IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

A joint study by the Australian Bureau of Statistics, the Department of Employment and Industrial Relations, the Department of Environment, Housing and Community Development, the Department of Industry and Commerce and the Industries Assistance Commission

EMPIRICAL ESTIMATION OF THE CRESH PRODUCTION FUNCTION

by

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elasticities and substitution elasticities are all low.[1] A more detailed analysis of these results in terms of their consequences for the structure of Australian agriculture is beyond the scope of this paper.

SUMMARY AND CONCLUSIONS

The CRESH production function offers considerably more scope for testing hypotheses relating to ES *vis-a-vis* the C.E.S. functional forms assuming the analyst is interested in the *relative differences* in ES among factors. Of particular interest is the parsimonious parameterization the closed form CRESH model provides, requiring only n parameters to be estimated in order to generate $\frac{1}{2}n(n-1)$ different ES. This has important statistical advantages over flexible forms such as the translog function which, in order to generate $\frac{1}{2}n(n-1)$ ES of a homothetic function, requires the estimation of $\frac{1}{2}n(n+1)$ parameters thereby substantially reducing the number of degrees of freedom available.

The advantages accruing to the CRESH specification relative to current functional forms are not without some constraints imposed by the necessary econometric methodology. The analyst requires access to a suitable FIML simultaneous equation package in order to estimate and test his CRESH specification adequately.

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1. A similar set of results for the Australian agricultural sector has been obtained by Vincent (1977) from a model specification involving the direct estimation of ES from factor input demand functions derived from cost functions. In this approach, the functional form of the underlying production function remains unspecified.

CONTENTS

In Table 1 the substantive results of the empirical study are presented. A likelihood ratio (LR) test under the null hypothesis of no serial correlation, (R = 0), (Hendry (1971)) against the alternative diagonal AR specification when the AR parameters are constrained to equality produced a value of 15.1 which is significant at one per cent ($\chi_1^2 = 6.63$ at .01). When the AR parameters are not constrained to equality the stochastic structure is misspecified and the parameter estimates will vary according to which equation is deleted.[1] Further in our case a test of the equality of the AR parameters of R gave a value of 0.48 ($\chi_1^2 = 6.63$ at 1 per cent) which implies that it is not possible to discriminate between the alternative AR specifications, and hence the derived ES (Table 2) and the own and cross price elasticities of factor demands[2] (Table 3).

The results suggest that factor inputs into Australian Agriculture over the period studied have not been very responsive to changes in their relative prices. Both own and cross price input demand

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1. Berndt and Savin caution that considerable care must be exercised in performing and interpreting LR tests of AR specification in singular equation systems. A test of the overall AR specification can only be performed in terms of the correct specification of the AR matrix.

2. Hanoch (1971) shows that for the CRESH specification, the Allen-Uzawa partial ES between factors i and j (σ_{ij}) are;

$$\sigma_{ij} = -\frac{1}{q_i-1}\;\frac{1}{q_j-1}\;\frac{1}{\sum\limits_{i'=1} S_i^*} \quad \text{where} \quad S_i^* = S_i\,\frac{1}{q_i-1} .$$

The definition of the Allen-Uzawa substitution elasticities also implies that the output compensated cross price elasticity of demand for factor i with respect to changes in the price of input j is $\eta_{ij} = \sigma_{ij} S_j$. Similarly the own price elasticity of factor demand for i is

$$\eta_{ii} = \frac{1}{q_i-1}\left(1-\frac{S_i/q_i-1}{\sum\limits_{i'=1}^{n} S_{i'}^*}\right) .$$

TABLE 3 : OWN AND CROSS PRICE RESPONSIVENESS OF DEMAND

(i) Basic Model

Per cent response one year later in demand for	Input whose price changes by 10 per cent: land	labour	capital
land	- 0.866 (2.56)	0.151 (0.37)	0.715 (2.89)
labour	0.132 (0.37)	- 0.898 (1.87)	0.766 (2.25)
capital	0.144 (2.89)	0.177 (2.25)	- 0.321 (2.94)

