

IND. NO.	COMMODITY GROUP NUMBER															TOTAL	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15		
106																1.000	1
107																1.000	1
108																1.000	1
109																	0



**IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA**

A joint study by the Australian Bureau of Statistics, the Department of Employment and Industrial Relations, the Department of Environment, Housing and Community Development, the Department of Industry and Commerce and the Industries Assistance Commission

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ESTIMATES OF HOUSEHOLD DEMAND ELASTICITIES  
 FOR THE ORANI MODEL  
 by  
 Ashok Tulpuñé and Alan A. Powell  
 Industries Assistance Commission

Preliminary Working Paper No. OP-22 Melbourne September 1978

*The views expressed in this paper do not necessarily reflect the opinions of the participating agencies, nor of the Australian government.*



IND. NO.

COMMODITY GROUP NUMBER

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	TOTAL
76								0.775		0.011					0.214	1
77								0.013		0.053	0.821				0.113	1
78								0.038		0.661					0.301	1
79															1.000	1
80								0.001		0.115					0.884	1
81															1.000	1
82															1.000	1
83															1.000	1
84																0
85																0
86															1.000	1
87															1.000	1
88											1.000					1
89															1.000	1
90												0.540		0.445	0.015	1

\*

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IND. NO.	COMMODITY GROUP NUMBER	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	TOTAL
61		1.000															1
62									0.362								1
63									0.982								1
64							0.055		0.608								1
65									0.020	0.955							1
66									1.000								1
67																	0
68																	1
69									0.416							0.584	1
70									1.000								1
71									1.000								1
72									0.763	0.237							1
73									1.000								1
74									1.000								1
75									1.000								1

IND. NO.	COMMODITY GROUP NUMBER														TOTAL	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
46															1.000	1
47															1.000	1
48										0.215					0.785	1
49															1.000	1
50															1.000	1
51															1.000	1
52															1.000	1
53										0.016	0.897				0.087	1
54										0.982					0.018	1
55										1.000						1
56										0.500					0.500	1
57																0
58															1.000	1
59															1.000	1
60															1.000	1

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ESTIMATES OF HOUSEHOLD DEMAND  
ELASTICITIES FOR THE ORANI MODEL

by  
Ashok Tulpuñé and Alan A. Powell \*

1. INTRODUCTION

The purpose of this paper is to provide estimates of consumer demand elasticities for the ORANI<sup>1</sup> model using up-to-date data on consumer expenditure in Australia. Estimates of expenditure elasticities are made using Betancourt's<sup>2</sup> Twice Extended Linear Expenditure System (TELES). (For a derivation of Betancourt's system using Howe's<sup>3</sup> short-cut method, the reader is referred to a recent paper by Tulpuñé<sup>4</sup>.) In the present paper the TELES system is estimated for eight commodity groups using Australian time-series data for 1964-65 to 1975-76. Estimates of parameters of the eight commodity TELES model are used to obtain expenditure elasticities for the 109 input-output industries.

The plan of the paper is as follows. In Section 2 a brief account of the consumer demand specification in ORANI is given. Section 3 contains a notation for the TELES model, with the aid of which a strong prior

\* The authors would like to thank Tony Lawson for helpful comments which materially improved our presentation. Responsibility for remaining flaws rests with the authors.

1. P. Dixon, B. Parmenter, G. Ryland and J. Sutton, ORANI, A General Equilibrium Model of the Australian Economy: Current Specification and Illustrations of Use for Policy Analysis - - First Progress Report of the IMPACT Project, Vol. 2 (Canberra: Australian Government Publishing Service, 1977).

2. R. Betancourt, "Household Behaviour in a Less Developed Country: An Econometric Analysis of Cross-sectional Data," University of Maryland, August 1973 (mimeo).

3. H. Howe, "Development of the Extended Linear Expenditure System from Simple Saving Assumptions," European Economic Review, Vol. 6 (1975), pp. 305-310.

4. A. Tulpuñé, "Empirical Estimation of Labour Supply Elasticities," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. BP-12, Industries Assistance Commission, Melbourne, July 1978 (mimeo).

2. constraint on the value of the Frisch parameter is introduced. This is done in order to ensure that the average elasticity of substitution among commodities, which proves very elusive to estimate on the basis of any one set of time series data, is consistent with the weight of the evidence on this matter. In Section 4 a brief account of the data is given, and in Section 5 the results of the maximum likelihood estimation of the 8-commodity TELES system are presented. In Section 6, estimates from an earlier estimation of a further six commodity split of one of these eight commodities - - food - - are integrated with the results for the eight commodity system. Using less formal methods, we also disaggregate our estimates for the public transport group into three sub-groups thought to differ substantially in their expenditure elasticities. As a result of the splits made in Section 6, the initial 8-commodity results have been disaggregated to cover a total of 15 commodity groups. In Section 7 these fifteen expenditure elasticities are further expanded so that they span 109 domestically produced commodities, plus non-competitive imports. Section 8 is a brief recapitulation of the main features of the paper.

## 2. HOUSEHOLD DEMAND IN ORANI

In the ORANI model, ordinary price elasticities,

$\partial \ln x_i / \partial \ln p_j \equiv \eta_{ij}$ , are defined as<sup>1</sup>

$$(2.1) \quad \eta_{ij} = \epsilon_i \alpha_j (1 + \epsilon_j / \omega) \quad \text{for } i \neq j ;$$

and

$$(2.2) \quad \eta_{ii} = \epsilon_i / \omega - \epsilon_i \alpha_i (1 + \epsilon_i / \omega)$$

( $i, j = 1, \dots, 109$ ),

1. See equations (5.22) and (5.23) in P. Dixon, B. Parmenter, G. Ryland and J. Sutton, (op. cit.) p.48.

IND. NO.	COMMODITY GROUP NUMBER	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	TOTAL
31		1.000															1
32		1.000															1
33										0.707							1
34						1.000											1
35						1.000											1
36									1.000								1
37										0.488							1
38										0.477							1
39										0.160							1
40										0.956							1
41																	1.000
42																	1.000
43																	1.000
44																	1.000
45																	1.000

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	TOTAL
16			0.896	0.104												1
17	0.033	0.046		0.139	0.583	0.199										1
18							1.000									1
19	0.962					0.020									0.018	1
20	0.997			0.003												1
21				0.975		0.025										1
22	0.125			0.342	0.066	0.308									0.159	1
23				0.884	0.116											1
24							1.000									1
25							1.000									1
26							1.000									1
27																0
28								1.000								1
29								1.000								1
30								1.000								1

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where  $\alpha_i$  is the average budget share in total consumer expenditure of the  $i$ th input-output commodity,  $\epsilon_i$  is the expenditure elasticity for the  $i$ th input-output commodity, and  $w$  is the Frisch parameter<sup>1</sup>. These formulae are based on a directly additive utility specification which is assumed, in the ORANI model, to underlie the behaviour of the representative consumer. Below we narrow down the specification further, making the assumption that the utility function is Klein-Rubbin.

