negligible impact on the amount of domestically owned income. This is because there is a negligible effect on the amount of domestically owned capital.[8] The explanation depends on two relationships. First, other things being equal, as domestic income (and thus savings) increases so does domestic ownership of capital. By substituting from (T32) into (T33) and by assuming that $(s - \pi) = x_d^R$, we obtain

$$(4.2) \qquad q + k = \frac{\Gamma(1 + U)}{\tau U} x_d^R ,$$

where $q + k$ is the percentage change in the domestic ownership of capital. Second, other things being equal, as domestic ownership of capital increases, so does domestic income. Starting from

$$(4.3) \qquad X_d^R = X^R - (1 - Q)KW_K^R ,$$

where X_d^R is real domestic income, X^R is total real income (GDP), W_K^R is the real rental on units of capital and K is the capital stock, we obtain, in percentage change form,

$$(4.4) \qquad x_d^R = x^R\left(\frac{X^R}{X_d^R}\right) - \left(k + w_K^R\right)\left(\frac{(1 - Q)KW_K^R}{X_d^R}\right) + q\left(\frac{QKW_K^R}{X_d^R}\right) .$$

From our data base, $\Gamma = 0.16$, $(1 + U)\tau U = 2.1$, $(X^R/X_d^R) = 1.066$, $\left[(1 - Q)KW_K^R/X_d^R\right] = .066$ and $(QKW_K^R/X_d^R) = .197$. With a 0.40 per cent reduction in capital stock, we have found that $x^R = -0.10$, and $w_K^R = 0.31$. Thus, with $k = -0.40$, (4.2) and (4.4) reduce to

$$(4.5) \qquad q = 0.40 + 0.34\, x_d^R ,$$

and

$$(4.6) \qquad x_d^R = -0.10 + .197q ,$$

implying that $q = .396$, $x_d^R = -.01$ and $(q + k) = .004$.

The final result in column 2 of Table 7 which warrants explanation is for the change in the balance of trade. This is close to zero because the percentage change in real domestic absorption turns out to be close to the percentage change in real GDP. In our miniature model real domestic absorption is made up of consumption and investment. The result in column 2 for real consumption has already been explained. The percentage change in real investment (y^R) is equal to the percentage change in aggregate capital by assumption. In our data base, the shares of consumption and investment in absorption are 0.75 and 0.25. Thus, we have

$$(4.7) \qquad a = .75(0.05) - .25(.40) = -0.06 ,$$

where a is the percentage change in real absorption.

In simulation 1 the change in the balance of trade is set exogenously at zero and the gdp result is very close to that in simulation 2. Thus, aggregate absorption is approximately the same in simulations 1 and 2. However, the allocation of absorption between consumption and investment is different. In simulation 1, both consumption and investment are forced to move with GDP. Consequently, compared with simulation 2, simulation 1 shows a reduction in the consumption-investment ratio. In fact, the increase in the domestic ownership share of capital (and thus in domestically accruing income) is greater in simulation 1 than in simulation 2, but there is a marked fall in the average propensity to consume in simulation 1.

4.3 An illustration of Euler's method in computing MO81 results

The results reported in the previous subsection were computed from MO81 using the Johansen method, (4.1). We can have confidence in this method if the exogenous shock is small. In this subsection, we examine the effects of a large shock : complete elimination of the tariff on good 2. Our interest is in how sensitive the results are to the method by which they are computed. We will compare results computed using Euler's method, (2.21) - (2.23), for various values of n (the number of steps) including $n=1$ (Johansen's method).

Table 8 contains results for the 100 per cent reduction in the tariff on good 2 computed under the same assumptions as in simulation 2 in Table 7. Results in column 1 of Table 8 (i.e., the Johansen-style results) could be calculated from the results in column 2 of Table 7 by multiplying the latter by $-100/3.4 = -29.4$. In columns 2 - 6 of Table 8 we give Euler-style results for n equal to 2,4,8,16 and 32.

Table 8 - Selection of Results for an Elimination of the Tariff on Good 2 Computed by Euler's Method : Standard Long-run Simulation Available with the Complete Version of MO81