(ii) First Order AR

Per cent response one year later in demand for	Input whose price changes by 10 per cent: land	labour	capital
land	- 0.578 (2.39)	0.097 (2.83)	0.481 (2.29)
labour	0.085 (2.83)	- 0.553 (2.27)	0.468 (2.18)
capital	0.097 (2.29)	0.108 (2.18)	- 0.205 (2.25)

(iii) First Order AR with $\phi_1 = \phi_2$

Per cent response one year later in demand for	Input whose price changes by 10 per cent: land	labour	capital
land	- 0.590 (2.42)	0.099 (2.88)	0.491 (2.30)
labour	0.086 (2.88)	- 0.563 (2.30)	0.477 (2.19)
capital	0.099 (2.30)	0.110 (2.19)	- 0.209 (2.26)

Ratios of estimated parameters to estimated asymptotic standard errors in brackets.

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INTRODUCTION

Giora Hanoch (1971) proposed an n factor production function called CRESH (Constant Ratio Elasticity of Substitution Homothetic) which yields $\frac{1}{2}n(n-1)$ symmetrical but different pairwise elasticities of substitution (ES). The CRESH property imposes the restriction that the ES vary proportionately as factor combinations change but as the function is also homothetic the ES remain invariant with the level of output. CRESH represents a generalization of the well known C.E.S. function subject to the constant ratio property and is itself a special case of the constant ratio of elasticity of substitution (CRES) model.[1] These forms may be contrasted with constant difference elasticity of substitution models derived from cost functions (Hanoch (1975)).

Given the inherent generality of CRESH, it is somewhat surprising to find that there have been no reported attempts at empirical estimation some five years or more following its introduction. This situation contrasts with the burst of activity which followed the introduction of the C.E.S. function.[2] The main objective of this paper is to remedy this

1. There is no clear *a priori* preference for the constancy of ES or that all factors are substitutes as required by the C.E.S. specification. CRESH allows for constant relative differences among ES and also negative ES indicative of complementarity among factors (at least in a limited fashion).

2. Some studies have implied a CRESH specification. For example, the International Monetary Funds' Multilateral Exchange Rate Model specifies import substitution among intermediate inputs and final consumption goods from alternative sources according to a CRESH index (Dixon (1976)). In addition Dixon *et al.* (1977) make use of the CRESH specification to model input demand and product supply relationships in agriculture.

apparent deficiency by providing a framework for estimating the ES using the CRESH function. The estimating equations produced by the CRESH function are applied to an aggregate three factor production model of Australian Agriculture.[1]

THE CRESH FUNCTION

The CRESH function written implicitly is

$$\sum_{i=1}^{n} \left(\frac{X_i}{Y} \right)^{q_i} Q_i - 1 = 0 \qquad (1)$$

where :

X_i = level of factor input, i (i = 1, ..., n) ,

Y = level of output ,

Q_i, q_i = distribution and substitution parameters respectively of the CRESH function[2] .

From (1) if $q_i = \rho \; \forall_i$ then (1) reduces to a C.E.S. function of the form :

$$Y = \left[\sum_{i=1}^{n} X_i^{\rho} Q_i \right]^{1/\rho} \qquad (2)$$

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1. While our main objective is to illustrate an estimation procedure for CRESH the validity of our results depend upon the extent to which constant returns to scale (CRTS) is a satisfactory assumption for the Australian agricultural sector as a whole. As yet, there is no convincing empirical evidence to challenge the CRTS assumption. See for example, the review of Cobb/Douglas production function studies undertaken in Australian agriculture (Anderson and Powell (1973)).

2. In equation (1) Hanoch (1971) replaced Y with a function g(Y) in order to allow flexible scale effects. We assume constant returns to scale throughout this paper. In addition when $q_i = 0$ the term in brackets in equation (1) is replaced by the term log (X_i/Y) .