3. THE TELES MODEL

3.1 Notation

Our TELES model consists of the following set of ten equations :

$$(3.1) \quad v_i = p_i x_i = p_i y_i + \beta_i \left\{ a - \sum_{i=1}^8 p_i y_i + w y_h \right\} \quad (i = 1, \dots, 8) ;$$

$$(3.2) \quad s = \beta_s \left\{ a - \sum_{i=1}^8 p_i y_i + w y_h \right\} ;$$

and

$$(3.3) \quad -wH = -w y_h + \beta_L \left\{ a - \sum_{i=1}^8 p_i y_i + w y_h \right\} ;$$

where

1. R. Frisch, "A Complete Scheme for Computing all Direct and Cross Elasticities in a Model with Many Sectors," Econometrica, Vol. 27 (1959), pp. 177-96.

$v_i$  = total current dollar consumption expenditure on commodity  $i$  divided by the number of working persons ;

$x_i$  = total quantity of  $i^{th}$  good consumed (in dollars of constant purchasing power) divided by the number of working persons ;

$\gamma_i$  = socially accepted minimum (or 'subsistence') size of  $x_i$  ;

$\beta_i$  = marginal propensity to consume good  $i$  out of 'full income,' including the value of voluntary leisure - - i.e., of leisure in excess of  $\gamma_L$  - - imputed at the average after-tax hourly wage rate,  $w$  ;

$\beta_s$  = marginal propensity to save out of full income ;

$\beta_L$  = marginal propensity to consume leisure out of full income<sup>1</sup> ;

$\gamma_L$  = minimum acceptable leisure hours ;

$\gamma_h$  = maximum acceptable hours at work ;

$$\gamma_h = T - \gamma_L ;$$

$p_i$  = price of good  $i$  ;

$s$  = savings in dollars ;

1. The parameters of the underlying utility function may be normalized in a variety of ways which are observationally equivalent. The normalization used here, which underlies the interpretation of the  $\{\beta_i\}$ , is that

$$\sum_{i=1}^8 \beta_i + \beta_s + \beta_L = 1 .$$

TABLE A4

IND. NO.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
TOTAL	1.000	1.000	1.000	0.056 0.944	0.422 0.564 0.014	0.775 0.129									
1															
2															
3															
4															
5															
6															
7															
8															
9															
10															
11															
12															
13															
14															
15															

IND. NO.

COMMODITY GROUP NUMBER

## APPENDIX 4 : PROPORTIONS MATRIX

Appendix 4 shows the distribution of the product of each industry (local production plus imports) to the 15 consumption categories. A list of 109 industries is included in Appendix 3 and the 15 commodity groups are shown in Appendix 1. The figures in Appendix 4 are based partly on data supplied by the Australian Bureau of Statistics and the 1962-63 classifier.

These figures have no official status : they incorporate approximations and other judgemental elements which are the sole responsibility of the authors.

H = hours worked per working person per year ;  
 wh = after tax labour income ;  
 w = average after tax hourly wage rate ;  
 a = after tax non-labour income .

For later use we define :

$$v = \text{total consumption expenditure divided by the number of working persons} = \sum_{i=1}^8 v_i .$$

The TELES system consists of equations (3.1), (3.2) and (3.3).

One of the equations must be deleted before estimation to keep the variance-covariance matrix of the stochastic errors which are appended to the behavioural equations non-singular.<sup>1</sup> In this case, equation (3.2) is deleted. Thus there are eighteen parameters ( $\gamma$ 's and  $\beta$ 's in (3.1) and (3.3)) to be estimated simultaneously.

1. See Anton Barton, "Maximum Likelihood Estimation of a Complete Set of Demand Equations," European Economic Review, Vol. 1 (1969), pp. 7-73 ; Richard Parks, "Maximum Likelihood Estimation of the Linear Expenditure System," Journal of the American Statistical Association Vol. 66, No. 36 (December 1971), pp. 900-903 ;

Alan A. Powell, "Aitken Estimators as a Tool in Allocating Predetermined Aggregates," Journal of the American Statistical Association Vol. 64, No. 327 (September 1969), pp. 915-922 ;

Lujgi Solari, "Sur l'estimation du Systeme Linéaire de Dépenses Par La Méthode du Maximum de Vraisemblance," Centre D'Econometrie, Cahier FN/5243.1/3, Faculté des Sciences Economiques et Sociales, Université de Genève, Mars 1969 (mimeo), p. 40, and Theori des Choix et Fonctions de Consommation Semi-Agrégées, Modèles Statistiques (Geneva : Dros, 1971).

Arshad Zaman, "Formulation and Estimation of a Complete System of Demand Equations," Department of Economics, Michigan State University, Econometrics Workshop Special Report No. 3, September 1970, pp, vi + 227.

3.2 Prior Restriction on the Frisch Parameter

In the linear expenditure family of models - - LES<sup>1</sup>, ELES<sup>2</sup> and TELES - - the Frisch 'parameter'  $\omega$  in fact is not a parameter at all, but a variable. Conceptually,  $\omega$  is the elasticity with respect to total expenditure of the marginal utility of an additional dollar's worth of consumption (on the assumption that the latter is optimally allocated among commodities). Within these models

$$\omega = - \frac{\sum_{i=1}^8 v_i}{\sum_{i=1}^8 (v_i - p_i \gamma_i)}$$

It is conventional in empirical work to evaluate  $\omega$  at sample means; thus if a superscript  $\hat{\phantom{x}}$  indicates the estimated value, and a superscript bar indicates evaluation at the sample means, the Frisch parameter is estimated by<sup>3</sup>

$$\hat{\omega} = \frac{\bar{v}}{\bar{v} - \sum p_i \gamma_i}$$

Experimental empirical analysis using different commodity aggregation indicates that the estimates of  $\gamma$ 's are unstable depending

1. Richard Stone, "Linear Expenditure System and Demand Analysis : An Application to the Pattern of British Demand," Economic Journal, Vol. 64, No. 255 (September 1954), pp. 511-527.
2. Constantino Lluich, "The Extended Linear Expenditure System," European Economic Review, Vol. 4 (April 1973), pp. 21-52.
3. Some minor ambiguity arises in applying this formulae :  $\bar{v}$  could be interpreted either as total consumption as actually observed at the sample mean, or alternatively, as the fitted value at the sample means of the exogenous variables. In practice, however, the difference is usually negligible.

Industry number	Input-Output Code	Industry Name	Elasticity Estimate
097	61.03	OTHER INSURANCE	1.19426
098	61.04	INVESTMENT, REAL EST	1.19500
099	61.05	OTHER BUSINESS SERV	1.19500
100	61.06	OWNERSHIP OF DWELLG	1.88800
101	71.01	PUBLIC ADMIN	1.19500
102	72.01	DERENCE	0
103	81.01	HEALTH	1.19500
104	82.01	EDUCATION, LIBRARIES	1.19500
105	83.01	WELFARE SERVICES	1.19500
106	91.01	ENTERTAINMENT	1.19500
107	92.01	RESTAURANTS, HOTELS	1.19500
108	93.01	PERSONAL SERVICES	1.19500
109	99.01	BUSINESS EXPENSES	0
110		NON-COMPETITIVE IMPORTS	1.43900

Source : See text.

