



IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

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ESTIMATION OF THE FIRM HOUSEHOLD MODEL

by

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1. Introduction

In this paper we are interested in analyzing the behaviour of a typical firm-household which enters into decisions on consumption, production, investment and financing. The assumption of an imperfect capital market means that these decisions are necessarily interdependent - for example, consumption competes directly with investment for available funds. The emphasis of the model is with the long-term growth or decline of the enterprise. The model is designed to be applied to panel-type data, and so is meant to be indicative of individual enterprises each involved in one of a broad range of activities.

To allow concentration on long-term changes, the model is specified as a free-endpoint type - at each point in time the enterprise is allowed to reorganize its portfolio of capital assets to give an optimal configuration. Two points about this assumption need to be made. First, application is aimed at the micro level so that supply of capital (even land) to each individual enterprise is assumed infinitely elastic and general equilibrium considerations are avoided. This is in contrast to Dixon, Vincent and Powell (1976) where aggregative data is used, and such problems are overcome by holding capital assets fixed, thereby avoiding the intertemporal aspect. Secondly, however, general equilibrium aspects cannot be ignored completely, since otherwise a single model would predict only one pattern of production, clearly at variance with reality. This problem is overcome by considering broad classes of models based on broad categories of activities and the corresponding production functions. Thus, for example, a dairy farmer is constrained to face a dairying type production function, and so will not suddenly change to wheat if wheat prices rise. In effect, supply aspects are introduced by differentiating between land suitable for certain activities. These constraints are technical in nature, and best imposed from outside the model. Thus the apparent ease of reallocation introduced by the free-endpoint assumption is in fact constrained within the model by the prior specification of the production function.

The model is specified in the next section, and the rest of the paper is devoted to estimation. A more complete derivation of the model, detailing its relationship with previous models, can be found in McLaren (1976).

2. The Model Specification

Consider the following production structure:

an m -vector of capital goods K' with price vector c' ;
 an l - " " variable inputs L' " rental " w' ;
 a k " " outputs Q' " price " s' ;

subject to a technology given by the scalar production frontier $f(Q, K, L) = 0$. The distinction between capital goods K and variable inputs L is that capital goods require a conscious long-term committment of funds (eg. land, machinery, buildings etc.) whereas variable inputs can be financed within the production period (eg. contract labour, fertilizer etc.).

With K given, the variable profit function is defined by

$\pi(s, w, K) = \max(s'Q - w'L)$ subject to $f(Q, K, L) = 0$;
 see Diewert (1974). Maximization of π is in the nature of a short-run optimizing decision. There is a duality between π and f , and the output supply and input demand functions are given by

$$Q_i = \frac{\partial \pi}{\partial s_i} \qquad L_i = \frac{\partial \pi}{\partial w_i}$$

In the longer run, there is the possibility of reallocating total capital investment $X = c'K$ into alternative configurations of fixed capital assets. Thus consider the revenue function defined by

$$R(s, c, w, X) = \max_K \pi(s, w, K) \quad \text{subject to } c'K = X$$

There will be a duality among R , π and f . From the point of view of an integrated consumption-production-investment-financing model, the function R is the appropriate specification of the productive sector of the model, and $\frac{\partial R}{\partial X}$ represents the marginal return of \$1 invested in the production sector. In the literature, attention is concentrated on functional forms for f or π . Thus functional forms for R will have to be derived from these, or specified directly subject to appropriate conditions (e.g., increasing in s ; decreasing in c, w ; increasing in X ; homogeneous of degree 1 in s, c, w, X).

Similarly on the consumption side if we consider an n - vector of commodities q' with prices P' , and direct utility function $U(q)$, we can define the indirect utility function

$$V^*(P, E) = \max_q U(q) \text{ subject to } P'q \leq E .$$

V^* is the appropriate specification of the consumption sector for an intertemporal model, and individual demand functions can be derived if required by results from duality theory. (See Diewert 1974.)