TABLE 2 : (ALLEN-UZAWA) PAIRWISE SUBSTITUTION ELASTICITIES[1]

(i) Basic Model

	labour	capital
land	0.094 (0.37)	0.102 (2.89)
labour		0.110 (2.25)

(ii) First Order AR

	labour	capital
land	0.060 (2.83)	0.069 (2.29)
labour		0.067 (2.18)

(iii) First Order AR with $\phi_1 = \phi_2$

	labour	capital
land	0.061 (2.88)	0.070 (2.30)
labour		0.068 (2.19)

Ratios of estimated paramenters to estimated asymptotic standard errors in brackets.

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1. The substitution elasticities in Table 2 have been calculated using mean input cost shares S_j .

The data described above was used as input to equation (9) and the autoregressive (AR) system (9) and (12) with R diagonal and yielded the following FIML estimates[1] of the $\{q_i\}$, together with the AR coefficients $\{\phi_i\}$. The estimates are contained in Table 1.

TABLE 1 : CRESH PARAMETER ESTIMATES

	Basic Model	First order AR	First order AR with $\phi_1 = \phi_2$
q_1 (land)	-9.11 (1.44)	-14.04 (2.30)	-13.73 (2.33)
q_2 (labour)	-8.44 (1.06)	-14.46 (2.17)	-14.17 (2.20)
q_3 (capital)	-7.62 (0.40)	-12.49 (1.73)	-12.20 (1.70)
ϕ_1	---	0.34 (2.90)	0.38 (4.03)
ϕ_2	---	0.43 (3.96)	
log likelihood determinant	11.52	11.84	11.83

Ratios of estimated parameters to estimated asymptotic standard errors in brackets.

1. See Wymer (1973), to whose outstanding RESIMUL package we were fortunate to have access.

We shall assume that producers are interested in cost minimization[1] and choose $X_i (i = 1,\dots n)$ to minimize total costs $C = \sum_{i=1}^{n} P_i X_i$ (where P_i are given factor prices) subject to the CRESH production function (1). An equivalent problem is the minimization of the following Lagrangean expression :

$$(3) \qquad L(X_i, \lambda) = \sum_{i=1}^{n} P_i X_i - \lambda \left[\sum_{i=1}^{n} \left({}^{X_i}/_{Y} \right)^{q_i} Q_i - 1 \right] .$$

First order conditions for a minimum of (3) imply ,

$$(4) \qquad P_i = \lambda \, q_i \left({}^{X_i}/_{Y} \right)^{(q_i - 1)} {}^{Q_i}/_{Y} \qquad (i = 1, \dots, n) ,$$

together with (1) .

Our objective is to obtain estimates of the $\{q_i\}$ and hence the Allen-Uzawa pairwise ES and the implied output compensated own and cross price elasticities of factor demands from (1) and (4).

From the view point of empirical application the system (1) and (4) is intractable in two respects; (i) it is non-linear in the variables[2] and (ii) it contains the non-observable Lagragean multiplier, λ .

We therefore proceed to eliminate the unobservables and to linearize the first order conditions.

1. In this illustrative study in which we concentrate on input demand relationships, cost minimization rather than profit maximization is appropriate. Output could however have been made endogenous by simultaneous estimation of the input demand system of the CRESH function with the corresponding product supply system derived from a function yielding suitably behaved product transformation frontiers. See for example Dixon, Vincent and Powell (1977).
2. While non-linear FIML estimation procedures do exist, our experience to date in the use of such procedures suggested to us that the chances for successful estimation of parameters in the system (1) (4) would be slight.

Eliminating λ in (4), the $(n - 1)$ expansion path equations are obtained as

$$(5) \qquad \left(P_i/P_j\right) = \left(q_i/q_j\right) \left(X_i/Y\right)^{(q_i-1)} \left(X_j/Y\right)^{-(q_j-1)} Q_i/Q_j$$

$$(i = 1, \ldots, n - 1, \quad i \neq j) \quad ,$$

which may be linearized using logarithmic differentials to give :

$$(6) \qquad x_i - \left(\frac{q_j-1}{q_i-1}\right) x_j = \left(\frac{1}{q_i-1}\right) p_i - \left(\frac{1}{q_i-1}\right) p_j + \left(1 - \frac{q_j-1}{q_i-1}\right) y$$

