IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

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MODELLING THE PRODUCTION SECTOR OF AN ECONOMY : A SELECTIVE SURVEY AND ANALYSIS

by

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1. Introduction

The purpose of this paper is to present a selective survey of the literature and a discussion of the issues involved in formulating a model of the production sector of an economy and estiming the model. The survey is not intended to be complete or comprehensive, since the emphasis is on indicating ways of formulating and estimating models rather than detailing the history of the development of the state of the art. Surveys have been provided by Walters (1963), Nerlove (1967a, 1967b), Solow (1967), Nadiri (1970), and Kennedy and Thirwall (1972), Jorgenson (1971), Klein (1974), and Diewert (1974b) on various aspects of production functions.

In section 2 the theory of production is briefly presented and the implications of duality for the empirical estimation of the technology are indicated. Duality theory increases the range of estimation models depending on which variables are regarded as exogenous and which are regarded as endogenous to the problem.

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Section 3 is concerned with functional forms for production and cost functions with special emphasis on the class of "flexible" functional forms which allow greater flexibility in modelling the substitution possibilities among inputs and the scale effects relating inputs to outputs.

The problem of modelling technical change is taken up in Section 4. We consider various restrictions which can be imposed on the nature of technical change to yield an empirical model including the embodiment hypothesis and also discuss endogenous theories and their empirical implications.

Section 5 is devoted to the adjustment of inputs to current prices and output in a dynamic setting. Quantity adjustment models of the flexible accelerator type are discussed within a cost of adjustment framework while price expectations models are also considered. It is shown that there is duality between such models.

The analyses of Sections 2-5 are brought together in Section 6 to consider the specification and estimation of an empirical model of the Australian economy as outlined by Dixon (1975). Various possibilities are suggested and the data and estimation requirements outlined. Finally, an alternative approach to modelling the production sector is suggested in the event that data requirements for the Dixon model are too severe.

2. Production Theory and Estimation

In this section we are concerned with the general approach to the estimation of the transformation frontier relating feasible inputs to outputs using observed data, economic theory, and statistical methods.

The purpose is to briefly provide a description of the theoretical model which forms the basis of empirical research and to indicate various methods of estimation. The emphasis is on the approach to estimation rather than the details of stochastic specification, estimation technique, functional form and data construction. Nevertheless, some of the implications for these of the general approach are indicated.

We begin by being general and then become more specific and concrete by considering a special case which is most popular in applied research.

a) General Approach

The general approach is to assume the existence of a production possibility set which contains all feasible input-output points given the technology, that is, given a certain state of knowledge. The boundary of this set may be represented by the transformation frontier

$$t(y) = 0$$

where y is a vector of net outputs (plus if output, minus if input). Economic theory imposes certain restrictions on the production possibility set and hence upon the transformation frontier $t(y) = 0$. In reality we do not know t but do have observations $y_1, \ldots, y_T$ which refer to different points of time or different production units or both. The problem is to use this empirical information to estimate t.

One crucial problem is that the state of knowledge may not be constant over the observations. We either i) assume that it is constant over the observations, raising the possibility of a misspecification of the model underlying the observations, or ii) assume that the transformation frontier shifts in a particular way over the observations. We consider this problem in detail in a later section. Here we ignore the problem for simplicity, making assumption i).

Assuming all observations come from a common transformation frontier we can estimate that frontier in two ways. The first is to use a nonparametric method developed by Farrell (1957) and further analysed by Afriat (1972). This method has been used by Hanoch and Rothschild (1972), Geiss (1971), Timmer (1971) and Aigner and Chu (1968). We do not consider this method further. The second approach is to postulate that the observations were generated from the transformation frontier but subject to random disturbances. Given the density for the disturbances and an assumed functional form for t the parameters of this functional form may be estimated and statistical inferences made. The choice of density and functional form impose restrictions on the model in order to make it operational, that is make it feasible to empirically estimate the frontier. They necessarily force the researcher to trade-off between generality of functional form and stochastic specification and simplicity of execution. There does not seem to exist a consistent set of rules to guide the researcher in his choice though, as Mundlak (1973) points out, the choice depends to a large extent upon the use to which the results are to be put. With these general observations in mind let us now consider various alternative ways of estimating the technology.

Rather than estimate the technology directly as suggested above we might impose behavioral assumptions which indicate more precisely the manner in which the observations on y were generated. These usually involve the idea of economic choice and may take many forms. One class of behavioral assumptions involves the maximization of a linear function p_1y_1 over the set of y_1 such that $t(y_1,y_2) = 0$ where y_1 is a vector of choice variables, p_1 is the corresponding price vector and y_2 is a vector of fixed "variables". The "profit" function

$$(2) \quad \Pi(p_1,y_2) \equiv \max_{y_1} \{p_1y_1 : t(y_1,y_2) = 0\}$$

is dual to the transformation frontier t in the sense that each may be completely derived from knowledge of the other. Certain regularity conditions are required, of course, leading to different duality theorems. For a sampling of the literature on duality see Shephard (1953, 1970), Uzawa (1964), Diewert (1971, 1974b), Lau (1969, 1972) and Jorgenson and Lau (1974).

In particular, different theorems apply depending upon the nature of y_1 and y_2. If y_1 refers to a set of inputs then we refer to $\Pi(p_1,y_2)$ as (the negative of) a cost function which may be a total cost function if y_2 refers to outputs only or a variable cost function if y_2 includes some fixed factors. If y_2 refers to fixed inputs only then $\Pi(p_1, y_2)$ is called a variable profit or gross profit function and if y_2 does not exist then it is called a profit function. If y_2 includes only primary inputs Π is a value added function and if y_2 includes only outputs it is called a revenue function.

The importance of the theory for our purposes is that, under certain regularity conditions, $\Pi(p_1,y_2)$ and $t(y_1,y_2) = 0$ are two equivalent representations of the technology.[1] Accordingly, if the behavioral assumption underlying Π is valid then we may use either representation to estimate the technology. To estimate t we need observations on y, a functional form for t and a stochastic specification. To estimate Π we need observations on Π, p_1 and y_2, a functional form for Π and a stochastic specification.

It should be noted that a functional form for Π and a corresponding stochastic specification imply the functional form for t and its stochastic specification and vice versa. That is, "self duality" where the profit function and transformation function belong to the same family is the exception rather than the rule.

Another theoretical result with empirical implications is that the optimal choice for y_1 and the shadow prices for y_2 may be derived from a differentiable profit function as

(3) $y_1 = \partial\Pi(p_1,y_2)/\partial p_1$

(4) $p_2 = \partial\Pi(p_1,y_2)/\partial y_2$.

Since the function $\Pi(p_1,y_2)$ is linear homogeneous in p_1 it is evident that the net supply functions (3) completely characterize the profit function. To show this we note that the profit function may be derived from the net supply functions using Euler's theorem on homogeneous functions.

(5) $p_1 \; \partial\Pi(p_1,y_2)/\partial p_1 \equiv \Pi(p_1,y_2)$.

A third way of estimating the technology, therefore, is to estimate (3). It should be noted that (3) involves the same behavioral assumption as contained in (2), including the assumption that y_2 is fixed. The net supply function (3) may be estimated given observations on y_1,y_2, and p_1 together with a functional form for Π and a stochastic specification.

If $\Pi(p_1,y_2)$ is linear homogeneous in y_2 and we observe the shadow prices p_2 for y_2, as in a constant returns to scale economy with a competitive market for the fixed inputs y_2 for example, then we may estimate the technology by estimating the shadow price functions (4) since, again using Euler's theorem, the profit function may be obtained from (4) as

(6) $y_2 \; \partial\Pi(p_1,y_2)/\partial y_2 \equiv \Pi(p_1,y_2)$.

A fifth approach using the same behavioral assumptions underlying (2) is to estimate the first-order conditions for the maximization involved in (2). The first order conditions are, after eliminating the Lagrange multiplier,

$$(7)\quad p_1/p_1y_1 = \partial t(y_1,y_2)/\partial y_1 \div (\partial t(y_1,y_2)/\partial y_1)y_1.$$

If we add the market equilibrium condition that the market for y_1 clears and the demand for (supply of) y_1 is fixed, then if t is an industry transformation frontier, y_1 may be taken as fixed and p_1 as the endogenous price vector. Thus, given observations on fixed quantities y_1 and y_2 and an endogenous price vector p_1, we may estimate (7).

If this additional assumption is not made then (7) may be solved, in principle, for y_1 as a function of p_1 and y_2. But in practice this may be difficult or impossible so that the derivation of (3) from Π is obviously to be preferred. Thus duality theory enables us to easily obtain supply and demand functions consistent with the theory simply by specifying a valid functional form for Π and differentiating with respect to the price vector. This point has been emphasized by Diewert (1971, 1974b) and Lau (1974).

The foregoing indicates that there are many approaches to the estimation of the technology or the part of it that one is interested in, each one requiring a special stochastic specification, a special functional form and different data. Each has been used in empirical work. Direct estimation of the transformation function was first undertaken by Cobb and Douglas (1928) while Nerlove (1963) appears to be the first to estimate the cost function. The factor demand functions approach was used by Arrow et al. (1961) while Klein (1953) suggested the factor shares approach.

b) Production and Cost Functions

While the analysis of the preceeding subsection was cast in general terms to emphasize the basic structure of theory based upon maximizing behavior, this subsection will be devoted to a specific case where the firm minimizes the input cost subject to producing a given single output. This is done to make clearer the various ways we can estimate the technology in a context that is most often used by applied researchers.

The firm is assumed to have a production function

(8) $y = f(x)$

relating output y to the vector of inputs $x = (x_1, \ldots, x_N) \geqq 0$.[2] The function f is assumed to be a non negative, quasi-concave, nondecreasing continuous function defined over the non-negative orthant. The cost function is

(9) $C(w,y) \equiv \min_x \{wx: f(x) \geqq y\}$.

Diewert (1971), for example, has shown that under the assumptions on $f(x)$ the cost function $C(w,y)$ is a non-negative, continuous function defined over the positive price orthant (it can be extended over the whole non-negative orthant). In addition it is concave and nondecreasing in w and nondecreasing in y. Furthermore, given C we may reconstruct f.[3] Thus the cost and production functions are dual in the sense that each may be derived from the other. Moreover, given any valid functional form for C we can derive, in principle, the production function which yields C as its cost function. There is no loss of information or generality in using the cost function as the point of departure for empirical research (or theoretical research).