Consider now the following intertemporal optimization model:

$$\text{maximize } \int_0^{\infty} e^{-\delta t} V^*(P, E) dt$$

$$\text{subject to } R(s, c, w, X) + \dot{B} = E + \dot{X} + r(Z).B$$

$$\text{and } X(0) - B(0) = W_0 \text{ given ,}$$

where B is total borrowing with rate of interest r ,

$$Z = \frac{B}{X} \text{ and } r'(Z) > 0 .$$

Structure to the problem so far is given by the intertemporal additivity of the utility functional and the additive structure of the accounting identity. Further structure will be given by specifying functional forms for V^* , R and r , but the major behavioural aspects of the model are contained in the specification above. Briefly, utility is generated only by consumption spending, so that, for example, a large farm is not desirable per se, but only in so far as it generates revenue to allow purchase of consumption goods. Prices of consumption goods, P , outputs, s , variable inputs, w , and capital goods, c , are assumed constant in planning time, independent of decisions of the enterprise. The enterprise borrows from external sources, and pays an average rate of interest given by the $r(Z)$ schedule. Therefore although the actual rate of interest is determined by the actions of the enterprise, and so endogeneous, the schedule itself is subject to the same assumptions as prices. Thus strictly speaking the model will generate demand schedules for funds, capital goods and variable inputs, and supply schedules for outputs, and so is only part of a general equilibrium specification. Such a complete model would jointly determine all prices and quantities and answer questions such as, for example, the effects that low beef prices, high labour costs, high interest rates, particular levels of tariffs on machinery, etc., would have on prices of land suitable for beef production, on spending decisions, etc. Such a general equilibrium approach is

not followed here, since the model is to be used for a data base consisting of panel data on individual enterprises, and so an assumption of constant prices may be reasonably appropriate, allowing treatment of demand and supply schedules as reduced forms.

One problem generated by the assumption of constant prices is the implied perfect second-hand market for capital goods, clearly an unrealistic assumption. For example, a well cannot be sold. This problem can be partially offset by appropriate specification of the production function, so that instead of specifying land, fences and wells as separate capital goods, they would be lumped together as "improved land".

In the basic model there are two sources of funds, net revenue from productive activities and net new borrowing, and three uses of funds, consumption spending, net investment and interest payments. A maintained assumption is that all enterprises are net borrowers, and this is assumed in the specification of $r(Z)$. The assumption that the average rate of interest increases with the debt-asset ratio is somewhat arbitrary, but is probably not too unrealistic. To make the basic model operational, refinements such as taxes, tax credits, depreciation and external income sources would be introduced.

To derive first order necessary conditions for an optimum, set up the Hamiltonian

$$H(t, X, B ; E, A ; \psi_1, \psi_2) \\ = e^{-\delta t} V^*(P, E) + \psi_1 (R(X) + A - E - r(Z).B) + \psi_2 A$$

where $\dot{B} = A$ is a dummy control law, the states are X, B , and controls E, A . Two further derived state variables are $Z = \frac{B}{X}$ and $W = X - B$. W is in fact the basic state variable, specifying at each point in time the net worth of the entity, and giving the left-hand endpoint condition $W(0) = W_0$ and right hand endpoint condition $\lim_{t \rightarrow \infty} W(t) \geq 0$. In fact it will turn out that while mathematically W, X, B and Z are all states, economically X, B , and Z will be control (decision) variables. This is because the free endpoint assumption together with continual replanning means that for each change in exogenous variables X and B must be brought into an optimal relationship.

First order necessary conditions are:

$$(i) \quad \dot{X} = H_{\psi_1} = R(X) + A - E - r(Z).B;$$

$$(ii) \quad \dot{B} = H_{\psi_2} = A;$$

$$(iii) \quad \dot{\psi}_1 = -H_X = -\psi_1 [R'(X) + r'(Z).Z^2];$$

$$(iv) \quad \dot{\psi}_2 = -H_B = \psi_1 [r(Z) + r'(Z)Z];$$

$$(v) \quad H_E = e^{-\delta t} \frac{\partial V^*}{\partial E} - \psi_1 = 0;$$

$$(vi) \quad H_A = \psi_1 + \psi_2 = 0.$$

First note that conditions (iii), (iv) and (vi) give

$$(2.1) \quad R'(X) = r(Z) + r'(Z)Z(1-Z),$$

solution of which gives the optimal debt/asset ratio Z^* , depending on the form of R and r , but completely independent of the consumption sector. In principle this equation could be solved to give B and X in terms of W . But an extremely important case is that in which $R(X) = h(c, s, w).X$ so $R'(X) = h$ is not a function of X , and the resulting Z^* is constant in planning time. This would be the case if the production structure exhibited constant returns to scale, and this assumption is maintained in the sequel.