$$(i \neq j; \; i = 1, \ldots, n - 1) \quad ,$$

where

$$x_i = dX_i/X_i \; , \qquad y = dY/Y \; , \qquad p_i = dP_i/P_i \quad .$$

Similarly, equation (1) may be linearized using logarithmic differentials to give :

$$(7) \qquad \sum_{i=1}^{n} S_i x_i - y = 0 \quad ;$$

where

$$S_i = \left[q_i \left(X_i/Y\right)^{q_i} Q_i \Big/ \sum_j q_j \left(X_j/Y\right)^{q_j} Q_j/Y \right] = \frac{X_i P_i}{\sum_j X_j P_j} \quad .$$

The $(n-1)$ linear expansion path equations (6) together with the identity (7) constitute a system of $\underline{n}$ linear homogenous equations with the $\underline{n}$ factor demands as endogenous variables.[1]

1. If one is prepared to specify one of the factors as always exogenous (such as capital in the short run) or output as endogenous then the system of (n-1) expansion path equations only is required. The implication is that maximum likelihood estimators of the reduced system vary according to the choice of factor assumed to be exogenous.

We make use of serial data on prices, inputs and outputs for Australian agriculture over the period 1920/21 to 1969/70 compiled by Powell (1974). Briefly, the specification of variables (all expressed in logarithmic differences) is as follows :

- x_1 : unimproved land area (acres);
- x_2 : hired labour inputs (man-years);
- x_3 : depreciated capital stock placed on a constant price basis and consisting of structures and plant and machinery at replacement cost valuation. The implied assumption is that the flow of capital services is proportional to the depreciated capital stock;
- y : value added defined as gross value of farm production adjusted for changes in livestock inventories less non factor expenses;
- p_1 : rental price of land calculated as the perpetuity associated with its market value;
- p_2 : price of hired labour (average earnings of rural employees);
- p_3 : price of capital calculated as value added less payments for land rents and to hired labour expressed as a proportion of the capital stock.[1]
- S_1, S_2, S_3 : expenditure shares of land, labour and capital respectively in total cost.

1. It follows from the behavioural assumptions - cost minimisation in the context of a linear homogeneous production function with output pre-determined - that the sum of factor expenditures will equal factor income. In Australian agriculture, it is reasonable to regard the return to the owner operators' capital and labour as a residual which reflects the instabilities of the agricultural income stream due to variable export price and output effects. Box Jenkins methods were used to identify, fit and diagnostically check the appropriate model to extract the permanent component of this price series. An autoregressive moving average (ARMA) model of order three was identified as follows:

$(1-\phi L)P_t = \alpha_0 + (1-\phi L^3)\varepsilon_t$ where ϕ is an AR coefficient (0.68), α_0 is a constant (0.05), θ is a MA parameter (0.33), and L^3 is a lag operator of order 3. These procedures are exogenous to the FIML methods used to estimate the substitution parameters.

with AR structures similar to (12). They showed that (11) and (12) together imply a priori restrictions on the AR coefficients of the form :

(13) $S'R = kS'$

where k is a constant.

The Berndt and Savin results relating to restrictions on the AR parameters (13) in a reduced form singular system apply to our CRESH specification. In particular, Berndt and Savin show that when R is specified as a diagonal matrix, the diagonal elements of R must be constrained to equality in order to preserve the invariance property of the parameters. Below we provide empirical evidence which supports the Berndt and Savin result in the context of our AR CRESH reduced form system.

Empirical Analysis

We have chosen to use our CRESH estimating system to investigate input demand and production relationships in Australian agriculture. Similar studies have been carried out assuming a C.E.S. specification distinguishing only two factors of production (capital) and labour. In this analysis three factors (land, labour and capital) are distinguished and three different ES among the factor inputs are obtained. The traditional capital labour disaggregation of factor inputs is unnecessarily restrictive since it implies that land is either strictly complementary to the use of capital and labour or is perfectly substitutable for one of capital and labour depending upon whether land is excluded from the measured input indexes, or alternatively added to one of them.