If $C(w,y)$ is differentiable in w then the factor demand functions coincide with the partial derivatives of c with respect to the factor prices, a result due to Shephard (1953):

(10) $x_i = C_i(w,y) \equiv \partial C(w,y)/\partial w_i \qquad i = 1,\ldots,N.$

If we define $s_i \equiv w_i x_i / \sum_j w_j x_j$ as the share of factor i in the cost of production the share equations may be obtained from (10) or, writing the cost function in log form, as

(11) $s_i = \partial \ln C(w,y)/\partial \ln w_i \equiv S_i(w,y) \qquad i=1, \ldots, N.$

We now consider alternative ways of estimating the technology:

A. We may attempt to estimate the parameters of the production function f directly if we have sample data on input quantities and output. Usually a disturbance term is appended to (8) in either additive or multiplicative form and it is assumed that x is independent of this disturbance. There are several disadvantages to estimating (8). First, it is unlikely that x will be independent of the disturbance particularly if the inputs are chosen by the firm. Second, multicollinearity is likely to be a problem especially with time series data and more than two inputs. If one is interested in the curvature of f then the associated parameters may therefore be estimated with large standard errors. Third, although price data and behavioural assumptions do not appear to be required they are usually needed to construct indices of input quantities since aggregation is usually mandatory. See Jorgenson and Griliches (1967), for example. Fourth, if cost minimization does take place we are not using full information unless we model such behavior.

B. If we have sample data on input prices, output and the cost of production we may estimate the cost function assuming, of course, cost minimization. At least for "small" industries the assumption that w is independent of the disturbance may be reasonable since the industry may regard w as given to it by market forces beyond its control. As with estimation of f(x), estimation of C(w,y) may suffer from collinearity among factor prices and so the parameters may have large standard errors. Also, it remains true that all information is not being used if in fact data on input quantities are available.

The main differences between estimating the cost function and estimating the production function are the data used, and the assumptions regarding how the data were generated.

C. If we have data on output, input prices and quantities of inputs then the factor demand functions (10) or perhaps some transformation of them, say the factor share functions (11), may be estimated. Other transformations of (10) may of course be used as estimating equations. For example, we might wish to divide (10) by output y and to get input-output coefficients x_i/y as dependent variables, or we might estimate ratios of the factor demand functions, or we might estimate expenditure equations by multiplying (10) by w_i. However, rather than consider all possibilities we concentrate here upon the estimation of (10) and (11).

It is clear that (10) and (9) are equivalent representations of the technology since (10) is obviously derived from (9) and since (9) may be constructed from (10) using Euler's theorem on homogeneous functions. Suppose we add disturbances u_i $(i=1,\ldots,N)$ to the demand equations (10) to represent errors in cost minimization. Then, since the x_i are jointly chosen to minimize input costs, the errors in minimization should be jointly distributed. That is, the disturbance u_i in equation i has a density which depends upon the disturbance u_j in equation j. If we can reasonably assume that w and y are independent of these disturbances then the factor demand functions may be estimated as a multi-variate regression model with jointly distributed disturbances. Thus estimation of the factor demand functions requires more sophisticated technique than estimation of the production function or cost function which are single equation models. The gain, however, is that more information is used hopefully enabling "better" estimation of the parameters. And nowadays the computational burden is not prohibitive nor even a major concern if the demand functions are linear and the disturbances are distributed normally, since there are efficient programs available for the solution of such problems, for example, Chow and Fair (1971).

D. If data on input shares, input prices and output are available then the share equations (11) may be estimated. The share equation approach may be attractive for two reasons. First, to avoid problems of heteroskedasticity when assuming a normal distribution for the parameters it is often convenient to assume that the shares are distributed normally with constant variances (and covariances) since scale effects are then taken into account. Indeed, this is why input output coefficients might be estimated rather than (9). Second, if the underlying production function is homothetic the shares are independent of the level of output meaning y does not appear in (11). Then without data on output estimation of the technology may still be undertaken by estimating the share equations. Obviously, scale effects cannot be so estimated but the representative isoquant can be estimated; if interest centres on the substitution possibilities and not upon scale effects then there is no loss by estimating the share equations. That some information is lost by the share equations is evident since the cost function cannot be constructed from the share equations.

This observation may be very important not only when data on y are not available but when y is intrinsically a non-observable concept. For example, the formulation of the industry production function proposed by Dixon (1975) involves such non-observable concepts. This formulation is that industry output is a fixed coefficient function of various inputs some of which are aggregates of more basic inputs. In general terms we have that that the production function is weakly separable in a group of inputs x_2.

(12) $y = f(x_1, g(x_2))$.

Since the number of inputs is large (they include intermediates) estimation of f using the methods outlined above may not be feasible. However, even though $g(x_2)$ is not observed we may estimate most of the parameters of g using the share technique if $g(x_2)$ is linear homogeneous. For example, x_2 may include the primary resources so $g(x_2)$ is linear homogeneous. For example, x_2 may include the primary resources so $g(x_2)$ is real value added which is not observable. This approach has been suggested by Arrow (1972) and used extensively by, for example, Burgess (1974a, 1974b), Berndt and Christensen (1973a, 1973b) and others. To relate (12) to the above approach using cost functions we note that the cost function is separable in prices if the production function is (12) with $g(x_2)$ linear homogeneous

(13) $C(w_1, w_2, y) = C^* (w_1, h(w_2), y)$

where $h(w_2) \equiv \min \{w_2 x_2 : g(x_2) \geqq 1\}$ is a unit cost function. The parameters of g may be estimated by estimating $h(w_2)$ or the share equations $s_{i2} = \partial \ln h(w_2)/\partial \ln w_{i2}$ $\qquad i = 1, \ldots, N$.

When share equations are estimated, account has to be taken of the fact that they are dependent and sum identically to unity. Thus, if disturbances u_i are added to the share equations (11) their sum $\sum_{i=1}^{N} u_i \equiv 0$ which implies that the joint density function over $(u_1, \ldots, u_N)$ is completely represented by the density function over $(u_2, \ldots, u_N)$. Thus one equation may be (must be, using most algorithms) dropped and the remaining N-1 equations estimated as an ordinary multivariate regression model. Maximum likelihood estimates will obviously be independent of the equation dropped so any one may be dropped. This adding-up property highlights the jointness of the disturbances; they cannot be independent.

In addition to the joint dependence of the disturbances, another reason why joint estimation of the demand or share equations is required is that the same parameters will appear in different equations. If these cross-equation parameter restrictions were ignored the estimated demand or share functions would not integrate into a unique cost function.

One final note on the share equation approach. If all parameters of the cost function are required then the cost function itself must be estimated along with the share equations and, of course, this requires observations on output.

E. Methods B,C and D require that w and y be the exogenous variables. However, if in the industry in question the input quantities are fixed along with output then it is the shadow price vector w for inputs which is endogenous. Assuming cost minimization by competitive firms within the industry we get the following inverse share equations from the production function

$$(14) \quad s_i \equiv \frac{w_i x_i}{\sum_j w_j x_j} = x_i \partial f(x)/\partial x_i / \sum_j x_j \; \partial f(x)/\partial x_j \equiv S^*_i(x).$$

If data on input quantities and prices are available then estimation of (14) may proceed in the same manner as the estimation of (11). Again, to estimate all the parameters of f(x) the production function would have to be estimated along with these share equations.

F. There is one other method for estimating the parameters of the production function (indirectly) when output data are not available. This approach suggested by Diewert (1973, pp. 6-8) is to estimate the parameters of the reciprocal indirect production function

$$(15) \quad g(w/M) \equiv \min_x \{1/f(x) : wx \leqq M\}$$

which is dual to the production function f(x). Using Roy's Identity we get the input demand functions

$$(16) \quad x_i = \frac{\partial g(v)/\partial v_i}{\sum_j v_j \partial g(v)/\partial v_j} \qquad i = 1, \ldots, N$$

where $v_j \equiv w_j/M$ and $M = \Sigma w_j x_j$ is the cost of production. Given observations on x_i and w_i (hence M), a stochastic specification and a functional form for g(v) we can estimate its parameters by estimating (16). If M is given exogenously, as perhaps with a public utility with fixed budget, then estimation of (16) as a multivariate regression model proceeds smoothly. If M is endogenous however the estimation must take its dependence upon the x_i's and hence upon the disturbances into account. This nontrivial complication added to the fact that (16) will be generally nonlinear in parameters makes this approach dominated by the share approach considered above.

3. Functional Forms

a) General Comments. The analysis of section 2 was general in that no special assumptions were made about the functional forms used for the production, cost, demand or share equations. In this section we review various functional forms which may be used in empirical research.

The choice of functional form should depend on two properties. First, the functional form should be capable of representing a wide range of technologies in order to minimize the prior assumptions imposed on the estimating equations. Second, the functional form should be tractable within the assumptions of the model. That is, the estimating equations should be simple enough to carry out the estimation with minimal computational burden and with ease of interpretation. In reality any choice is a compromise between these two objectives and such a choice must be based upon value judgements in general.

It is with respect to the first criterion that simple functional forms for production and cost functions such as the Cobb-Douglas (CD), Leontief (L), and constant elasticity of substitution (CES) forms are dominated by more general forms such as the flexible functional forms to be discussed in detail below. For example, it is well known that the Cobb-Douglas production function has a Hicks-Allen elasticity of substitution (AES) which is unity for all input pairs and under cost minimization implies that factor shares are constant. Since the substitution elasticities are measures of curvature around an isoquant it is evident that the shape of the isoquant is severly restricted by assuming a Cobb-Douglas function. This is highlighted by the implication that shares are constant. The apparent success of the CD function in applied work seems to be due to two reasons.

First, using aggregated time series the direct estimation of the CD function is reasonable since the substitution effects are not well identified by the highly collinear data. Second, Fisher (1969) argues that the constancy of factor shares of labour and capital in aggregate data fits the CD hypothesis. This does not necessarily mean that there exists an aggregate CD production (or cost) function but that we have yet to explain this constancy.