$$\text{Now define } g = h + r'(Z^*)Z^{*2} = r(Z^*) + r'(Z^*)Z^*$$

so, from (iii), it follows that

$$\dot{\psi}_1 = -g\psi_1, \text{ giving } \psi_1(t) = \psi_0 e^{-gt} \text{ with } \psi_0 \text{ a constant. Equation (v)}$$

then allows solution of $E(t)$ in terms of ψ_0 and h , the explicit solution depending on the functional form chosen for V^* . If for example U is Klein-Rubin, $U(q) = \beta' \ln(q-\gamma)$, then the indirect utility function is

$$V^*(P, E) = \text{constant} + \ln(E-P'\gamma) - \sum \beta_i \ln P_i.$$

With this V^* , (v) is

$$\frac{e^{-\delta t}}{E-P'\gamma} = \psi_0 e^{-gt},$$

$$\text{or } E = P'\gamma + \frac{1}{\psi_0} e^{(g-\delta)t}.$$

From this point on, only optimal Z appears, so the $*$ is deleted.

Now

$$\begin{aligned}\dot{W} &= \dot{X} - \dot{B} = hX - E - r(Z)B \\ &= \frac{h}{1-Z} \cdot W - E - r(Z) \frac{Z}{1-Z} W.\end{aligned}$$

$$\begin{aligned}\text{But } h - r(Z)Z &= r(Z) + r'(Z)Z - r'(Z)Z^2 - r(Z)Z \\ &= (1-Z)[r(Z) + r'(Z)Z] \\ &= (1-Z)g;\end{aligned}$$

$$\text{so } \dot{W} = gW - \frac{1}{\psi_0} e^{(g-\delta)t} - P'\gamma$$

with general solution

$$W(t) = \frac{e^{(g-\delta)t}}{\psi_0 \delta} + \frac{P'\gamma}{g} + C_1 e^{gt}.$$

Transversality gives $C_1 = 0$ and $\frac{1}{\psi_0} = \delta[W(0) - \frac{P'\gamma}{g}]$ giving the open loop solution for $W(t)$, $\psi_1(t)$ and $E(t)$. In fact

$$\frac{1}{\psi_1(t)} = \delta(W(t) - \frac{P'\gamma}{g})e^{\delta t},$$

giving the closed loop solution for $E(t)$

$$(2.2) \quad E = P'\gamma + \delta(W - \frac{P'\gamma}{g}).$$

The optimal closed loop solutions for B and X are simply stated in terms of Z :

$$(2.3) \quad X = \frac{1}{1-Z} \cdot W;$$

$$(2.4) \quad B = \frac{Z}{1-Z} W.$$

Similarly for \dot{X} and \dot{B} , using

$$\dot{W} = (g-\delta)[W - \frac{P'\gamma}{g}], \text{ it follows that:}$$

$$(2.5) \quad \dot{X} = \frac{1}{1-Z} \cdot (g-\delta)[W - \frac{P'\gamma}{g}];$$

$$(2.6) \quad \dot{B} = \frac{Z}{1-Z} (g-\delta)[W - \frac{P'\gamma}{g}].$$

Note that (2.5) and (2.6) are valid in planning time only, since in calendar time we can expect X and B to have discrete jumps caused by discrete changes in exogeneous variables requiring discrete changes in Z . Relations (2.3) and (2.4) take account of this, and so are more appropriate. For further elaboration see McLaren (1976) Section 6. Equations (2.1) to (2.6) provide a complete characterization of the closed loop solution to the optimization problem, with unknowns δ and the parameters of V^* , R and r . Note also that (2.1) to (2.6) are not independent - in fact a subset such as (2.1), (2.2) and (2.3) is sufficient. It remains to specify r and R , which will then determine h , g and Z .