Industry number	Input-Output Code	Industry Name	Elasticity Estimate
049	27.04	PHARMACEUTICALS	1.19500
050	27.05	SOAP + DETERGENTS	1.19500
051	27.06	COSMETICS, TOILETRY	1.19500
052	27.07	CHEMICAL PRODS NEC	1.19500
053	27.08	Oil + COAL PRODUCTS	1.19925
054	28.01	GLASS	1.56896
055	28.02	CLAY PRODUCTS	1.57600
056	28.03	CEMENT	1.38500
057	28.04	READY-MIXED CONCRETE	0
058	28.05	CONCRETE PRODUCTS	1.19500
059	28.06	NON-METAL MIN PRODS	1.19500
060	29.01	BASIC IRON + STEEL	1.19500
061	29.02	OTHER BASIC METALS	1.19500
062	31.01	STRUCTURAL METAL	1.33281
063	31.02	SHEET METAL PRODS	1.56900
064	31.03	METAL PRODUCTS NEC	1.37698
065	32.01	MOTOR VEHICLES, PARTS	1.20081
066	32.02	SHIP + BOAT BUILDING	1.57600
067	32.03	LOCOMOTIVES	1.57600
068	32.04	AIRCRAFT BUILDING	1.19500
069	33.01	SCIENTIFIC EQUIPT	1.35347
070	33.02	ELECTRONIC EQUIPT	1.57600
071	33.03	HOUSEHOLD APPLIANCES	1.57600
072	33.04	ELECTRICAL MACHINERY	1.48524
073	33.05	AGRICULTURAL MACH.	1.57600
074	33.06	CONSTRUCTION EQUIPT	1.57600
075	33.07	OTHER MACHINERY	1.57600
076	34.01	LEATHER PRODUCTS	0.49951
077	34.02	RUBBER PRODUCTS	1.20189
078	34.03	PLASTIC PRODUCTS	1.41223
079	34.04	SIGNS, WRITING EQUIPT	1.19500
080	34.05	OTHER MANUFACTURING	1.23813
081	36.01	ELECTRICITY	1.19500
082	36.02	GAS	1.19500
083	37.01	WATER, SEMERAGE	1.19500
084	41.01	RESIDENTIAL BUILDING	0
085	41.02	BUILDING NEC	0
086	46.01	WHOLESALE TRADE	1.19500
087	48.01	RETAIL TRADE	1.19500
088	48.02	MOTOR VEHICLE REPAIR	1.19500
089	48.03	OTHER REPAIRS	1.19500
090	51.01	ROAD TRANSPORT	0.20513
091	52.01	RAILWAY TRANSPORT	0.40358
092	53.01	WATER TRANSPORT	0.30000
093	54.01	AIR TRANSPORT	2.39971
094	55.01	COMMUNICATION	1.19500
095	61.01	BANKING	1.19500
096	61.02	FINANCE + LIFE INS	1.19500

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on the level of disaggregation used, and, at a fixed level of disaggregation, on the particular aggregation scheme used. Consequently, the estimates of the Frisch parameter  $\omega$  are unstable and in all cases higher than a recent estimate given by Williams<sup>1</sup>.

Sato has interpreted  $(-1/\omega)$  as the average elasticity of substitution among commodities<sup>2</sup>. Because (as in the case of production analysis) there is some difficulty to be expected in estimating such a parameter<sup>3</sup>, it is important to ensure that the estimate of  $\omega$  is consistent with the weight of the evidence on this matter. Therefore, following a procedure analogous to that used by Ryan<sup>4</sup>, we enforced the following condition on the demand equations (3.1) to (3.3) :

$$(3.4) \quad -\omega = 1.82 = \bar{v} / \left[ \bar{v} - \sum_{i=1}^8 \bar{p}_i \bar{y}_i \right]$$

The value - 1.82 is a weighted average of values of  $\omega$  for different types of Australian households as reported by Williams<sup>5</sup>. His estimates in turn are based partly on pooled international evidence<sup>6</sup>.

1. Ross Williams, "The Use of Disaggregated Cross-Section Data in Explaining Shifts in Australian Consumer Demand Patterns Over Time," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. SP-13, Industries Assistance Commission, Melbourne, May 1978 (mimeo).
2. Kazuo Sato, "Additive Utility Functions with Double-Log Consumer Demand Functions," Journal of Political Economy, Vol. 80, No. 1 (January/February 1972), pp. 102-124.
3. See Alan A. Powell, Empirical Analytics of Demand Systems (Lexington, Massachusetts : D. C. Heath, 1974), p. 70.
4. David Ryan, "Effect of Ethnic Origin on Household Consumption Patterns in Australia," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. SP-10, Industries Assistance Commission, Melbourne, June 1977 (mimeo).
5. Ross Williams, op. cit..
6. C. Luch, A. Powell and R. Williams, Patterns in Household Demand and Savings (New York : Oxford University Press, 1977), pp. 74-81.

APPENDIX 3 : CONSUMER EXPENDITURE ELASTICITY ESTIMATES FOR INPUT OUTPUT INDUSTRIES

8. With (3.4) imposed there are only seventeen free parameters to be estimated. We chose to eliminate  $\gamma_8$  from the list of free parameters, recovering its value from

$$(3.5) \quad \gamma_8 = \left[ 0.45055\bar{v} - \sum_{i=1}^7 \bar{p}_i \gamma_i \right] / \bar{p}_8,$$

which follows from (3.4). In these equations,  $\bar{v}$  is the average over the sample period of  $v$ , and  $\bar{p}_i$  is the sample average price of commodity  $i$ .

The marginal budget shares and total expenditure elasticities given in this paper are obtained by fitting equations (3.1) and (3.3) (subject to condition (3.5)) to data on consumer expenditure in Australia for the period 1964-65 to 1975-76.

4. THE DATA

The following data series are required for estimating the

TELES model described above :

- (i) Consumer expenditure at current prices on eight commodities ( $p_i x_i$ ) ;
- (ii) Eight price indices ( $p_i$ ) ;
- (iii) After tax income from wages and salaries ( $wH$ ) ;
- (iv) Total disposable income ( $y$ ) ;
- (v) Total hours worked ( $H$ ) ;
- (vi) Hourly wage rate ( $w$ ) ;
- (vii) Number of working persons .