The CES function permits the AES to deviate from unity but does require it to be constant by construction. The CES function thus generalizes the CD and Leontief (L) functions which assume a common constant elasticity of substitution which is unity and zero respectively. Since the cost function for a CES production function belongs to the same family of functions (means of order -b), that is they are self-dual, we may illustrate the function by examining the cost function which is

$$(1) \quad C(w,y) = y^{1/r} \{\sum_{i=1}^{N} a_i w_i^{-b}\}^{-1/b} = y^{1/r} c(w) \qquad r>0$$

$$a_i \geq 0, \quad b>-1$$

In this expression r denotes the degree of homogeneity of the production function and $b = 1-\sigma$ where σ is the common constant substitution elasticity. The factor demand functions are

$$(2) \quad C_i(w,y) = y^{1/r} \ a_i w_i^{-\sigma} \ c(w)^{\sigma} \qquad i=1,\ldots,N$$

If $\sigma=0$ and $r=1$ then the demand functions are $a_i y$ which imply the fixed coefficient model of Leontief; if $\sigma=1$ then factor shares are a_i which implies the Cobb-Douglas function.

The CES function by definition is restrictive in the nature of substitution permitted. In particular, all factors are "equally substitutable" with each other, a restriction which has no theoretical justification but which simplifies the empirical work considerably. If there are just 2 inputs then this restriction may not be very hard to take since it simply means that the single cross elasticity of substitution is constant. But to extend the constancy to a multi-factor technology and assume that the AES between electricity and machines is the same as between unskilled labour and materials, for example, seems to be unreasonable until verified empirically. Even with two inputs one may not wish to assume that the AES remains constant around the isoquant. We conclude therefore that a priori the CES function is restrictive in that it restricts the elasticities of substitution (a) to be constant, and (b) to be the same constant for every pair of inputs. Such restrictions should be tested not imposed a priori.

The L and CD functional forms being special cases are even more restrictive in that they specify a priori that the common AES is zero and unity respectively. Clearly, the discovery of the CES functional form provided a significant advance in applied production work in permitting the estimation of the elasticity of substitution. However important that advance it is of limited comfort in the empirical estimation of multiple input production functions.

Another feature of the CES function is that it is additive as illustrated by the basic CES form given by c(w) in (1). This special form of separability is not independent of the constancy of the AES since Berndt and Christensen (1973c) and Russell (1975) have shown that homothetic weak separability is equivalent to having certain sets of AES equal. Accordingly, not only should a functional form have variable AES but homothetic weak separability should be avoided since it does impose certain equality restrictions which may not be desirable. As Mundlak (1973) points out separability should be put to an empirical test before it is imposed on the model. Unfortunately, it is difficult to construct a tractable functional form which may be used to successfully test for separability.

Given the a priori presumption against the CES form because it implies that (a) the AES are all constant, (b) the AES are the same for all pairs of inputs and (c) the function is additive, there has been considerable effort made to obtain less restrictive functional forms. One obvious approach is to make the common AES σ a function of some variable such as the level of output or the factor ratio or factor share etc. Such generalizations have been called Variable Elasticity of Substitution (VES) functions and have been discussed by Revankar (1971), Lee and Fletcher (1968), Sato and Hoffman (1968) and Lovell (1968, 1973a, 1973b).

A recent spurt of functional forms owes its origin to Diewert (1971) who generated a functional form that is linear in parameters and which provides a second order approximation to any arbitrary twice differentiable function. The Generalized Leontief (GL) functional form, so called because when used as a cost function yields the Leontief cost function as a special case, was quickly followed by the translog (TL) functional form developed by Christensen, Jorgenson and Lau (1971), and Sargan (1971), the Generalized Cobb-Douglas (GCD) developed by Diewert (1973) and a generalization of GL by Denny (1974) and Kadiyala (1972). In addition Diewert has developed functional forms for special functions such as revenue and variable profit functions as well as indirect utility functions and indirect production functions. It is to a consideration of these flexible functional forms that we now turn.

b) Functional Forms Flexible in Prices

We concentrate attention on functional forms for cost functions since it is unlikely that the production function approach will be useful at the industry level of disaggregation. In this subsection attention is focussed on functional forms which are flexible in the sense of providing second order approximations in input prices to an arbitrary continuously differentiable cost function. In the following subsection we direct attention to the output or scale effects, but in this subsection we assume constant returns to scale to highlight the differences between functional forms in their ability to describe a wide range of substitution possibilities for a multi-input technology.

Consider the following unit (output) cost functions:

(3) GL $$c(w) = \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij} w_i^{1/2} w_j^{1/2} \qquad b_{ij} = b_{ji}$$

(4) GCD $$\ln c(w) = b_{oo} + \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij} \ln(w_i + w_j) \qquad b_{ij} = b_{ji}, \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij} = 1$$

(5) TL $$\ln c(w) = b_{oo} + \sum_{i=1}^{N} b_{oi} \ln w_i + 1/2 \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij} \ln w_i \ln w_j$$

$$b_{ij} = b_{ji}, \sum_i b_{oi} = 1, \quad \sum_{j=1}^{N} b_{ij} = 0 \quad i=1,\ldots,N$$

The first two functional forms are due to Diewert (1971, 1973) and the third is due to Christensen et al. (1971). We will refer to these as the Generalized Leontief (GL), Generalized Cobb-Douglas (GCD) and the transcendental logarithmic or translog (TL) forms. Each unit cost function is linear homogeneous in prices as theory requires. Diewert has shown that GL and GCD are nondecreasing concave functions if the b_{ij} are non-negative while GL is positive if some $b_{ij}>0$ as well and GCD is positive if $b_{oo} > -\infty$. Thus under certain parameter restrictions the GL and GCD functional forms satisfy all of the conditions required of a cost function. The TL form satisfies all these conditions globally only if $b_{ij}=0$ $i,j=1,\ldots,N$ in which case it reduces to a Cobb-Douglas function. Nevertheless, for applied work, while global regularity may be desirable it is not essential since it usually suffices that the function be regular (satisfies the required conditions) in the subset of the sample space that is empirically relevant. Most researchers simply check whether the estimated function is regular at each of the sample points.

Each of these functional forms is capable of providing a second order approximation in prices to an arbitrary continuously differentiable cost function. It is this property which has suggested the term "flexible functional form" and which distinguishes this class of functions from other "generalized" functional forms. If we have an arbitrary cost function $c^*(w)$ with first and second order partial derivatives $c^*_i(w^*)$ and $c^*_{ij}(w^*)$ at a point w^* then there exists a GL, GCD and a TL cost function $c(w)$ such that the first and second derivatives at w^* coincide with those of c^*. That is, there is a choice of parameters for each of the functional forms such that

(6) $c(w^*) = c^*(w^*)$, $c_i(w^*) = c^*_i(w^*)$ and $c_{ij}(w^*) = c^*_{ij}(w^*)$ all i,j.

Since the AES may be expressed in terms of the first and second partial derivatives of the cost function, as shown by Uzawa (1962),

(7) $\sigma_{ij}(w) = c(w)\ c_{ij}(w)\ /c_i(w)\ \ c_j(w)$ all i,j

it follows that there exists GL, GCD and TL cost functions which will accurately reproduce <u>any</u> matrix of AES at the point of approximation. It is in this sense that flexible functional forms are capable of modelling a wide range of technologies.

Each functional form has $N(N+1)/2$ free parameters which is enough to provide a second order approximation to an arbitrary differentiable unit cost function.

There are two distinct approaches to the use of flexible functional forms. First, one can choose a form from this family, assume that the true cost function has this form, and then estimate its parameters. Second, one can choose a form from this family, assume that the true cost function has unknown form, and then estimate the second order approximation to the unknown production function. Presumably the stochastic specification should take account of truly random disturbances and the unknown approximation error. In the ensuing analysis the former approach is assumed.

One of the important implications of the development of flexible functional forms is that the problem of choosing a functional form is simplified. By definition any flexible functional form is capable of a second order approximation so any form from this family may be chosen. Of course there may be other differences which are important: estimation procedures, ability to model the technology over a wide range of data, etc.

It was suggested above that if data is available the input demand functions or share functions should be estimated. While each of the functional forms considered here are linear in parameters facilitating estimation of the cost function, they differ when it comes to estimating demand and share equations. The GL function yields, by Shephard's (1953) Lemma, the demand functions

$$(8) \quad \text{GL} \quad c_i(w) = \sum_{j=1}^{N} b_{ij}\, w_j^{1/2}\, w_i^{-1/2} \qquad i=1,\ldots,N$$

which are linear in parameters, permitting estimation using multivariate linear regression for which efficient algorithms for maximum likelihood or two-stage Zellner (1962) estimation exist. The share functions are ratios of terms each linear in parameters and therefore require a slightly more sophisticated estimation technique. The GCD and TL forms, on the other hand, yield linear share equations

$$(9)\quad \text{GCD} \quad s_i(w) \equiv \frac{w_i c_i(w)}{c(w)} = \sum_{j=1}^{N} 2\, b_{ij}\, w_i (w_i + w_j)^{-1} \qquad i=1,\ldots,N$$

$$(10)\quad \text{TL} \quad s_i(w) = b_{oi} + \sum_{j=1}^{N} b_{ij} \ln w_j \qquad i=1,\ldots,N$$

but yield nonlinear (ratios of linear terms) demand functions. Thus the choice of functional form effectively narrows depending upon whether one wishes to estimate share or demand equations.