For the revenue function R , one approach would be to specify a form for $f(Q, K, L)$ and derive $R(s, c, w, X)$ by constrained maximization. This approach is feasible for a Cobb-Douglas or C.E.S. - C.E.T. specification, but while these are useful in that they parameterize on the elasticities of substitution and transformation, they are too restrictive for more than two inputs or two outputs. Generalization, for example to a CRETH-CRESH formulation, would not seem to enable an explicit derivation of R , although we should note that the method developed in Dixon, Vincent and Powell (1976) may allow $\partial R/\partial X$ to be estimated as a shadow price, even if the explicit analytical form is unknown. Alternatively, deriving R from a specification of $\pi(s, w, K)$ is a simpler constrained optimization problem. Thus consider Diewert's translog variable profit function (where $v = s, w$):

$$(2.7) \quad \ln \pi(v, K) = \alpha_0 + \sum \alpha_i \ln v_i + \frac{1}{2} \sum \sum \gamma_{ih} \ln v_i \ln v_h \\ + \sum \sum \delta_{ij} \ln v_i \ln K_j + \sum \beta_j \ln K_j \\ + \frac{1}{2} \sum \sum \phi_{jk} \ln K_j \ln K_k,$$

(see Diewert (1974)).

The following constraints are sufficient to ensure homogeneity of degree 1 in v :

$$(i) \quad \sum_i \alpha_i = 1$$

$$(ii) \quad \sum_i \delta_{ij} = 0 \quad \text{for all } j$$

$$(iii) \quad \gamma_{ih} = \gamma_{hi}$$

$$(iv) \quad \sum_i \gamma_{ih} = 0 \quad \text{for all } h .$$

Homogeneity of degree 1 in K requires

$$(v) \quad \sum_i \beta_j = 1 ,$$

$$(vi) \quad \sum_j \delta_{ij} = 0 \quad \text{for all } i ,$$

$$(vii) \quad \sum_j \phi_{jk} = 0 \quad \text{for all } k ,$$

$$(viii) \quad \sum_k \phi_{jk} = 0 \quad \text{for all } j .$$

Solution for the form of R is facilitated by the further constraint

$$(ix) \quad \phi_{jk} = 0 \quad \text{for all } j,k .$$

This constraint is justified only on the basis of mathematical tractability, and its implications are considered below. Using this constraint, and maximizing (2.7) subject to $c'K = X$ gives the set of conditions

$$\lambda c_j K_j = (\beta_j + \sum_i \delta_{ij} \ln v_i) ;$$

and using conditions (v) and (vi) gives $\lambda = \frac{1}{X}$, so that

$$(2.8) \quad \frac{c_j K_j}{X} = (\beta_j + \sum_i \delta_{ij} \ln v_i) .$$

Thus the proportion of X allocated to capital good j depends on prices of variable inputs and outputs. This means that the effect of the constraints $\phi_{jk} = 0$ is to constrain the elasticity of substitution between capital inputs to be unity. However the specification is more general than Cobb-Douglas in capital inputs, as the above proportion is not constant. Hotellings Lemma gives the following share equations for variable inputs and outputs.

$$(2.9) \quad S_i = \alpha_i + \sum_h \gamma_{ih} \ln v_h + \sum_j \delta_{ij} \ln K_j .$$

Substitution of (2.8) into (2.7) gives $\ln R$, and it can be seen that

$$\begin{aligned}
 (2.10) \quad \ln \frac{\partial R}{\partial X} &= \alpha_0 + \sum \alpha_1 \ln v_1 + \frac{1}{2} \sum_i \sum_h \gamma_{ih} \ln v_i \ln v_h - \sum_j \sum \delta_{ij} \ln v_i \ln c_j \\
 &+ \sum_j \sum \delta_{ij} \ln v_i \ln(\beta_j + \sum_i \delta_{ij} \ln v_i) \\
 &- \sum_j \beta_j \ln c_j + \sum_j \beta_j \ln(\beta_j + \sum_i \delta_{ij} \ln v_i) .
 \end{aligned}$$

This equation gives $R'(X)$ ($\lambda=h$).

Turning now to $r(Z)$, the simplest specification would seem to be

$$(2.11) \quad r(Z) = r_1 + r_2 Z$$

since at least two parameters seem necessary to capture the two aspects of "general level" and "effect of security". Then (2.1) gives

$$\begin{aligned}
 h &= r_1 + 2r_2 Z - r_2 Z^2 \\
 \text{and } Z &= \frac{2r_2 \pm \sqrt{4r_2^2 - 4r_2(h-r_1)}}{2r_2} .
 \end{aligned}$$

Since we are interested in $0 < Z < 1$, the "minus" solution is clearly appropriate, giving the two boundaries for Z :

$$\begin{aligned}
 Z &= 0 && \text{for } h = r_1 \\
 Z &= 1 && \text{for } h = r_1 + r_2
 \end{aligned}$$

Simplifying the solution for Z :

$$(2.12) \quad Z = 1 - \left(\frac{r_2 - h - r_1}{r_2} \right)^{\frac{1}{2}}$$

3. Some Econometric Considerations

Clearly a full systems implementation would be desirable and fully efficient, but hardly practicable. A multi-step estimation procedure, based on the multistage nature of the optimization, may represent enough saving in computing to offset any inefficiency.