Industry number	Input-Output Code	Industry Name	Elasticity Estimate
001	01.01	SHEEP	0.52000
002	01.02	CEREAL GRAINS	0
003	01.03	MEAT CATTLE	0.52000
004	01.04	MILK CATTLE	0.04534
005	01.05	POULTRY	0.01700
006	01.06	OTHER FARMING	0.54862
007	02.00	SERVICES TO AGRIC	0
008	03.00	FORESTRY	1.19500
009	04.00	FISHING	0.55493
010	11.01	IRON	1.19500
011	11.02	OTHER METALLIC MINS	1.19500
012	12.00	COAL + CRUDE OIL	1.19500
013	14.00	NON-METALLIC NEC	1.19500
014	16.00	SERVICES TO MINING	0
015	21.01	MEAT PRODUCTS	0.52038
016	21.02	MILK PRODUCTS	0.05980
017	21.03	FRUIT + VEG PRODUCTS	0.45536
018	21.04	MARGE,OILS + FATS	0.45800
019	21.05	FLOUR + CEREAL PRODS	0.15673
020	21.06	BREAD,CAKES	0.13190
021	21.07	CONFECTIONERY	0.42974
022	21.08	FOOD PRODUCTS NEC	0.52549
023	21.09	SOFT DRINKS,CORDIALS	0.43422
024	21.10	BEER + MALT	0.56500
025	21.11	ALCOHOLIC DRINKS NEC	0.56500
026	22.01	TOBACCO	0.49951
027	23.01	PREPARED FIBRES	0.29100
028	23.02	MAN-MADE FIBRES,YARN	0.29100
029	23.03	COTTON,SILK,FLAX	0.29100
030	23.04	WOOL + WORSTED YARNS	0.29100
031	23.05	TEXTILE FINISHING	0.29100
032	23.06	TEXTILE FLOOR COVERS	1.57600
033	23.07	TEXTILE PRODUCTS NEC	1.46424
034	24.01	KNITTING MILLS	0.29100
035	24.02	CLOTHING	0.29100
036	24.03	FOOTWEAR	0.29100
037	25.01	SAWMILL PRODUCTS	1.38074
038	25.02	PLYWOOD,VENEERS	1.37660
039	25.03	JOINERY + WOOD PRODS	1.25608
040	25.04	FURNITURE,MATTRESSES	1.55910
041	26.01	PULP,PAPER	1.19500
042	26.02	FIBREBOARD	1.19500
043	26.03	PAPER PRODUCTS NEC	1.19500
044	26.04	NEWSPAPERS + BOOKS	1.19500
045	26.05	COMMERCIAL PRINTING	1.19500
046	27.01	CHEMICAL FERTILISERS	1.19500
047	27.02	INDUSTRIAL CHEMICALS	1.19500
048	27.03	PAINTS,VARNISHES	1.27706

Table A2.2 : Other Fares

I-0 Industry	Sub-group		Total
	13 Air Transport	14 Other Public Transport	
1 to 89			
90		153.9	153.9
91			
92		116.5	116.5
93	201.8		201.8 *
94 to 109			

Notes \* Based on estimated number of air passengers  
Figures are in 1971-72 \$ million.

The last series is required to calculate the various averages (expenditure, income, etc.) for a "typical" consumer/worker<sup>1</sup>.

(1) Consumer expenditure : Disaggregated data on private final consumption expenditure at current prices for 1964-65 to 1975-76 are given in the Australian National Accounts<sup>2</sup>. For the purposes of this paper the commodities are grouped into the following eight major groups :

- 1) Food
- 2) Drink and Tobacco
- 3) Clothing and Footwear
- 4) Housing
- 5) Household Durables
- 6) Private Transport
- 7) Public Transport
- 8) Other Goods and Services .

(ii) Price indices : The Australian National Accounts<sup>3</sup> (ANNA) also include estimates of private final consumer expenditure at average 1966-67 prices. Price indices for the eight consumption categories are obtained by dividing the current price expenditure estimates by the corresponding constant price estimates.

(iii) After tax income from wages and salaries : Data on after tax wages and salaries are not available for 1964-65 to 1975-76. Estimates of before tax income from wages, salaries and supplements and of income from

1. The increased workforce participation of married women over the period 1964-65 to 1975-76 raises the question of validity of the concept of a typical or representative consumer/worker.

2. Australian Bureau of Statistics, Australian National Accounts, National Income and Expenditure 1975-76 (ABS Ref. No. 7.1), Canberra, 1977.

3. Ibid.

other sources are given in the Australian National Accounts.<sup>1</sup> As well as estimates of total income tax paid are given. Estimates of after tax income from wages and salaries are obtained by applying the average tax rates on total income (other than government transfer payments) calculated from the total income and total tax series.

(iv) Total disposable income : Figures are published in the Australian National Accounts.<sup>2</sup>

(v) Total hours worked : Estimates of total hours worked are obtained as a sum of four quarterly figures. For each quarter the total hours worked are estimated as the product of number of employed persons, average hours per employed person per week and number of weeks per quarter.

Estimates of the number of employed persons<sup>3</sup> and average hours worked per week are obtained from various labour force survey reports.<sup>4</sup> As the average hours figure is based on the month in which the survey was conducted, it will be biased as an estimate of the quarterly figure. This error will be most severe in the November and February quarters, as the very low average hours worked in December and January will not be reflected in the November and February figures. The above method thus gives an over-estimate of total hours worked as it does not fully take into account annual holidays. It is not currently possible to make an allowance for this bias.

1. Ibid.

2. Ibid.

3. Employed persons comprised all those who, during the survey week did any work for pay, profit, or payment of any kind; worked 15 hours or more without pay in a family business or farm, or had a job but did not work because of illness, leave, holidays, etc..

4. Australian Bureau of Statistics, The Labour Force (ABS Ref. No. 6.22). Figures for 1964 to 1968 are from Historical Supplement to the Labour Force and for other years from the annual reports. Estimates of average hours per week are not available for 1964-65. The figures used are derived from the distribution of number of persons by hours worked.

APPENDIX 2 : ALLOCATION OF ABS COMMODITY GROUPS, FOOD  
AND OTHER FARES TO DIFFERENT CONSUMPTION SUB-GROUPS AND INDUSTRIES

Table A2.1 : Food Group

I-0 Industry	Food Sub-group Number						Total
	1	2	3	4	5	6	
1	15.1 (100)						15.1 (100)
2		9.3 (100)					9.3 (100)
3		1.6	26.8				28.4
4		(5.6)	(94.4)				(100)
5		68.0	91.0	2.3			161.3
6		(42.2)	(56.4)	(1.4)			(100)
7 & 8					339.6 (85.8)	56.3 (14.2)	395.9 (100)
9		56.8 (100)					56.8 (100)
10 to 14							
15		1246.7 (98.7)			8.1 (0.6)	8.1 (0.6)	1262.9 (100)
16			644.3 (89.6)	74.7 (10.4)			719.0 (100)
17	11.0 (3.3)	15.4 (4.6)		46.3 (13.9)	194.1 (58.3)	66.1 (19.8)	333.0 (100)
18						75.4 (100)	75.4 (100)
19	116.9 (98.0)					2.4 (2.0)	119.3 (100)
20	495.8 (99.7)				1.5 (0.3)		497.3 (100)
21				183.5 (97.5)		4.8 (2.5)	188.3 (100)
22	39.2 (14.9)			107.2 (40.6)	20.6 (7.8)	96.9 (36.7)	263.9 (100)
23				182.9 (88.4)	24.0 (11.6)		206.9 (100)
24 to 109							

Note 1. Figures are in 1971-72 \$ million.