The GL cost function reduces to a linear function in factor prices if $b_{ij}=0 \quad i\neq j$ which implies that input-output coefficients (8) are fixed, independent of factor prices. Thus the Leontief production function may be tested by testing these parameter restrictions. Both GCD and TL reduce to a Cobb-Douglas cost function under certain parameter restrictions, for GCD these are $b_{ij}=0 \quad i\neq j$ and for TL they are $b_{ij}=0 \quad i,j=1,\ldots,N$. A more general functional form permitting a wider range of special cases has been provided by Denny (1972) and Kadiyala (1972):

$$(11)\quad c(w) = \{\sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}\, w_i^{r/2}\, w_j^{r/2}\}^{1/r} \qquad b_{ij}=b_{ji}$$

This reduces to GL when r=1 and to the CES form when $b_{ij}=0$ $i\neq j$ which in turn reduces to the CD and L forms as limiting cases. However, the share and demand functions are nonlinear in parameters reducing its attractiveness somewhat.

c) Output and Returns to Scale

A common assumption in theoretical and empirical research is that the production function is linear homogeneous meaning that there are constant returns to scale. The justification for this assumption at the industry level is that in the long run a perfectly competitive industry with price taking firms and free entry and exit will be able to produce any output simply by doubling all inputs. Thus average cost is constant over all output levels. Since free entry and additivity of production by firms (absense of externalities) imply nondecreasing returns to scale for the industry (Debreu (1959,p.41)) we must also assume that firms face nonincreasing returns to scale technologies to get that the industry production function has constant returns to scale. If these assumptions are appropriate then constant returns to scale may be imposed *a priori* on the estimation model. The cost function is $C(w,y) = y\ c(w)$ where $c(w)$ is a unit cost function for which functional forms were presented above.

Scale effects at the industry level may occur if externalities related to industry output are present or if entry and exit are not free within the period and firm production functions are not linearly homogeneous. The second point is relevant to industries which have few firms. It is also relevant to the question of whether a production function has (decreasing) "scale" effects because the scale effects are genuinely decreasing or because some ignored factor is fixed. In these circumstances, in the absence of a specific model of the structure of the industry, it may be desirable to allow for non-constant returns to scale.

Homothetic production functions are those that may be written as

(12) $y = f(x) = F(h(x))$

where $h(x)$ is a linear homogeneous function and F is a positive increasing function. It is readily shown that the cost function factors as

(13) $C(w,y) = g(y)\ c(w)$

where $g \equiv F^{-1}$ is assumed to exist. The function $g(y) = y^{\beta}$ is a 1-parameter representation which yields β as the constant elasticity of cost with respect to output, a representation implicitly used in CD and CES production function models. Zellner and Revankar (1969) and Lovell (1973c) use the notion of a returns to scale function (elasticity of cost with respect to output in our context) which depends upon the output level to generate a class of functional forms for $g(y)$. The returns to scale function they suggest is

(14) $r(y) \equiv \alpha(1+\beta y)^{-1}$ $\qquad \alpha>0$

which reduces to $r(y) = \alpha$ if $\beta=0$[4]

To provide a second order approximation to $C^*(w,y)=g^*(y)\ c^*(w)$ we require an extra 2 parameters in addition to those used to provide a second order approximation to $c^*(w)$. Thus the Zellner-Revankar returns to scale function will in general provide such an approximation. Another candidate is

(15) $g(y) = a+y+by^2$.

However, for cases (14) and (15) the demand functions will turn out to be nonlinear in parameters in general.

We now consider second order approximations to an arbitrary differentiable cost function $C(w,y)$ which is not necessarily homothetic. Our choice is certainly not complete; we simply indicate forms which are convenient. The choice is restricted to forms linear in parameters and to forms which respect the homogeneity of C with respect to w. Consider the following forms:

(16) GL $C(w,y) = y \sum_i \sum_j b_{ij} w_i^{1/2} w_j^{1/2} + y^2 \sum_i b_{oi} w_i + b_{\infty} \sum_i w_i$ $\qquad b_{ij}=b_{ji}$

(17) GCD $\ln C(w,y) = b_{oo} + \sum_i \sum_j b_{ij} \ln(w_i+w_j) + \ln y \sum_i b_{yi} \ln w_i + b_{oy} \ln y + b_{yy} \ln y \ln y$

$$b_{ij}=b_{ji}, \ \sum_i \sum_j b_{ij}=1, \ \sum_i b_{yi}=0$$

(18) TL $\ln C(w,y) = b_{oo} + \sum_i b_{oi} \ln w_i + 1/2 \sum_i \sum_j b_{ij} \ln w_i \ln w_j + \ln y \sum_i b_{yi} \ln w_i +$

$$b_{oy} \ln y + b_{yy} \ln y \ln y$$

$$b_{ij}=b_{ji}, \ \sum_i b_{oi} = 1, \ \sum_j b_{ij}=0, \ \sum_i b_{yi}=0.$$

Clearly these forms use (3)-(5) as their basis with additional terms to attain a second order approximation in output and prices. Each has $K = N(N+1)/2 + N+1$ free parameters which is enough to provide a second order approximation to an arbitrary differentiable cost function. This compares with $N(N+1)/2$ required for a unit cost function (or one linearly homogeneous in y) and $N(N+1)/2 + 2$ for a cost function for a homothetic production function. Thus an extra N-1 parameters are required for non-homotheticity.

The demand functions for GL and the share equations for GCD and TL are linear in parameters (as are the cost functions or their log):

(19) GL $C_i(w,y) = y \sum_j b_{ij} (w_j/w_i)^{1/2} + y^2 b_{oi} + b_{\infty}$ $\qquad i=1,\ldots,N$

(20) GCD $S_i(w,y) = 2 \sum_j b_{ij} w_i (w_i+w_j)^{-1} + \ln y \, b_{yi}$ $\qquad i=1,\ldots,N$

(21) TL $S_i(w,y) = \sum_j b_{ij} \ln w_j + \ln y \, b_{yi} + b_{oi}$ $\qquad i=1,\ldots,N$

thus facilitating estimation.

These forms permit various tests regarding the output effects. For example, in both GCD and TL homotheticity may be imposed by requiring $b_{yi}=0$ $i=1,\ldots,N-1$ (implying that $b_{yN}=0$). If, in addition, $b_{yy}=0$ then homogeneity of arbitrary degree $1/b_{oy}$ is imposed. Linear homogeneity in output is attained by further requiring that $b_{oy}=1$. Thus this formulation is rich in testable hypotheses.

The GL formulation in (19) is not so rich since it is only possible to impose homotheticity without imposing linear homogeneity if we give up the second order approximation in prices. If all $b_{oi} = 0$ and $b_{oo} = 0$ we have imposed linear homogeneity; if all $b_{ij} = 0$ and $b_{oo}=0$ then we have imposed homogeneity of degree 1/2 with a unit cost function linear in prices (Leontief).

d) Other Functional Forms

A major problem with flexible functional forms is that the number of parameters K increases rapidly as the number of inputs increases. In general, we require $K = \frac{(N^2+N)}{2} + N + 1$ parameters to provide a second order approximation to a cost function C(w,y) where N is the number of inputs. If, as occurs inevitably with time series data, the number of sample observations is limited then a problem develops. If we are committed to N inputs (the Hicks or Leontief conditions for aggregation are not met, say) then there are two approaches to the parameter problem. First, one can impose constraints on the parameters of a flexible functional form. The difficulty is in deciding what constraints to impose since any constraint reduces the flexibility of the functional form in a specific way. One might set certain parameters equal to zero (or some other constant) or impose separability conditions to reduce the number of free parameters. As a second approach we might abandon the flexible functional form and generate another functional form according to some other principle. For example, Mukurji (1963) and Gorman (1965) and Hanoch (1971) generate a production function in which the ratios of AES are constants, a generalization of the CES functional form. In this class are the variable elasticity of substitution functions already mentioned above. In an interesting paper, Hanoch (1975) uses the concept of direct and indirect implicit additivity to generate functional forms. He defines the reciprocal indirect production function g(w/M) to be implicitly additive if it can be written

(22) $$G(z,y) = \sum_{i=1}^{N} G_i(z_i,y) \equiv b$$

where y is output and $z_i \equiv w_i/M$ where M is the cost of production. Interpreting z as a vector of inputs we have the direct definition of implicit additivity. Under special functional form restrictions Hanoch derives forms which generalize the functions developed by Mukurji and Gorman and develops a new class of functions in which differences in AES are constant. These forms have fewer than 3N parameters. Finally, Mundlak (1973) reviews the functional form literature and suggests that non separable functional forms with 2n+2 parameters may be generated as $f(x) = g(x)* h(x)$ where * denotes an operation and $g(x)$ and $h(x)$ may be simple additive forms. A form in which the log of $f(x)$ is the product of two loglinear forms is suggested but this form does not appear to be very useful.

4. Technical Change

a) General Comments

Perhaps the most difficult problem in applied production analysis is that of distinguishing movements along the production function from movements from one production function to another. Suppose we observe that the input-output point moves from $(x(1), y(1))$ to $(x(2), y(2))$ from one period to the next. If $y(1) = y(2)$, is the movement from $x(1)$ to $x(2)$ a movement along the isoquant of a given production function (substitution) or a movement from one production function to another (technical change)? If $x(2) = \lambda x(1)$, $\lambda>0$, is the movement from one isoquant to another along the same production function (scale effect) or a movement from one production function to another (technical change)? In general it is difficult if not impossible to separate the effects of scale and substitution from technical change. To distinguish these effects in practice requires considerable structure to be imposed on the estimating procedure.[5]

In this section technical change is examined from the point of view of the researcher interested in formulating a model of the production sector. General surveys have been provided by Nadiri (1970) and Kennedy and Thirwall (1972). We first consider restrictions on the way in which technical change enters the production function, including special emphasis on the embodiedment hypothesis, and then consider models of endogenous technical change. While interest centres on empirical specifications these can only be derived and evaluated within a theoretical framework.

b) Restrictions on the Form of Technical Change

Technical change involves the shifting of the instantaneous production function so that the full production function may be written as

(1) $y = f(x,a)$ or $y = g(x,t)$

where a is a vector of knowledge. The second representation assumes we know the time path a(t) for knowledge. We need to be more specific about how a or t enter the production function if we are to undertake empirical work: saying that $\partial g/\partial t > 0$, for example, does not restrict the functional form sufficiently. In this section we examine various types of restrictions which may be imposed on the nature of technical change. It should be noted that most are not derived from any solid theory of technical change. Rather they provide convenient characterizations.

i) Neutrality A particular type of neutrality occurs when a special function or relationship is independent of technical change. For example, if the marginal rate of substitution between each pair of inputs is independent of technical change then the technical change is said to be Hicks neutral (HN) according to the definition used by Blackorby, Lovell and Thursby (1975). This is equivalent to saying that the inputs form a weakly separable group, that is that

(2) $y = f(h(x),a)$ or $y = g(h(x),t)$.