Denote observations by O_{ijt} , meaning enterprise i , involved in activity j at time t . Assuming all activities and individuals face the same $r(Z)$ schedule, r_1 and r_2 can be estimated for each time point t , thus generating a time series of r_1 , r_2 .

This is achieved by fixing t , say \bar{t} , and pooling observations across enterprises and activities, i.e., $O_{ij\bar{t}}$, $W_{i,j}$. Observations on the average rate of interest r_{ij} and debt-asset ratio Z_{ij} allow regression estimates of the interest rate function:

$$r_{ij} = r_1 + r_2 Z_{ij}$$

for $t = \bar{t}$, giving estimates $\hat{r}_1^{\bar{t}}$, $\hat{r}_2^{\bar{t}}$. Repeating the above process for $t = 1, \dots, T$ generates the set of estimates \hat{r}_1^t , \hat{r}_2^t ; $t = 1, \dots, T$.

Consider now the following implied stacking of optimization decisions:

- (a) given W choose X ;
- (b) given X choose the individual K_j ;
- (c) given the vector K' , choose L_i and Q_k .

The recursive nature of these optimization problems allows a recursive estimation procedure, and is our justification for beginning with a variable profit function and deriving the implied revenue function, rather than specifying the revenue function directly.

Now we pool observations across enterprises and time, holding the activity fixed, i.e., $O_{ij\bar{t}}$, $W_{i,t}$. For activity \bar{j} , estimation of equations (2.8) and (2.9), subject to appropriate restrictions, generates estimates of most of the parameters of the variable profit function. Clearly (2.8) poses no problems, since only prices appear as regressors. In (2.9) the variables K_j appear as regressors, and this is justified by the recursive nature of the optimization. Alternatively, once (2.8) is estimated, it could be used to generate a "purged" K series for use in (2.9). Only α_0 remains, and can easily be estimated by moving all variables to the left of (2.7). Incorporation of a time trend would allow for neutral technical progress over the sample period. Equation (2.10) then allows the generation of a series for $R^i(X)=h$ across time for activity \bar{j} . Repeating for all activities $j=1, \dots, J$ generates a series for h , \hat{h}_{jt} .

The two separate poolings of observations have thus generated estimates \hat{h}_{jt} , \hat{r}_1^t , \hat{r}_2^t . Equation (2.12) then allows the estimation of the implied optimal debt-asset ratio across time and activities, \hat{Z}_{jt}^* , and hence a series for g , \hat{g}_{jt} .

At this stage the possibility exists of using actual observations on Z to improve the Z_{jt}^* series, but it is not at all clear how (or whether) this should be done. Note that behavioural equations (2.3) and (2.4) have not been used. To a certain extent these are tautological, deriving their only economic content from Z^* . This simply reflects the fact that $Z = B/X$ is the appropriate control variable, rather than B and X separately, which are then defined by (2.3) and (2.4). The \hat{g}_{jt} series is now used in the estimation of the consumption function (2.2). This allows estimates of the parameters of the utility function, γ , and the rate of time preference δ , by regression under constraints non-linear in parameters i.e.,

$$E = P\gamma + \delta W - \frac{P}{g} \cdot \beta$$

$$\text{subject to } \underline{\beta} = \delta \underline{\gamma} .$$

We will finish this section with a comment on the possible use of observations on Z . Apart from the use of observations on Z in estimating r_1 and r_2 , it would seem that the appropriate place to use Z as a dependent variable is equation (2.12). Some manipulation gives

$$r_2 - r_1 - r_2(1-Z)^2 = h$$

and h could be replaced by its definition from (2.10). Thus a series generated by r_1 , r_2 and Z would be regressed on prices, generating a direct estimate of the h series. If this approach were to be followed, it would seem more appropriate to specify the R function directly, thus simplifying the functional form for h . Equations for L , Q and K could be derived from duality theory, and joint estimation would increase efficiency. Dependence on the recursive nature of the optimization process would be eliminated. It may be of some interest to compare the two approaches.

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