2. Figures in brackets are percentages.

3. The Figures are based on the Monash 1962-63 classifier (See Harrower, op. cit.)

## APPENDIX 1 : COMMODITY CLASSIFICATION.

Eight Commodity TELES Classification	Expanded TELES Classification	ANA Goods, ABS Classification
1. Food	1. Bread & cereals 2. Meat & fish 3. Dairy & eggs 4. Sugar, preserves, confectionery 5. Fruit & vegetables 6. Other food	1. Food
2. Drink & tobacco	7. Drink & tobacco	2. Cigarettes & tobacco 3. Alcohol
3. Clothing & footwear	8. Clothing & footwear	4. Clothing & footwear
4. Housing	9. Housing	6. Rent including imputed rent
5. Household durables	10. Household durables	8. Furniture & floor coverings 9. Electrical goods 10. Hardware & other 16. Purchase of motor vehicles 17. Operation of motor vehicles
6. Private transport	11. Private transport	14. Rail, tram & bus 15. Other fares
7. Public transport	12. Rail, tram & bus fares 13. Air transport 14. Other public transport	7. Health 8. Gas, electricity & fuel 11. Newspapers, books, etc. 12. Toys, travel and sporting goods 13. Other goods 18. Postal & telephone services 19. Education services 20. Life insurance 21. Entertainment 22. Other services 23. Expenditure overseas
8. Other goods & services	15. Other goods & services	

(vi) The hourly wage rate : Estimated as the ratio of after tax wage and salary income to total hours worked. As the hours worked are over-estimated, the hourly wage rate is under-estimated. Moreover, since the definition of employed persons includes persons other than wage and salary earners, this produces a further downward bias in the wage rate series.

(vii) Number of working persons : For this variable a simple average of the four quarterly figures is used as the annual value.

#### 5. EMPIRICAL ESTIMATION OF EIGHT COMMODITY TELES MODEL

When the value of the Frisch parameter  $w$  is fixed, one of the 18 parameters in equations (3.1) and (3.3) becomes a function of the other parameters as shown in (3.5). Therefore there are only 17 parameters  $\{\beta_1, \dots, \beta_8 ; \gamma_1, \dots, \gamma_7 ; \gamma_n\}$  to be estimated. Thus,

$\gamma_8$ , i.e., the subsistence parameter for 'other goods and services', does not appear as a coefficient but can be obtained from (3.5). Similarly,  $\beta_8$  can be recovered from the normalization condition (footnote 1, page 4)<sup>1</sup>.

The system was estimated using the RESIMUL package<sup>2</sup>. The solution gives numerical estimates of  $\beta_1$  which represents the marginal budget shares out of 'full income'.<sup>3</sup> They can be used to estimate the elasticities of interest in the ORANI model, viz., the 'total expenditure' elasticities, defined as follows :

1. In ELES (and TELES)  $\gamma_s$  is identically zero -- see Howe, op. cit., and Tulpulé, op. cit..

2. C. R. Wymer, "Computer Programs : RESIMUL Manual," International Monetary Fund, Washington, D.C., 1977 (mimeo).

3. See page 4.

$$(5.1) \quad \epsilon_i^0 = \frac{\partial v_i}{\partial v} \cdot \frac{v}{v_i}, \quad i = 1, \dots, 8;$$

where, we remind the reader<sup>1</sup>

$$v = \sum_{i=1}^8 v_i.$$

An expression for  $v$  can be obtained by summing over the eight commodities in (3.1).

$$(5.2) \quad v = \sum_{i=1}^8 P_i Y_i = \sum_{i=1}^8 P_i Y_i + \left[ a - \sum_{i=1}^8 \beta_i \right] \left[ a - \sum_{i=1}^8 P_i Y_i + \gamma_h^w \right];$$

$$= (1 - \tilde{\beta}) \sum_{i=1}^8 P_i Y_i + \tilde{\beta} (a + \gamma_h^w),$$

where

$$(5.3) \quad \tilde{\beta} = \sum_{i=1}^8 \beta_i.$$

A small change in total expenditure may be caused by a small change in a (non-wage income) or  $w$  (hourly wage rate) or both  $a$  and  $w$ , if there are no price changes. Consider a situation in which both  $a$  and  $w$  change by the same proportion and denote these changes by  $\delta a$  and  $\delta w$ .

That is, assume that

$$(5.4) \quad \frac{\delta a}{a} = \frac{\delta w}{w} \quad \left[ \text{or } \delta a = \frac{\delta w}{w} \cdot a \right].$$

1. In formulae (5.1) and in places below, we use the affix <sup>0</sup> to indicate that the level of aggregation involved is higher than the final 109 input-output commodities for which we finally give elasticity estimates in Appendix Table 3.

TABLE 6 : ESTIMATED VALUES OF ITEMS OF NON-COMPETITIVE IMPORTS (1971-72) AND ASSUMED ELASTICITIES

Item	Value in (\$m)(a)	Elasticity
Food	42.6	0.458 (same as other food)
Furniture & floor covering	1.5	1.576 (same as durables)
Other goods	19.5	1.195 (same as other goods)
Motor vehicles	5.4	1.193 (same as pvt. transport)
Entertainment	1.2	1.417 (based on 'services' in Monash Project)
Expenditure overseas	85.0	2.0 (assumed)
Total	155.2	1.439

Source of (a) : Lawson, Tony, 'An Input-output Table for Australia, 1971-72', Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper, Industries Assistance Commission, Melbourne, 1978 (forthcoming).

### 8. CONCLUSION

This paper provides estimates of total expenditure elasticities for the 109 input-output industries and for non-competitive imports for use in the ORANI model. The elasticities for eight broad items of consumer expenditure are based on time series estimates of the TELES model using the latest available Australian National Accounts data with the value of the Frisch parameter  $\omega$  fixed a priori at 1.82. Those eight elasticity estimates were expanded first into 15 and then into 109 categories. The latter expansion involved the assumption that the input-output composition of the 15 consumption aggregates is stable. In some cases, elasticities have been derived on the basis of other relevant information or judgement.

from industry 1.05 directly to household consumption in 1968-69 (and 1971-72) were predominantly eggs. Therefore the elasticity estimate for industry 1.05 is assumed to equal the elasticity for the food sub-group dairy and eggs, namely 0.0170.

For three more industries, 11.01, 23.01 and 32.03, the procedure gave an elasticity estimate of zero. This is because according to the Q matrix there were no sales from these industries to final consumption in 1971-72. But the Q matrix was derived from mapping the vector of household final consumption expenditure from a commodity x industry input-output table at purchaser's prices with imports allocated indirectly into 23 consumption groups. Since it was based on a commodity x industry (absorption) table, there were no problems with secondary production. However the input-output data base for the ORANI model is derived from industry x industry input-output tables. In the case of these three industries there was some very minor secondary production, part of which was sold to household consumption. Therefore the ORANI model required elasticity estimates for these three industries. The estimates shown in Appendix 3 are guesses based on the likely composition of the secondary production in these industries. <sup>1</sup>

#### Non-Competitive Imports

The 109 input-output categories do not include non-competitive imports, the value of which in 1971-72 was \$155.2 million. The break-down of this into 5 of the 23 commodities is given in Table 6 along with assumed total expenditure elasticities. (The elasticity values are established by comparing the types of goods in each category with one of the 15 commodity groups in Table 5.) The weighted average elasticity for non-competitive imports is 1.439.