If, in addition production is input homothetic, the production function may be written

(3) $y = A(t)\ h(x)$

where $A(t) = A(a(t))$ is an index of technical change. Thus homotheticity and HN imply severe restrictions on the production function which may be taken into account and even tested in empirical work.

Most empirical work on production functions assumes the existence of Hicks neutral technical change. In combination with the homotheticity assumption it implies that factor shares and factor ratios are independent of technical change and so, if interest centres on these relations between inputs technical change may be ignored in the estimating procedure.

This holds irrespective of the time path for the rate of technical progress.

This property of Hicks neutrality was used by Berndt and Christensen (1973, 1974), Burgess (1974a, 1974b) and Berndt and Wood (1975) to estimate share equations without having to concern themselves with the time path of technical change. The advantage in terms of simplicity in estimation is clear. There are disadvantages to the assumption however. If we are interested in the shift of the production function over time then we need to estimate the time path of A(t). And, of course, the assumption may not be correct in which case we have made a specification error which will bias our estimates of the parameters of h(x). Estimation of a fixed exponential rate of technological change has been undertaken in most empirical studies such as those by Arrow et al. (1961), Coen and Hickman (1970), Kotowitz (1968), Tsurumi (1970), Ferguson (1965), McKinnon (1962), Woodland (1974) and many others.

Other types of neutrality may be assumed, the most popular one being Harrod neutrality and Solow neutrality, these being formally equivalent. Technical change is said to be Harrod neutral if the relationship between the marginal product of capital and the capital-output ratio is independent of the technical change. This only appears to be an interesting definition under constant returns to scale. In this case with two inputs Harrod neutrality is equivalent to representing the production function as

(4) $y = G(a_L(t)\ L,\ K)$

That is, it is equivalent to purely labour augmenting technical change. This has been shown by Uzawa (1961), Sàto and Beckmann (1968) and extended to multiple inputs by Burmeister and Dobell (1969). Solow neutrality occurs when the relationship between the labour-output ratio and the marginal product of labour is independent of technical change, which is equivalent to capital augmented technical change. In addition, Sato and Beckmann (1968) have generated several other definitions of neutrality of technical change. For each definition an invarient relationship between variables was specified and the implications for the form of the production function were derived, thus providing a framework within which the various types of neutrality may be exposed to the data. Sato (1970) has empirically estimated some of these production functions and rates of technical change using this framework.

By definition neutrality of technical change leaves certain relationships unchanged. Accordingly, these relationships may be estimated using time series data without making any assumption about the time path of such neutral technical change. While such an assumption is convenient, it should be tested. Under constant returns to scale Hicks' neutrality and Harrod-Solow type neutrality may be tested by considering each as special cases of the following factor augmenting specification (discussed below):

(5) $y = g(\alpha_1(t)\, x_1, \ldots\ldots, \alpha_N(t)\, x_N)$

where g is linear homogeneous. Hicks' neutrality occurs when $\alpha_i(t) = \alpha(t)$ for all $i=1,\ldots,N$; Harrod-Solow type neutrality for factor j occurs when $\alpha_i(t) = \alpha(t)$ for all $i \neq j$, and $\alpha_j(t)$ = constant say unity. For example, if $\alpha_i(t) = e^{\lambda_i t}$ then a test for Hicks' neutrality is equivalent to testing restriction $\lambda_i = \lambda$ for all i; and when $\lambda_i = \lambda$ for all $i \neq j$ and $\lambda_j = 0$ then we have Harrod-Solow neutrality for factor j.

ii) Separability[6] Hicks' neutrality is equivalent to the existence of an aggregator function for inputs in the production function, specifically the production function has the following representation

(6) $y = f(h(x), a)$.

That is, the inputs form a separable group so that the marginal rates of substitution between the inputs is independent of a and the marginal rates of substitution between the elements of a depend only upon $h(x)$ not the vector x itself.

An assumption often implicitly made in the literature is that the elements of a form a separable group so the production function has the representation:

(7) $y = f(x, \alpha(a))$

where $\alpha(a)$ is a consistent index of technical change. This restriction implies that the marginal rates of substitution between elements of vector a are independent of the elements of x and that the marginal rates of substitution between the inputs depend upon the index of technical change not the individual elements of a. If we could identify and quantify the elements of a (e.g. various indices of the level of various sorts of education, cumulated research output, number of volumes in the national library, etc.) then we could estimate the parameters of f using time series data. Then the assumption of separability would impose restrictions on the functional form which might be tested given a suitable functional form.

In a more general context separability has important implications. Consider the case where a firm may use inputs to produce outputs and additional knowledge. In general we might write the transformation function as

(8) $h(y,z,x,a) = 0$

where y denotes outputs, x inputs, a knowledge, and z additions to knowledge. Static profit maximization yields solutions for y,z, and x given vector a and prices p,q and w. If we assume the representation

(9) $h(y,x_y,a,g(z,x_z,a)) = 0$

where $x = (x_y, x_z)$ we imply that the production of knowledge is separable from the production of goods and services. This further implies a two stage maximization process in which the first stage is to maximize research profits, then given that solution the second stage is to maximize production profits. If the two activities are non-joint then we have two frontiers

(10) $h(y,x_y,a) = 0$ and $g(z,x_z,a) = 0$

in which case maximization occurs quite independently. That is the case with the Kennedy model of induced technical change discussed below. In particular, we can derive the cost function for technical change $C(w,z;a)$ which may be used to express the set of efficient choices of z available for a given research budget.

iii) Factor Augmentation A popular restriction on technical change is to assume that there exist indexes $\alpha_i(a)$ $i=1,\ldots,N$ representing the efficiency of factor i relative to some base, such that the production function has representation

(11) $y = f(\alpha_1(a)x_1,\ldots,\alpha_N(a)x_N) = f(x^*)$

where $x_i^* \equiv \alpha_i(a)x_i$. Here each factor is augmented by the efficiency index. The variable x_i^* is a measure of factor i in efficiency or base period units.

There are two approaches for empirical work. First, one might try to calculate $\alpha_i(a)$ and hence x_i^* and, once that is done, estimate the parameters of f. This is the approach taken by Griliches and others who adjust the labour force data for quality using an index of the education level. Second, we might estimate the parameters of $\alpha_i(a)$ given observations on y,x and a and a functional form for the α_i and f functions. Typically this is done by assuming a time path for $a(t)$ such that $\alpha_i(a(t)) = e^{\lambda_i t}$. This is the approach taken by David and Van de Klundert (1965) and implicitly by Duncan and Binswanger (1974a, 1974b, 1974c) for example.

A more general representation than (11) is

(12) $y = f(x_1^*(a,x_1),\ldots,x_N^*(a,x_N))$

where x_i^* is a function of a and x_i. Representation (11) is obviously a special case when x_i^* is linear homogeneous in x_i. Representation (12) effectively permits the index of efficiency to depend upon x_i. One example is the linear form $x_i^* = x_i + \lambda_i t$ used by Parks (1970) and Woodland (1975) in which there is a constant addition to efficiency per period.

In almost all of the above mentioned papers which provide estimates of rates of augmentation, the null hypothesis that all factor augmentation rates are the same (neutral technical change in the sense of Hicks[7]) is rejected by the data. For example, David and Klundert (1965) find that for the private domestic sector of the U.S. technical progress has been labour-saving during the present century. In particular they estimate the difference in the exponential rates of augmentation to be 0.0072 in labour's favour, which, though small, is significantly different from zero. They then go on to suggest that the period 1900-1960 is made up of 3 sub-periods the first of which (1900-1918) was a labour-saving period, the second (1919-45) was an average neutral in bias, the last sub-period being significantly labour saving in bias.[8]

Duncan and Binswanger (1974a, 1974b) find significant differences in rates of augmentation for Australian manufacturing industries. Sato (1967) tests for Hicks neutrality in the capital index of his separable production function and rejects it. Again Sato (1970, p. 200) concludes that "Technical progress is, on the average, nonneutral. The labor efficiency tends to rise faster (2 percent) than the capital efficiency (1 percent)" for the U.S. Private Nonfarm Sector. Fishelon (1974) finds that labour efficiency has grown relative to capital efficiency in Israel.

The only test of the augmentation hypothesis seems to have been made by Griliches (1964) who estimated a production function with education as an argument and found that it entered the production function in the same way as labour, thus enabling the production function to be expressed as a function of the product of labour and education.

iv) Embodied Technical Change Most empirical work assumes that technical change is disembodied. However, various economists, notably Solow, have suggested that embodied technical change is theoretically and empirically important. The idea is that such technical change is embodied in, say, capital in the form of a modification to a machine; machines produced before the technical change will not have increased efficiency. A disembodied technical change, on the other hand, is one where machines of all vintages have their efficiency increased, for example, by improved layout of the plant.

Essentially the embodiment hypothesis is that technical change is manifested in the construction of new capital forms or new labour forms or even new product forms. There is the presumption that each new form is a close substitute for the old forms and that they differ in some characteristic. Most emphasis has been placed upon technical change embodied in capital, though the same analysis applies to labour and produced goods. In the following we consider capital embodiment.

One approach to embodiment is to define the production function in the space of characteristics. Then a new technical advance would be indicated by the availability of different combinations of characteristics. But this approach has not been used. Rather, the approach has been to express the production function on the input space introducing the problem of an increasing dimension for the input space. To obviate this problem assumptions sufficient to ensure the existence of an aggregate of all vintages have been made in empirical work. The problem of embodied technical change is therefore part of the more general problem of aggregation which has been discussed by many economists including Hicks, Leontief, Green, Solow and Fisher.

The aggregation approach is as follows. Let there be one type of labour and one type of capital which undergoes embodied technical change. Then the production function is

(13) $y=F(L,K_1,\ldots,K_N)$

where y is output, L is labour input and K_i is the capital input for vintage i. The aggregation hypothesis is that there exists an index $J(K_1,\ldots,K_N)$ such that

(14) $y=G(L,J(K))$.