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	COMPUTED PROJECTIONS(a)						EXTRAPOLATIONS(b)		PERCENTAGE ERRORS	
	1 step	2 step	4 step	8 step	16 step	32 step	1-2 step	16-32 step	Johansen $\left[\frac{(1)}{(8)} - 1\right]100$	1-2 step extrapolation $\left[\frac{(7)}{(8)} - 1\right]100$
gdp	1.79	1.33	1.05	0.89	0.82	0.77	0.86	0.73	143.3	16.7
u	-0.70	-2.47	-3.54	-4.12	-4.43	-4.59	-4.24	-4.75	-85.2	-10.7
c^R	-1.39	-2.29	-2.83	-3.12	-3.28	-3.36	-3.18	-3.44	-59.5	-7.4
y^R	11.75	12.98	13.80	14.28	14.55	14.69	14.20	14.83	-20.7	-4.2
m	8.14	9.40	10.22	10.70	10.95	11.09	10.67	11.22	-27.4	-4.9
e	8.02	9.07	9.71	10.06	10.25	10.34	10.13	10.44	-23.2	-3.0
ΔB	-0.04	-0.11	-0.18	-0.21	-0.24	-0.25	-0.19	-.26	-84.7	-26.3
$\xi^{(3)}$	-8.88	-9.50	-9.85	-10.03	-10.13	-10.17	-10.13	-10.22	-13.2	-0.9
z_1	13.90	15.61	16.71	17.34	17.68	17.85	17.33	18.03	-22.9	-3.9
z_2	-9.63	-11.03	-11.90	-12.39	-12.66	-12.79	-12.43	-12.93	-25.5	-3.8
q	-11.87	-12.56	-13.01	-13.27	-13.41	-13.49	-13.25	-13.56	-12.5	-2.3
t	-66.35	-77.80	-82.55	-84.35	-85.08	-85.39	-89.24	-85.71	-22.6	4.1

(a) Column (1) contains results from a Johansen-style solution computed according to equation (2.24). Columns (2)-(6) were computed via the n-step Euler method (2.21) - (2.23) for n = 2, 4, 8, 16 and 32 respectively.

(b) Column (7) was calculated from columns (1) and (2) via (4.9) with n = 1. Column (8) was calculated in an analogous way from columns (5) and (6). The results in column (8) are assumed to be free from linearization errors and are used as the exact solution to the model in calculating the percentage errors given in columns (9) and (10).

The n-step results were computed by breaking the change in the exogenous variable (T_2) into n equal parts. The initial value of T_2 in our data (Table 6) is 1.42. A 100 per cent cut in the ad valorem tariff rate on good 2 reduces this to 1.00. Thus, in the first step of the n-step procedure, we applied (4.1) to compute the effects on the endogenous variables of the exogenous shock

$$(t_2)_0^n = -\left(\frac{0.42/n}{1.42}\right) 100 .$$

The results of this computation were used in updating the entire data base. This involved the updating of input-output flows (defined as the product of a price and a quantity) according to the formula

$$(\text{Flow})_1^n = (\text{Flow})_0 \left(1 + \frac{p_0^n}{100}\right)\left(1 + \frac{q_0^n}{100}\right) ,$$

where $(\text{Flow})_0$ and $(\text{Flow})_1^n$ are the initial and updated values of the flow, and p_0^n and q_0^n are the relevant components of the x_1 and x_2 vectors in the first application of (4.1). Using the updated flows, we re-evaluated the matrix $-A_1^{-1} A_2$. Then, at the second step of the n-step procedure this re-evaluated matrix was used to derive the effects on the endogenous variables of the exogenous shock

$$(t_2)_1^n = -\left(\frac{0.42/n}{1.42 - 0.42/n}\right) 100 .$$

The cycle of data updates, re-evaluations of the matrix $-A_1^{-1} A_2$, recalculations of the exogenous shock $(t_2)_q^n$, and recomputations of solution-values for the endogenous variables was then repeated a further $n - 2$

times. The final results in the n-step column in Table 8 are of the form

$$(x)^n = \prod_{q=0}^{n-1} \left(1 + \frac{x_q^n}{100}\right) - 1 ,$$

where x_q^n is the solution value or exogenous input for variable x at the $(q+1)^{th}$ step of the procedure.

Columns 1 - 6 of Table 8 reveal that for any endogenous variable the results of the different Euler computations exhibit (approximately) the following relationship

$$(x)^{2n} - (x)^n = 2\left[(x)^{4n} - (x)^{2n}\right] . \quad (4.8)$$

This suggests that we can calculate $(x)^\infty$ by extrapolating from results generated by Euler-style computations using small numbers of steps. Relationship (4.8) implies the extrapolation rule

$$(x)^\infty = (x)^{2n} + \left[(x)^{2n} - (x)^n\right] . \quad (4.9)$$

If (4.8) were to apply exactly in our results we could obtain $(x)^\infty$ by extrapolation from a 2-step Euler computation and a 1-step computation. In fact (4.8) applies only approximately, with the results conforming more closely to the relationship as n increases. In columns 7 and 8 of Table 8 we report two sets of extrapolations (generated via (4.9). Those in column 7 were computed with $n=1$ and for those in column 8 we used $n=16$. Column 8 is assumed to be free from linearization errors, i.e., to be an accurate solution to the non-linear form of MO81. Columns 9 and 10 contain the percentage deviations of the Johansen results (column 1) and of the results generated via (4.9) with $n=1$ (column 7) from the linearization-error-free results (column 8).