1. A good indication of the secondary production can be obtained from Table 1 of Australian Bureau of Statistics, Australian National Accounts - - Input-output Tables, 1968-69, Catalogue No. 5209.0.

When a and w change as above then the total expenditure v will change by  $\delta v$ .

An expression for  $\delta v$  can be obtained from (5.2).

$$(5.5) \quad \delta v = \tilde{\beta} (\delta a + \gamma_h \delta w) , \\ = \tilde{\beta} \left( \frac{a}{w} + \gamma_h \right) \delta w .$$

When the total expenditure, v, changes by  $\delta v$ , let expenditure on the  $i^{\text{th}}$  commodity change by  $\delta v_i$ . Using (3.1),  $\delta v_i$  can be expressed as

$$(5.6) \quad \delta v_i = \beta_i \delta a + \beta_i \gamma_h \delta w , \\ \text{(from (5.4))} \\ = \beta_i \left( \frac{a}{w} + \gamma_h \right) \delta w .$$

Substituting from (5.5) into (5.6), taking the limit as  $\delta v \rightarrow 0$ , and converting into elasticity terms, the required total expenditure elasticity in (5.1) will be given by

$$(5.7) \quad \frac{\partial v_i}{\partial v} \cdot \frac{v}{v_i} = \frac{\beta_i}{\tilde{\beta}} \left/ \frac{v_i}{v} \right. \\ = \frac{\beta_i}{\tilde{\beta}} \left/ \text{(average budget share of } i^{\text{th}} \text{ commodity)} \right. .$$

A value of  $\tilde{\beta} = 0.56756$  was obtained by fitting the eight commodity TELES model to Australian data for 1964-65 to 1975-76. Estimates of  $\beta_i$ , the average budget shares, and the expenditure elasticities calculated from (5.7), are shown in Table 1.

TABLE 1 : ESTIMATED MARGINAL BUDGET SHARES AND TOTAL EXPENDITURE ELASTICITIES

Commodity	$\beta_i$	$\frac{\beta_i}{\beta}$	Average budget share*	Total expenditure elasticity
1. Food	0.03638 (0.00198)	0.06410 (0.00300)	0.1915	0.335 (0.016)
2. Drink & Tobacco	0.02945 (0.00145)	0.05189 (0.00090)	0.0919	0.565 (0.010)
3. Clothing & Footwear	0.01517 (0.00276)	0.02674 (0.00402)	0.0919	0.291 (0.044)
4. Housing	0.14163 (0.00643)	0.24954 (0.00458)	0.1322	1.888 (0.035)
5. Household Durables	0.07101 (0.00330)	0.12512 (0.00635)	0.0794	1.576 (0.080)
6. Private Transport	0.07341 (0.00462)	0.12935 (0.00531)	0.1084	1.193 (0.049)
7. Public Transport	0.01566 (0.00066)	0.02759 (0.00082)	0.0322	0.857 (0.026)
8. Other Goods & Services	0.18484 (0.00736)	0.32568 (0.00418)	0.2725	1.195 (0.015)
Sum	0.56756 (0.02272)	1.00	1.00	

(\* Average budget shares at the mean of the sample period.)

Figures in parentheses are the estimated asymptotic standard errors.

$E_{ij}$  is the elasticity of consumer demand for products originating in industry  $i$  with respect to an expansion of the demand for the  $j^{\text{th}}$  consumption category. In the absence of further information, we assume that  $E_{ij} = 1$  for all  $i$  and  $j$ , and calculate the 109 total expenditure elasticities from (7.5) as :

$$(7.5) \quad \epsilon_i = \frac{\partial \log x_i}{\partial \log C} = \sum_{j=1}^{15} \frac{x_{ij}}{x_i} \cdot \epsilon_j \quad (i = 1, \dots, 109)$$

Estimates of the shares of particular consumption items in the total demands placed on industry  $i$  by consumption,  $\frac{x_{ij}}{x_i}$ , are given in Appendix 4. Together with the estimates of elasticities  $\epsilon_j$  given in Table 5 they are used to calculate the 109 elasticities  $\epsilon_i$ , which are given in Appendix 3. Under our procedure more than one input-output industry may have the same elasticity.<sup>1</sup> More disaggregated time series data would be necessary to allow any substantial improvement in this regard.

In the case of one industry, 1.05 Poultry, the above procedure was not used. According to the matrix classifier in Appendix 4 the output of this industry that is sold to household consumption is distributed to three different food groups: meat and fish, dairy and eggs, and sugar, preserves and confectionery. It will be recalled that a 1962-63 Q matrix was used to expand the food group into six sub-groups. According to the 1962-63 input-output tables most of the output of the poultry industry was sold to household consumption. However in the 1968-69 input-output tables there were significant sales from industry 1.05 to industry 21.01, Meat Products. It appears that in the more recent input-output table chicken meat is sold to household consumption via industry 21.01 and so the sales

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1. There are 46 distinct elasticity values.

The 109 x 15 Q matrix converted into a proportions matrix is shown in Appendix 4. This proportions matrix is used in estimating the 109 elasticities as follows:

Let

$X_{ij}$  = quantity index (viz., base period dollar value) of consumption of type  $j$  originating in industry  $i$

( $i = 1, \dots, 109$ )  
( $j = 1, \dots, 15$ );

$x_i$  = consumer demand for the products of industry  $i$ ;

$\delta x_i$  = change in consumer demand for goods produced by industry  $i$  due to a change in total consumption from  $C$  to  $(C+\delta C)$ .

Now we can write

$$(7.1) \quad \delta x_i = \sum_{j=1}^{15} \frac{\delta x_i}{\delta x_j^0} \delta x_j^0 \quad (i = 1, \dots, 109),$$

where

$x_j^0$  = total consumption expenditure in consumption category  $j$  ( $j = 1, \dots, 15$ ).

(7.1) can be written as

$$(7.2) \quad \delta x_i = \sum_j \frac{\partial x_i}{\partial x_j^0} \frac{\partial x_j^0}{\partial C} \delta C,$$

and

$$(7.3) \quad \frac{\delta x_i}{x_i} = \sum_j \frac{\partial x_i}{\partial x_j^0} \cdot \frac{x_j^0}{x_i} \frac{\partial x_j^0}{\partial C} \frac{\delta C}{C}.$$

Now, taking limits as  $\delta C \rightarrow 0$ , and converting into the elasticity form, (7.3) can be written as

$$(7.4) \quad \frac{\partial \log x_i}{\partial \log C} = \sum_j \left[ \frac{\partial x_i}{\partial x_j^0} \frac{x_j^0}{x_i} \right] \left[ \frac{\partial x_j^0}{\partial C} \frac{C}{x_j^0} \right],$$

$$= \sum_j E_{ij} \cdot \frac{x_{ij}}{x_i} \cdot e_j^0.$$

Comparison of TELES Results with Other Estimates

In Table 1,  $\beta_i$  is the marginal propensity to consume the  $i^{\text{th}}$  commodity out of full income. To obtain the marginal propensity  $\mu$  to consume out of actual income, these  $\beta_i$  values are summed and renormalized to yield

$$\mu = \sum_{i=1}^8 \beta_i / (1 - \beta_1).$$

The marginal propensities to consume and to save as obtained from this study and an earlier one<sup>1</sup> are compared below.