The conditions on F for such aggregation or weak separability are that marginal rates of substitution between capital vintages be independent of L. As a special case these marginal rates of substitution could be constant, that is the marginal products could differ by fixed proportions. If one unit of vintage i capital was "worth" $1+\mu > 1$ units of vintage i-1 capital then it seems reasonable to define the total amount of capital as

(15) $$J_t(K) = \sum_{i=t-N}^{t} K_{it}(1+\mu)^{-(t-i)}$$

where K_{it} is the amount of vintage i capital existing in period t. The marginal rate of substitution between K_i and K_j is $(1+\mu)^{j-i}$ which is independent of L. The constant μ is to be interpreted as the rate of embodied technical change. It is evident that technical progress is factor augmenting in the sense that this year's vintage is worth more than last year's vintage.

The aggregation problem is typically posed in a more special way by postulating vintage production functions $y_i=F_i(L_i,K_i)$ where the i refers to vintage i. If labour is allocated efficiently over all vintages subject to $\Sigma L_i=L$ then we obtain (13) as the aggregate production function. Then the conditions for aggregation of capital may be expressed in terms of the vintage production functions. Fisher (1968, p. 269) has proved that "in the two-factor, constant returns case, an aggregate capital stock, J, exists if and only if all technical change is capital augmenting", that is if $y_i=F(L_i,b_iK_i)$. In this case the production functions in terms of "efficiency units" is the same for all vintages. This result is true when there are many labour types all efficiently allocated. For a survey of the work that has been done on this aggregation problem see Fisher (1969).

The representation (15) allows the estimation of the rate of embodied technical progress by treating the capital stock as a parametric function of past capital formation. This approach has been taken by Solow and others using two factors and constant returns to scale. Empirical work has been done by Solow (1961, 1962), Intriligator (1965), Nelson 1964, Wickens (1970), Szakolczai and Stahl (1969) and Berglas (1965).

The parameters of G and the parameter μ of J may be estimated by nonlinear regression methods. To do this we need observations on output, labour and the stocks of each capital vintage in existence in each observation period. The parameters could be chosen to maximize the likelihood function given a random disturbance term in (14). Typically, several values of μ are tried and the best one chosen, as in Intriligator (1965), so that a full search over μ is not undertaken. Berglas (1965) estimates an ad hoc model again by trying different values for the rate of disembodied technical change, μ.

Wickens (1970) and McCarthy (1965) point out a basic identification problem in studies that use Cobb-Douglas production functions in an attempt to distinguish the rates of disembodied and embodied technical change. The problem is that there are three "rate" parameters, the two rates of technical change and the rate of depreciation which are not identified. If the rate of depreciation is known then identification of the rates of technical change follows but it should be clear that the estimates of these rates depend crucially upon the choice of the depreciation rate. Thus many answers can be obtained simply by varying the rate of depreciation. Since this is seldom known with accuracy intrepretation of the results should be made with caution. McCarthy shows that this identification problem does not occur with the CES function.

Jorgenson (1966) has argued that it is impossible to distinguish between embodied and disembodied technical change using the usual approaches to the construction of indices of technical change. Specifically, he shows that one may interpret errors in the measurement of the price index for capital services as a measure of embodied technical change and hence, by an appropriate choice of measurement, we can make the index of embodied technical change as large or as small as we like. And, since the price index for capital services enters the definition of the index of disembodied technical change, we can never distinguish between the two forms.

Wickens (1970) estimates a Cobb-Douglas model avoiding the nonlinearity of (15) by using a Taylor approximation and concludes that "once movements in output are removed we can find no evidence to support the embodiment hypothesis. Put another way, annual time series data do not contain enough information to enable us to distinguish between a model with a rather high rate of embodied technical progress (say 10 percent) and one with a very low rate (say zero)."

Other studies are reviewed in Kennedy and Thirwall (1972). For the Australian manufacturing industries Lydall (1968) could find little support for the embodiment hypothesis.

The embodiment hypothesis is closely associated with the concept of factor augmenting technical change and with the problem of aggregation. The Solow hypothesis is that different vintages of "essentially the same" capital good differ only in their efficiency, that is technical change is factor augmenting but is embodied in new capital. This permits aggregation according to the Leontief (1947) rules. The development of a truly new capital good cannot be handled by the Solow technique, and leads to the more difficult problem of aggregation over heterogeneous capital goods or to the replacement of and addition to the arguments of the production function.

In all empirical studies some aggregation of capital is essential. We can use the Solow technique and adjust the vintages for quality change, taking the rate of embodied technical change as known or as a parameter to be estimated. Or we can compute the index(es) of capital stock(s) in the usual way and estimate rates of factor augmentation. Which is best is in the final analysis an empirical question. Both methods may be applied since the data requirements are the same; in the embodiment case the capital stocks are functions of the basic data on depreciation and investment and the rate of augmentation. This complicates the estimation of the parameters of the system since (15) is nonlinear in μ. This nonlinearity may be avoided by approximations such as those used by Wickens (1970) or Nelson (1964).

c) Induced Technical Change

i) General Comments In this section we survey the main theoretical analyses of the generation of technical change. Technical change is not given exogenously to the economy but is generated within the economic system in a manner and degree partly determined by the structure of prices and by past experience. The works surveyed in this section attempt to formalize this idea.

As a general statement of the problem let us consider a firm producing commodities (one for simplicity) using various inputs. At any point in time the technology is fixed and the firm is therefore limited in its choice of input-output combinations. The firm may also be able to change the state of knowledge and hence alter the feasible set of input-output combinations in the following period. For example, the firm may be able to devote some of its resources to research activities which improve knowledge about production and hence lead to higher profits or lower costs next period. As a second example, the firm may be able to improve its knowledge about production simply by undertaking production, that is the firm may learn by doing. In this case no cost is attached to the learning process unless, of course, it is unprofitable to produce. These two examples identify the two approaches that have been taken in the literature.

The actual way in which research and development or learning by doing manifest themselves in the production function is not clear. Is the learning by doing undertaken by the workers or management? Presumably machines cannot learn but even so such technical change may be machine augmenting. Theory doesn't seem to help much in specifying precisely how learning by doing affects the technology. With research and development the possibilities are even greater since technical change may take the form of education of workers, improvements in machine efficiency, improved management, and the introduction of new types of machines and commodities.

Suppose the technology is described by the transformation equation $f(y(t), x(t), z(t), a(t)) = 0$ where y is a vector of outputs, x is a vector of inputs, a is a vector of states of knowledge and $\dot{a}(t) = z(t)$ is the change in the state of knowledge through time. This is very general since x may include research inputs which produce new knowledge, or an element of a may represent experience which may be increased by producing an output or using an input. If the firm faces an output price vector $p(t)$ and input price vector $w(t)$ (appropriately discounted) then the profit maximizing competitive firm will choose time paths $y(t)$, $x(t)$ and $z(t)$ and hence $a(t)$ to solve the following problem

$$\text{(16)} \quad \text{maximize} \int_0^T \{p(t)\; y(t) - w(t)\; x(t)\}\, dt$$

$$\text{subject to} \quad f(y(t), x(t), z(t), a(t)) = 0$$

$$\mathring{a}(t) = z(t)$$

$$a(o) \text{ given}$$

In setting up the firm's problem in this way we have made several important assumptions. First, we have ignored the fixed capital problem for simplicity. Second, the time paths for p and w are assumed to be known with certainty along with the technical change function g. A thorough analysis would require that attention be given to the inherent uncertainty regarding the future path of prices and, especially, the payoffs from research. Third, we have assumed that the firm is aware of the gains from investment in research activities hence that the optimal time paths reflect this. This is perfectly reasonable for the firm but at the industry level this assumption may not be so reasonable. For, suppose that experience comes from an industry wide technical change function and that the industry is perfectly competitive. Then no one firm will believe that it can change its production function by its actions alone. In this case the actual time paths chosen by the firms for the industry will be sub optimal since here we have a clear case of an economy external to the firm but internal to the industry. Another problem occurs when the economies are internal to the firm but the technical change is available to all firms in the industry at zero or minimal cost. Here the incentive to carry out research is lost. Consequently, for the industry to behave as if it solved the above optimization problem we require that all firms should be identical doing the same things and they should be aware of this.

We now proceed to review some special models and to see if these models can aid in the empirical estimation of the technology.

ii) Learning by Doing Noting the importance of the role of learning from experience as indicated by psychologists and by various researchers in industrial learning, Arrow (1962) formulated a simple model of learning by doing and examined the consequences for income distribution and economic growth. Arrow rejects the cumulative output as an index of experience and instead uses cumulative gross investment in capital goods as the index on the grounds that "Each new machine produced and put into use is capable of changing the environment in which production takes place, so that learning is taking place with continually new stimuli". He was however interested in a macroeconomic growth model in which "the possibility of continued learning in the sense, here, of a steady rate of growth in productivity" was plausible.

Arrow assumed that new capital goods were more efficient than older ones because more had been produced before and hence that technical change is embodied in capital. Assuming that labour and each type of machine are employed in fixed ratio under constant returns to scale, Arrow is able to show that output is a function of the total labour input and cumulative gross investment. He then shows that the competitive equilibrium is sub optimal in that a higher discounted value of consumption is attainable under central planning. The reason is that individual producers do not take account of the effect of their decisions on technical change and hence on their profits.

Arrow's model has been extended within the context of macroeconomic growth by Levhari (1966a, 1966b). It has also been used to provide a better understanding of the concept of an infant industry in international trade by Clemhout and Wan and Bardhan who derive optimum subsidy formulae needed to attain the first best solution under competitive behavior when experience is measured by cumulative output discounted by a "memory loss" factor.

We now consider the empirical implications of learning models. Suppose that experience is measured by cumulative industry output and that the firms in the industry do not take account of the effect of industry output upon their own set of feasible input-output combinations. Then decisions are made on the choice of output y_t and input x_t subject to $f(y_t, x_t, \alpha_t) = 0$ where $\alpha_t \equiv \int_0^t y(r)\, dr$ at time t. Given observations on y_t, x_t and the index of "experience" α_t over some sample period together with a functional form for f and a stochastic specification we may estimate the parameters of f econometrically. Of course we may wish to add further assumptions about behavior and estimate the cost, profit or factor demand functions which will depend upon α_t.