The errors associated with the Johansen-style results (column 9) average about 40 per cent. It must be noted that the exogenous shock in this experiment is very large in comparison to the types of shocks which are normally under consideration by policy makers. Our experience with MO81 subjected to more modest shocks is that the Johansen errors are typically much smaller than those evident in Table 8.[9] Column 10 of the table shows that extrapolation based on only the one-step and two-step Euler computations dramatically reduces the errors. With few exceptions, errors in column 10 do not exceed 5 per cent. Errors of this size could not normally be regarded as serious in comparison with those introduced by data inaccuracies and the simplifications of reality imposed by theoretical assumptions.

5. Conclusion

MO81 is a model of ORANI. In building ORANI, we have used MO81 and its predecessors to try out new ideas. Because ORANI is very large, we have found that it is sensible to devise small, simplified versions for experimenting with potential improvements and extensions before attempting to implement them in the main system. In this paper we have described two sets of experiments with MO81 : one concerned with the solution procedure, and the other concerned with long-run macroeconomic closure and foreign ownership of capital.

Originally, Johansen's linearization method was the solution procedure adopted for ORANI. This method has advantages in terms of flexibility and cheapness, but it introduces uncontrolled linearization errors. In subsection 4.3 we showed how Euler's method combined with extrapolation can be used to produce MO81 results in which the linearization errors are negligible. Encouraged by similar results from an earlier version of MO81, we committed resources to programming the Euler method for ORANI. Results from ORANI follow the same pattern as those in the miniature versions. Consequently, we have been able to effectively eliminate linearization errors from ORANI computations without either a significant loss of computing flexibility or a serious increase in costs. Examples for ORANI of Euler-style solutions with extrapolation are in DPSV, section 47. The theoretic justification for the method is given in DPSV, chapter 5. Our view is that the method could be applied successfully to many of the current generation of general equilibrium models.

In fact, it has been used with satisfactory results in recent work by Bovenberg and Keller (1981) on a model quite different from ORANI.

To date, our work on long-run closure and foreign ownership has gone no further than this paper. Results from MO81, extended to deal with these issues, have proved interpretable and have not indicated any obvious flaws in our approach. However, before proceeding to add a category VI to ORANI, we will carry out further experiments with MO81. It would, for example, be useful to know whether our conclusion that reductions in capital have little impact on the amount of domestically owned capital depends on particular features of the MO81 data base used in this paper or whether this is a more general result. It would also be useful to know more about the role of τ (the time between the base year and the solution year). So far we have fixed τ at 10 in all experiments. Would our results be very different if τ were 5 or 15?

Notes

1. Full documentation is given in Dixon, Parmenter, Sutton and Vincent (DPSV), 1982.

2. In fact, we have calculated cost shares, sales shares, etc., from base period input-output accounts without any explicit forward projection. These shares are unaffected by a uniform expansion of all flows.

3. We use lower-case letters for percentage changes in the variables denoted by the corresponding upper-case letters. In writing (2.8) and (2.9) we have assumed that none of the components of X is zero. It is convenient, for the present (and not limiting in any practical sense), to think of all the ORANI variables as being positive. Later we will see that for one variable, the balance of trade, it is the change (not the percentage) which appears in ORANI computations.

4. That is one plus the ad valorem rate.

5. The estimation of the B_j for ORANI is described in DPSV, section 29(g).

6. Notice that the variable ω appearing in (T25) is treated endogenously. It adjusts so that the y_j, $j=1,\ldots,h$, are consistent with the exogenously given value for y^R. (We assume that the coefficients $I_j^{(2)}$ are not all zero.) Where ω is endogenous, (T25) can be thought of as determining the allocation of investment across industries while leaving the overall level of investment to be determined elsewhere.

7. We value units of industry j's capital stock existing in the solution year at their cost of construction, $\Pi_j(\tau)$. In view of our assumption of a one period gestation lag in the creation of capital (see (3.26)) it might be considered appropriate to value existing capital at more than $\Pi_j(\tau)$ per unit. We have found it convenient to use $\Pi_j(\tau)$, i.e., we value capital at time τ assuming that rentals for the year have already been paid and that depreciation has already taken place. In fact, as we will see, the valuation chosen is not critical. Under the assumptions we will be adopting the $\Pi_j(\tau)$'s cancel out of (3.41) - (3.42).

8. Recall that employment of labor is held fixed. The argument in the text will establish that the reduction in capital has little effect on the amount of domestically owned capital. It follows that it has little effect on the amount of employed, domestically owned resources. Thus, there is little effect on the overall level of domestic income although there is a redistribution of income in favour of domestic capitalists and away from labor.

9. See DPSV, subsection 8.4.

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