	TELES This study	ELSES Earlier study
Marginal propensity to consume, $\mu$	0.784	0.796
Marginal propensity to save, $(1 - \mu)$	0.216	0.204

Clearly, the two models yield highly compatible results, even though the sample periods differ considerably<sup>2</sup>.

In Table 2, estimates of expenditure elasticities obtained in previous Australian studies based on data up to 1966-67 are compared with the TELES results. The classifications used are not exactly the same but most commodity groups are roughly comparable.

1. C. Luch, A. Powell and R. Williams (LPW), Patterns in Household Demand and Saving, op. cit.
2. Sample periods were: For LPW, 1955-56 to 1966-67; for the present study, 1964-65 to 1975-76.

TABLE 2 : EXPENDITURE ELASTICITY ESTIMATES FOR EIGHT BROAD COMMODITY GROUPS : TELES 1964-65 TO 1975-76 \* CONTRASTED WITH OTHER AVAILABLE ESTIMATES

Commodity	Monash Study - Powell(a), 1955-56 through 1966-67	World Bank Study Litch, Williams* (b) : 1955 through 1966	TELES - Tulpule, Powell : 1964-65 through 1975-76
1. Food	0.447	0.43	0.335
2. Tobacco & Drink	0.480	0.45	0.565
3. Clothing	0.519	0.45	0.291
4. Rent	1.887	1.73	1.888
5. Durables	1.222	1.06	1.576
6. Private Transport	2.217	1.70	1.193
7. Public Transport	0	1.70	0.857
8.1 (Services)	(1.417)		
8.2 (Other)	(0.536)	1.28	1.195
8.3 (Fuel, Gas & Elec.)	(1.044)		

\* TELES estimated with Frisch parameter at sample mean set a priori to - 1.8 (figure based on Williams, op. cit.; applicable to 1974-75).  
 \* Classification of commodities not strictly compatible with the other two studies.  
 (a) Alan A. Powell, "A Linear Expenditure System for Australia 1955-56 through 1966-67," Monash University, Department of Economics, Economic Analysis of Protection Project, 1973 (mimeo).  
 (b) C. Litch, A. Powell and R. Williams, Patterns in Household Demand and Saving (New York : Oxford University Press, 1977), Table 3.6, pp. 44-45.

1. The matrix is documented in J.D. Harrower, "A Matrix Classifier for Transforming Consumer Goods into Input-Output Industries", IMPACT of Demographic Change on Industry Structure in Australia, Research Memorandum (SNAPSHOT Module), Industries Assistance Commission, Melbourne, July 1976.

The Australian Bureau of Statistics has prepared a matrix (called the Q matrix in this paper) which shows the industry origin of 23 groups of consumption items for 1968-69. Thus a typical element  $q_{ij}$  in the 109 x 23 Q matrix gives the value of  $j^{th}$  commodity that was supplied by the  $i^{th}$  industry at 1968-69 market prices. The value includes local production as well as imports. The Q matrix was updated to 1971-72 values by IMPACT staff. Thus,

$$\sum_j q_{ij} = (\text{total final consumption in 1971-72})$$

A list of the 23 commodities is given in Appendix 1. All food items are included in one consumption category. The food column (109 x 1) was expanded into a 109 x 6 matrix by allocating the food expenditure in each industry group into six food sub-groups using information on a 1962-63 Q matrix used in the Monash University Econometric Analysis of Projection Project.<sup>1</sup> Similarly column 15 other fares, which includes air travel and other travel was split into two columns. The split of column numbers 1 and 15 into 6 and 2 columns respectively is shown in Appendix 2. Thus the original ABS (109 x 23) Q matrix was first expanded into a (109 x 29) matrix. The 29 columns were reaggreated into the 15 commodity groups shown in Table 5 for which expenditure elasticities have been calculated. The classification used in the allocation of the 29 commodities into the 15 groups is shown in Appendix 1.

The elasticity estimates obtained so far are summarized in Table 5.

TABLE 5 : ESTIMATES OF TOTAL EXPENDITURE ELASTICITIES

Item of Consumption	Elasticity
1. Bread and Cereals	0.131
2. Meat and Fish	0.520
3. Dairy and Eggs	0.017
4. Sugar, Preserves and Confectionery	0.429
5. Fruit and Vegetables	0.474
6. Other food items	0.458
7. Drink and Tobacco	0.565
8. Clothing	0.291
9. Housing	1.888
10. Household Durables	1.576
11. Private Transport	1.193
12. Rail, Tram and Bus Fares	0.100
13. Air Transport	2.500
14. Other Public Transport	0.300
15. Other Goods and Services	1.195

#### 7. ELASTICITY ESTIMATES FOR THE INPUT-OUTPUT INDUSTRIES

The 15 elasticity estimates are expanded into 109 commodity groups produced by the 109 input-output industries on the basis of the contribution of each industry to each of the 15 consumption categories. The procedure adopted in estimating the 109 elasticities is as follows :

The TELES results are similar to the earlier estimates with two major exceptions. Thus the elasticity of Clothing and Footwear (0.291) is much lower than in previous studies and so is the elasticity for private transport. In the case of clothing the decrease in the elasticity may be due to a change in taste. In the case of private transport the decrease could be partly due to an approach towards a saturation level of car ownership and also due to a change in taste.

The small increase in the elasticity for durables could be due to the introduction of several new types of consumer durables that have been introduced on the market in the last decade.

#### 6. DERIVED ESTIMATES FOR FOOD AND PUBLIC TRANSPORT SUB-GROUPS

The elasticities obtained for the eight broad commodity groups are used to derive separate estimates for the 109 commodities produced by the 109 input-output industries in two stages. Firstly, the elasticities for the food and public transport groups are expanded into six estimates for items in the food category and three for items in the public transport category respectively using estimates obtained in other studies, and secondly, these fifteen estimates are expanded to give separate elasticities for the 109 input-output industries. The second of these stages is covered below in Section 7.

The Food Sub-group

Suppose that the demand for the food group is given by

$$(6.1) \quad V_1 = P_1^* X_1 = P_1^* Y_1 + \beta_1^* \left[ v - \sum_{i=1}^8 P_i Y_i \right];$$

and within the food group, the demand for the  $j^{\text{th}}$  sub-group is given by a nested expenditure system<sup>1</sup>

$$(6.2) \quad V_{1j} = P_{1j}^* X_{1j} = P_{1j}^* Y_{1j} + \beta_{1j} \left[ v_1 - \sum_{j=1}^6 P_{1j} Y_{1j} \right], \quad (j=1, \dots, 6);$$

where  $\beta_1^* = \beta_1 / \beta$ .