There has been little work done along these lines. Rapping (1965) estimates a production function linear in the logs of all the variables for shipping yards in the U.S. during World War II. He concluded from his results that economics of scale in production existed and that the best explanation of technical change was provided by the cumulated output. A time variable representing a constant rate of neutral technical change was inferior as an explanatory variable. Fellner (1969) used the time variable (time actually producing) successfully in the explanation of improved athletic performances. In addition there have been various studies at the micro level and which are discussed by Kennedy and Thirwall (1972, pp. 38-39).

Sheshinski (1967) estimated a CES production function in which the index of neutral technical change was specified as a function of time, representing "unlearned technical change", and experience, represented by (a) cumulative output and (b) cumulative gross investment. Thus he did not assume embodiment as did Arrow. The model was estimated using state cross-section data for 2-digit industries in the U.S. and using time series data for six countries including the U.S. and Australia. In general the results provided mild support for the Arrow hypothesis.

Berglas (1965) has estimated an ad hoc relationship between output per worker and capital per worker under three assumptions re technical change: all technical change is embodied in capital and is (a) time dependant, or (b) cumulative gross investment dependant and (c) technical change is disembodied and depends on cumulative gross investment. The results are not very clear cut or satisfactory however.

A major problem which suggests caution in applying the learning model at the industry level using time series data is that the index of experience, time and output are likely to be highly correlated. If this is the case then, the discrimination between learning by doing, exogenous technical change, and economies of scale must rely heavily upon the functional form adopted. Without good theoretical grounds for the choice of the functional form any discrimination is very weak and should be regarded with caution. In addition, while the model sounds plausible, it is a rather mechanical one in that learning is not a choice variable but occurs at a rate determined by the output or investment path which are, of course, choice variables.

iii) Choice of Technical Change[9] While technical change due to learning by doing is induced by our choice of input-output combinations in the ordinary production process, it is not the consequence of deliberate economic decisions taken with the objective of choosing a rate and direction for technical change. Here we briefly examine theoretical developments which emphasize such a choice.

This approach was first formulated by Kennedy (1964) though his work bears some relation to earlier work including that of Kaldor (1957). Kennedy formulated a myopic decision making model in which the direction of technical change is chosen to maximize the rate of reduction in the cost of production given that the rates of cost reduction for labour and capital are limited by an "innovation possibilities frontier". Let the unit cost be given by $\gamma = c(w^*)$ where $w_i^* = w_i/b_i$ and b_i is the index of efficiency for factor i under the factor augmentation hypothesis of technical change. Then the percentage reduction in unit cost due to a change in the b_i is

$$-\hat{\gamma} = \sum_i S_i(w^*) . \hat{b}_i \tag{17}$$

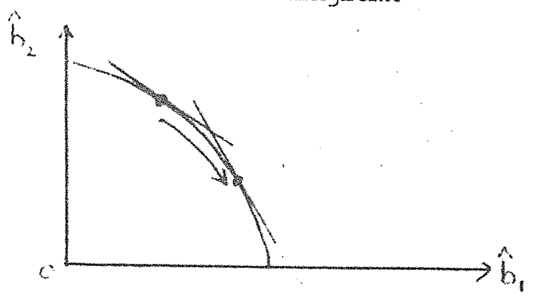
which is the objective to be maximized.[10] The constraint is that

$$g(\hat{b}_1, \ldots, \hat{b}_N) = 0 \tag{18}$$

which is the innovation possibilities frontier (IPF) in which the rates of augmentation are limited. The IPF is assumed to be concave. Treating the cost shares $S_i(w^*)$ as constants the conditions for maximizing the unit cost reduction given the IPF are

$$\frac{\partial g/\partial \hat{b}_i}{\partial g/\partial \hat{b}_j} = \frac{S_i}{S_j} \quad \text{and } g(\hat{b}_1, \ldots, \hat{b}_N) = 0. \tag{19}$$

When N=2, as in Kennedy's paper, it is evident that if S_1 increases for some reason then $\hat{b}_1$ will increase and $\hat{b}_2$ will decrease if g is concave as illustrated in the diagram:



Effect of an Increase in S_1

Kennedy argued that the cost shares were important in determining the direction of technical change and that his theory led to a theory of income distribution. He argued that in long run equilibrium $\hat{b}_1=\hat{b}_2$ and there will be a corresponding share ratio S_1/S_2 the equilibrium distribution of income. But he did not consider the stability of the adjustment path. Samuelson (1965) pointed out that stability requires that the elasticity of substitution between factors 1 and 2 $\sigma_{12} < 1$. This is because

(20) $\hat{S}_2-\hat{S}_1 = (\sigma_{12}-1)\ (\hat{b}_2-\hat{b}_1)$

if factor prices do not change. Thus if initially $\hat{b}_2 > \hat{b}_1$ and $\sigma_{12} < 1$ then the share of factor 2 will fall. This induces a reduction in $\hat{b}_2$ and an increase in $\hat{b}_1$ next period. If $\hat{b}_2 > \hat{b}_1$ still, then there will be a further reduction in S_2 until $\hat{b}_2=\hat{b}_1$. If $\sigma_{12} > 1$, however, $\hat{b}_2 > \hat{b}_1$ implies that S_2 will rise inducing a further increase in b_2 and a further increase in S_2 until $S_2=1$.

Kennedy was interested in the macro implications of his model, in particular the implications for income distribution. As Samuelson points out, Kennedy's analysis is deficient because he assumes that factor prices are constant, hence he cannot have a complete theory of distribution, and because he considers only myopic behavior. Samuelson goes on to generalize the model by assuming that factor prices are endogenous with factor endowments fixed, and then by considering an intertemporal decision-making model. He also allows endowments to change over time.

Others including Ahmad (1966), Nordhaus (1969) and Binswanger (1974) have criticized the Kennedy-Weizsacker-Samuelson model on various grounds. Ahmad argues that if the IPF constrains not the rates of change in the indices of augmentation $\hat{b}_i$ but the absolute changes $\dot{b}_i = \hat{b}_i\, b_i$ then the input shares are replaced by the input prices as the crucial determinants of the direction of technical change. This fits in with the original statements by Hicks (1932). Binswanger, following Nordhaus, argues that the specification of the IPF as a stable function is based upon unrealistically simple assumptions and that it is the choice of the position of the frontier not only of the point on the frontier that is the crux of the endogenous technical change problem. However as our formulation of the problem in (22) indicates this criticism is easily remedied.

At the micro or industry level where factor prices may be taken as given the Kennedy model is relevant and this approach has been set in an intertemporal framework by Kamien and Schwartz (1969). What are the implications of Kennedy's model for micro production functions? In the long run if $\sigma_{12} < 1$ we would expect to observe an equilibrium factor share which is constant at a value consistent with $\hat{b}_1 = \hat{b}_2$. Thus in the long run technical change is Hicks neutral with equal rates of augmentation for capital and labour.[11] This share is independent of the production function; only if the IPF shifts will the shares change. Of course, along the adjustment path rates of augmentation will not be equal. Indeed if $\sigma_{12} > 1$ then these rates diverge with the shares approaching zero or unity. If we restrict $\hat{b}_j \geq 0$ then if in equilibrium $S_2 = 1$ and $S_1 = 0$ we will have $\hat{b}_2 = 0$ and $\hat{b}_1 > 0$ and so we have either Harrod neutral technical change, $\hat{b}_L = 0$, or Solow neutral technical change, $\hat{b}_K = 0$. In short, the Kennedy model yields the Hicks, Harrod and Solow forms of neutrality as special long run solutions; this is of little help in deciding upon a choice between these forms of neutrality for production function estimation purposes. Moreover, in the short run these results may not be very relevant.

iv) An Empirical Model of the Kennedy Theory If the long run theory of induced technical change does not help in formulating restrictions on the form of technical change, it may be worthwhile considering whether we can estimate the complete model describing the input choice and the choice of direction of technical change simultaneously. While this seems a logical thing to do this problem does not appear to have been discussed in the literature. We now examine how a Kennedy type model might be estimated econometrically and used to test the theory of induced technical change and then consider several other approaches to the problem which have appeared in the literature.

The firm (industry) is assumed to make decisions regarding the choice of inputs for production given factor prices and the state of technology. The technology may be altered by devoting resources to research which will lead to increases in the indexes of input augmentation. The research and production technologies are separate or disjoint so the decisions are independent ones. The firm's cost function is $C(w,y,b) \equiv C^*(w^*,y)$ where $w_i^* \equiv w_i/b_i$ $i=1,\ldots,N$. This yields the input share equations for period t

$$(21) \qquad S_{it} = S_i(w_t,y_t,b_t) \qquad i=1,\ldots,N \quad [= S_i(w^*_t,y_t)]$$

where we recognize that b_t enters in a special way, that is as a vector of augmentation indices. According to the Kennedy model the rates of augmentation $\hat{b}_{it}$ are chosen to maximize the reduction in cost given factor prices and hence factor shares. That is, the firm solves the following problem

$$(22) \qquad r(S_t,M_t) \equiv \max_{\hat{b}_t} \{S_t\hat{b}_t : g(\hat{b}_t,M_t) \geqq 0\}$$

where M_t is the expenditure on research and development. The function $r(S_t,M_t)$ indicates the optimal cost reduction given S_t and M_t. It is formally equivalent to a revenue function as defined and discussed by Diewert (1974b) and is therefore a continuous function, nondecreasing concave and linear homogeneous in S_t, and nondecreasing in M_t. The optimal choice for $\hat{b}_{it}$ is derived from Hotelling's Lemma as

$$(23) \qquad \hat{b}_{it} = \frac{\partial r(S_t,M_t)}{\partial S_{it}} \equiv r_i(S_t,M_t) \qquad i=1,\ldots,N$$

whence, since $\hat{b}_{it} = (b_{it+1} - b_{it})/b_{it}$,

$$(24) \qquad b_{it+1} = [1+r_i(S_t,M_t)]\, b_{it} \equiv \rho_i(S_t,M_t)\, b_{it} \qquad i=1,\ldots,N.$$

If we could observe the vector of shares S_t, the vector of factor prices w_t, output y_t, research expenditures M_t and the vector of augmentation indexes b_t then we could estimate the parameters of the cost function and the revenue function (or, by duality the IPF) by estimating the share equations (21) and the technical change functions (24) . They form a nonlinear simultaneous equation system so the estimation procedure would need to take this into account. But, of course, we do not observe b_t.