In estimating this system, we treat the cost shares S_i as exogenous constants. Although such cost shares do change gradually through time, our judgement is that system (6), (7) can be reasonably well approximated by fixing the shares at their mean value. In effect, we are interpreting (6), (7) as a system which is precisely CRESH at co-ordinates corresponding to the mean values of S_i but which is elsewhere regarded as an approximation to CRESH. The ES, in the context of a CRESH system, vary proportionately to a weighted mean of $\{1/(q_i-1)\}$ where the individual S_i's are used as weights. The system together with a suitable stochastic specification of the residuals provides the basic estimating form for obtaining the CRESH parameters $(q_i, i = 1, \ldots, n)$.

Econometric Specification

It is convenient to re-write equations (7) and (6) as a system of n simultaneous linear equations as follows:

$$A x_t + B z_t = U_t , \qquad (t = 1,\ldots,T) ,$$

where

$$A = \begin{bmatrix} S_1 & S_2 & S_3 & \cdots & S_{n-1} & S_n \\ 1 & -\Theta_1 & & & & \\ & & 1 & -\Theta_2 & & 0 \\ & 0 & & \ddots & & \\ & & & & 1 & -\Theta_{n-1} \end{bmatrix} ;$$

$$x = \begin{bmatrix} x_{1t}, & x_{2t} & \cdots & , x_{nt} \end{bmatrix}'$$

(a n:1 column vector of changes in factor demands) ;

$$B = \begin{bmatrix} -1 & 0 & 0 & & & 0 \\ \theta_{1-1} & -\beta & \beta & & & \\ \theta_{2-1} & -\beta/\theta_1 & \beta/\theta_1 & & & \\ 0 & & & \ddots & & \\ \theta_{n-1} & & & & \frac{-\beta}{\theta_1\theta_2\cdots\theta_{n-1}} & \frac{\beta}{\theta_1\theta_2\cdots\theta_{n-1}} \end{bmatrix}$$

$$Z = \left[y_t, P_{1t}, P_{2t}, \ldots\ldots\ldots\ldots, P_{nt} \right]'$$

$$U_t = \left[0, U_{1t}, U_{2t}, \ldots\ldots\ldots\ldots, U_{n-1)t} \right]'$$

(the column vector of residuals) .

In equation (8),

$$\beta = \frac{1}{q_1-1} , \qquad \theta_j = \frac{q_{j+1}-1}{q_j-1} , \qquad (j = 1,\ldots,n-1) .$$

The contemporaneous disturbance-covariance matrix $E\left[U_{it}\ U_{it}'\right] = \Phi$ is singular since the first row and column of Φ is identically zero in order to accommodate the identity (7). However on substituting for the identity (7), FIML estimates may be obtained from the remaining (n-1) estimating equations. The M.L. parameter estimates obtained are invariant with respect to the choice of the (n-1) expansion path equations selected from the $\binom{n}{2}$ possibilities in the complete n equation system, provided the (n-1) paths selected are linearly independent (see Barten (1969) and Powell (1969)).

The reduced form model of (8) is

$$(9) \qquad x_t = \pi z_t + v_t \qquad (t = 1,\ldots,T) ,$$

where

$$\pi = -A^{-1} B ,$$

$$v_t = -A^{-1} U_t .$$

The definition of S_j following equation (7) implies that

$$(10) \qquad S'\pi = \underbrace{[1, 0, 0, 0,]}_{n \text{ terms}} ,$$

where $S' = \left[S_1\ S_2 \ldots\ldots\ldots S_n\right]$.

The singularity of the reduced form variance-covariance matrix is demonstrated by the fact that

$$(11) \qquad S'v_t = 0 \qquad (t = 1,\ldots,T) .$$

We also specify serial correlation in the residuals generated by the reduced form model (9) assuming that the autogressive (AR) disturbances follow a first order structure of the form :

$$(12) \qquad v_t = R\, v_{t-1} + \varepsilon_t \qquad (t = 2, \ldots, T) ,$$

where

R = n:n matrix of unknown AR parameters, and

ε_t = independently and identically normally distributed white noise error term.

Berndt and Savin (1975) have discussed the identification and estimation of singular equation reduced form systems