We evaluate the derivative of consumption of the  $j^{\text{th}}$  food group with respect to total consumption expenditure by

$$(6.3) \quad \frac{\partial V_{1j}}{\partial v} = \frac{\partial V_{1j}}{\partial v_1} \frac{\partial v_1}{\partial v}.$$

1. Note that it is the expenditure system which is nested and not the utility function (which remains directly additive). For a discussion of nested systems see Richard Stone, Mathematical Models of the Economy and Other Essays (London: Chapman and Hall, 1970), pp. 85-86.

passenger miles in 1960 and 1972 give an elasticity estimate of 2.67 . An estimate of 2.94 is obtained for the U.S. if 1965 and 1972 data are used.

The above calculations suggest that for air transport the elasticity would be high; a value of 2.5 is assumed here.

Other Travel

The expenditure elasticity for other travel (which includes taxi fares) is likely to be higher than that for rail and bus travel. The elasticity can now be estimated on the basis of the assumed values for rail, tram and bus travel and air travel, the average budget shares for the three items within the group and the elasticity for the public transport group as a whole (0.857) estimated in Section 3. The figures are summarized in Table 4.

TABLE 4 : EXPENDITURE ELASTICITIES FOR THE PUBLIC TRANSPORT SUB-GROUPS

Sub-group	Average budget share *	Elasticity
Rail, tram and bus	0.370	0.1
Air	0.287	2.5
Other	0.343	0.3
Total	1	

\* Our estimate

found to range from 0.08 to 0.57. In view of these estimates it is reasonable to assume a very low total expenditure elasticity for rail, bus and tram travel. A value of 0.1 is assumed here.

#### Air Transport

Data on internal (i.e., within Australia) passenger miles by fare paying passengers and number of passengers are available on a time series basis<sup>1,2</sup> but the share of government or business travel is not known. Between 1967-78 and 1975-76 passenger miles in Australia increased by 120 per cent and the number of passengers doubled. A crude estimate of the total expenditure elasticity without taking into account the own price effect or the cross price effects and assuming that the share of government and business travel is constant, can be obtained as follows :

$$\text{elasticity} = \frac{\log \left( \frac{\text{air travel in 1975-76}}{\text{air travel in 1967-68}} \right)}{\log \left( \frac{\text{Total expenditure at constant prices in 1975-76}}{\text{Total expenditure at constant prices in 1967-68}} \right)}$$

If passenger miles are used as a measure of air travel the above formulae yields an estimate of elasticity as 2.15, and the use of number of passengers as a measure of air travel gives an estimate of 1.86. A longer series is not available for Australia but similar calculations on US data<sup>3</sup> on

1. Australian Bureau of Statistics, Commonwealth Year Book, op. cit..
2. Australian Bureau of Statistics, Pocket Compendium, op. cit..
3. U.S. Department of Commerce and Bureau of Census, Statistical Abstract of the United States 1975, Washington, 1975.

Making use of (6.1) and (6.2), (6.3) can be written as

$$(6.4) \quad P_{1j} \frac{\partial x_{1j}}{\partial v} = \beta_{1j} \beta_1^*$$

Multiplying both sides of (6.4) by  $\frac{v}{P_{1j} x_{1j}}$  yields

$$(6.5) \quad \frac{\partial v}{\partial v} \frac{x_{1j}}{v} \cdot \frac{v}{P_{1j} x_{1j}} = \frac{\partial x_{1j}}{\partial v} \cdot \frac{v}{P_{1j} x_{1j}} = \beta_{1j} \beta_1^* / \xi_{1j}$$

where

$$(6.6) \quad \xi_{1j} = \frac{P_{1j} x_{1j}}{v} = \text{average budget share of } 1j^{\text{th}} \text{ item out of total expenditure } v.$$

Thus the elasticity of demand for the  $j^{\text{th}}$  sub-group within the food category with respect to total expenditure is given by (6.5).

The estimated value of  $\beta_1^*$  given in Table 1 is 0.06410. It is not possible to obtain estimates of  $\beta_{1j}$  and the average budget shares within the food group from recent national accounts. Instead, estimates given by Powell<sup>1</sup> for 1955-56 to 1966-67 are used. The elasticities for the food sub-groups thus obtained are given in Table 3.

1. A. Powell, "A Linear Expenditure System for Australia ...", op. cit..

obtained from assumed values for the first two categories and approximate estimates of budget shares for the three public transport sub-groups.

Rail, Bus and Tram Fares

In the absence of data on expenditure at constant prices it would be appropriate to look at the trends in passenger miles as they represent changes in real consumption. However, these data are not available for all the travel modes on a time series basis for Australia. Between 1967-68 and 1970-71 passenger miles on Victorian, South Australian, Tasmanian and Commonwealth railways increased<sup>1</sup> by about 3 per cent. Data on passenger miles by tram and bus are not available. However, the number of passengers decreased from 1967-68 to 1974-75<sup>1,2</sup> by about 13 per cent. Some of the decrease may be associated with rising costs of travel and some with increased use of private transport. Time series analysis of U.K. data<sup>3</sup> gave income elasticities of 1.00 and 0.28 for rail and bus travel respectively. In a number of U.S. studies<sup>4</sup> the income elasticity for public transport (transit expenditure) is

1. Australian Bureau of Statistics, Commonwealth Yearbook 1973 (No. 59), Canberra, 1973.
2. Australian Bureau of Statistics, Pocket Compendium of Australian Statistics 1977 (No. 62), Canberra, 1977.
3. A. Tulpułé, "Factors Affecting the Demand for Travel." Paper presented at the Australian Transport Research Forum, Sydney, 1975.
4. J. M. Morris and M. R. Wigan, "Transport Planning: A Family Expenditure Perspective," Australian Road Research Board Report AAR No. 71, 1978.

TABLE 3 : AVERAGE AND MARGINAL BUDGET SHARES WITHIN THE FOOD GROUP AND EXPENDITURE ELASTICITIES FOR THE FOOD SUB-GROUPS

Food sub-group	$\hat{\beta}_{1j}$	Average budget share within the food group	$\hat{\xi}_{1j}$	Elasticity
1. Bread & Cereals	0.0603	0.154	0.02949	0.1311
2. Meat & Fish	0.4488	0.289	0.05534	0.5199
3. Dairy & Eggs	0.0103	0.203	0.03887	0.0170
4. Sugar, Preserves, Confectionery	0.1704	0.133	0.02547	0.4289
5. Fruit & Vegetables	0.2334	0.165	0.03160	0.4735
6. Other	0.0766	0.056	0.01072	0.4581
Total	1	1	0.19149	

Public Transport Group

The public transport category includes expenditure on the following items :

1. Rail, bus and tram fares
2. Air travel
3. Other travel (shipping, ferry and taxi fares).

Estimates of expenditure on rail, tram and bus fares at current prices are given in the Australian National Accounts<sup>1</sup> but a constant price series is not available. Similarly, separate expenditure figures for air fares and other travel at current or constant prices are not available. In the absence of appropriate data series the estimates of elasticities for rail, bus and tram fares, and air travel are based on related series on volumes of traffic and elasticity estimates given in other studies. The elasticity for other travel is

1. Australian Bureau of Statistics, op. cit.