One approach is to linearize the share equations by a first-order Taylor approximation. Writing (21) in vector form as $S_t = S(w_t, y_t, b_t)$ and linearizing we get that

$$(25) \qquad \hat{S}_t = E(w_t,y_t,b_t)\ (\hat{w}_t-\hat{b}_t) = E(w_t,y_t,b_t)\ \hat{w}_t - E(w_t,y_t,b_t)\ R(S_t,M_t)$$

where $E(w_t,y_t,b_t)$ is a matrix of demand elasticities and $R(S_t,M_t)$ is the vector of derivatives $r_i(S_t,M_t)$ $i=1,\ldots,N$. If we could set E to some constant matrix $\bar{E}$ then the set of equations (25) could be estimated noting the econometric problems that sometimes arise in estimating sets of equations with lagged dependent variables in the list of regressors. Such a model would permit the estimation of $\bar{E}$ and the parameters of R. However, it is difficult to justify the assumption that E is fixed because it imposes severe restrictions on the cost function and because it would be an approximation for changes in w_t, y_t and b_t within a small range.

There is an attractive alternative however. First, note that the simultaneous system (21) and (24) is homogeneous of degree zero in the b_{it}'s. That is if we observed the time path described by the system for a given initial vector of augmentation indexes $b_o=\alpha_o$, then the same time path would be followed if we set $b_o=\lambda\alpha_o$ for any $\lambda>0$. Therefore we may normalize the indexes such that $b_t=1$ for some t without any loss of generality. Treating the period t=0 as the base period for the indexes we may estimate the complete system. The econometric model is

$$(26) \qquad S_{it} = S_i(w_t, y_t, b_t) + U_{it} \qquad i=1,\ldots,N$$

$$\text{where } b_{it} = \prod_{\gamma=1}^{t} \rho_i(S_{t-\gamma}, M_{t-\gamma})\, b_{io} \qquad b_{io} \equiv 1$$

where $U_t = (U_{1t}, \ldots, U_{Nt})$ is a vector of jointly dependent disturbances.

This model assumes that the technical change equations are non-stochastic but that the share equations are stochastic, an assumption which appears to be needed to get a tractable model. Given this formulation we may use observations on S_t, w_t, y_t, and M_t to estimate the system of share equations thus obtaining estimates of the parameters of the cost function and for the "revenue" function which is dual to the IPF. We may choose a functional form for $r(S_t, M_t)$, derived by Diewert (1974b), which may be interpreted as a second order approximation to an arbitrary differentiable revenue function. This form has the nice property that by setting certain parameters equal to zero the rates of augmentation are constants, independent of factor shares. Thus we are able to test the Kennedy model of induced technical change against the alternative that all rates are fixed exogenously. As a special case of the alternative we have Hicks' neutrality of augmentation when the constant rates are equal. Furthermore, the role of research expenditures may be tested.

What are the drawbacks of this empirical model? First, because it is nonlinear in the parameters the computational burden is considerable. Second, one would like a large number of observations in the sample since the number of parameters will be fairly large. Using flexible functional forms for C and r we will have about N^2+3N parameters.

Third, it assumes myopic behavior whereas it seems more realistic to assume that at each point in time the data are generated as the initial part of an intertemporal solution to an intertemporal model. This assumes that as new information becomes available (at each point in time) firms recalculate their optimal programs and embark on them.

The last deficiency of the Kennedy model has been examined by Kamien and Schwartz (1969). They examine the continuous time intertemporal problem.

$$(27) \qquad \max \int_{o}^{T} e^{-\rho t} \left[r(y(t)) - w\, x\,(t) - M(t)\right] dt$$

$$\text{s.t.} \quad y(t) = f(x(t), b(t)) \equiv f^*(x^*) \qquad \text{(say)}$$

$$g(\hat{b}) = h(M(t))$$

$$b(o) \text{ given}$$

where f^* is homothetic in x^* where $x_i^* = x_i/b_i$, ρ is a discount rate and $T = \infty$. Clearly at each point in time y and x will be chosen to maximize instantaneous profits so the problem may be rewritten as

$$(28) \qquad \max \int_{o}^{T} e^{-\rho t} \left[N(b) - M(t)\right] dt$$

$$\text{s.t.} \quad g(\hat{b}) = h(M(t))$$

$$b(o) \text{ given}$$

where $N(b) \equiv \max_{y,x} \{r(y) - wx : y = f(x,b) \equiv f^*(x^*)\}$ the w argument being ignored since it remains constant. Kamein and Schwartz examine the long run properties of the model and conclude that "optimal behavior is similar to that derived from a myopic objective, but tends to be less (more) biased than myopic behavior as $\sigma_{12} < (>1)\ 1$. Further, the myopic view point leads to underinvestment in technical advance if the firm size, in cost of sales, is growing, and to overinvestment otherwise".

Whether one can derive an empirical model based upon this optimization problem remains to be seen.

v) Empirical Work There appear to be few attempts to formally test the hypothesis of induced innovation of technical change. Morishima and Saito (1968) develop a two sector model in which some of the parameters (not σ) of the CES sectoral production functions are dependent upon the "scarcity" of the inputs. The scarcity of inputs is defined in terms of utilization rates. Obtaining results with a complex estimation procedure for the US private sector they find that the results of David and Klundert (1965) and Brown and Popkin (1962) regarding bias of technical change may be, in the large, explained by the induced technical change theory.

Lucas (1967a) formulated a microeconomic model of production and research which he analysed theoretically and then tested empirically using time series data for the U.S. manufacturing industry. The empirical model does not reflect the theory faithfully but is suggested by the theory. Lucas (p. 187) concludes that "statistically, then, our results reject quite strongly the view that the rate of technical advance in manufacturing is unaffected by market forces". Another study by Nordhaus (1969) finds little support for the IPF hypothesis and favours an alternative inventor hypothesis.

Another approach used is to rank factors by their estimated rate of augmentation and the rate of change in their prices over the sample period and to compare these rankings. This is the approach taken by Duncan and Binswanger (1974a, 1974b) in their work on production functions for Australian manufacturing industries. They claim mild support for the Hicks-Ahmad theory of induced technical change.

5. Imperfect Adjustment

a) General Comments

The preceding analysis has assumed that all adjustments of quantities to prices are instantaneous. In this section we consider various models of the adjustment process which have been proposed and estimated in the literature.

The production function $y = f(x)$ relates output per period to the inputs per period of the various factors of production. If we can measure the input quantities and the output quantities then we can use regression methods to estimate the parameters of f and so obtain estimates of the elasticities of substitution as functions of x. There is no need to consider any adjustment process. However, to use these estimates economists usually invoke some optimizing or other behaviour on the part of firms and this involves the implicit assumption of instantaneous (within the period) adjustment. Since our concern is with adjustment of quantities to price and other changes we cannot use the production function approach just outlined alone.

The most common approach, used extensively in the literature on investment functions, is to postulate that input quantities cannot adjust instantaneously but adjust at varying rates over a number of periods to some desired or long run level. The model is composed of two parts, one explaining the desired levels of inputs the other explaining the time path of response of inputs to the desired levels. Quantity adjustment models are discussed in the following subsection.

Another way of modelling the role of time in the behaviour of firms is to postulate that expectations about future prices are formed and it is these expectations which are used to calculate input-output choices. The expectations are presumably formed on the basis of past experience as well as feelings about the future. These models thus have two parts also; one model to explain expectations, another to explain input-output choices given these expectations. Such models are also discussed briefly below.

It should be emphasized that this whole area is extremely wide so that this brief survey cannot hope to be comprehensive. Nevertheless the main issues from an empirical viewpoint are presented. It should also be noted that the connection between theory and empirical work in this area is not as close as is desirable due to the complexity of intertemporal optimization models relative to static optimization models.

b) Quantity Adjustments

i) Adjustment Models. Since we are concerned with systems of input demand functions we will present the analysis in that context and so be only tangentially concerned with the voluminous literature on investment functions.

The typical *ad hoc* theory of adjustment goes as follows. Given a vector of input prices w_t and an output level y_t at time t the firm (industry) would like to choose a vector of inputs to minimize the cost of production. Let the cost minimizing vector of inputs be

$$(1) \quad x_t^* \equiv \nabla_w C(w_t, y_t)$$

where $\nabla_w C(w, y)$ is the vector of partial derivatives of C with respect to the elements of w. The input vector used in the previous period t-1 is x_{t-1} so the desired change in the input vector is $x_t^* - x_{t-1}$ which under perfect adjustment, would be attained. However, for various reasons it may be not possible to change x within the period so the input vector actually used in period t is $x_t \neq x^*$. The problem is to find a suitable adjustment mechanism relating x_t to x^* and x_{t-1}:

(2) $x_t = G(x^*, x_{t-1})$ (G is a vector-valued function)

such that $\lim (t \to \infty)\ x_t = x^*$. That is, we require that at least in the limit the desired vector of inputs is attained or, equivalently, that the difference equation system (2) is stable with solution x^*.

The special form of (2) which has been popular because of its relatively simple structure is

(3) $x_t - x_{t-1} = A(x_t^* - x_{t-1})$ or $x_t = Ax_t^* + (I - A)x_{t-1}$

where A is a matrix of "adjustment coefficients". It can be shown that the stable solution to (3) is $x = x^*$ and that (3) is stable if and only if every characteristic root of matrix A is in the open interval (0, 2) or, equivalently, the roots of (I - A) are less than unity in absolute value. The approach of x_t to x^* may be monotonic or oscillatory depending upon the nature of A or its characteristic roots. Clearly (3) is a generalization of the simple geometric lag in one variable.

The vector x_t^* is not observed so estimation of the parameters of the cost function C and matrix A must be done after combining (1) and (3) to get

(4) $x_t = A\nabla_w C(w_t, y_t) + (I - A)x_{t-1} + u_t$

where u_t is a vector of disturbances. Given observations over the sample period $t = 1,\ldots,T$ on the vector of inputs x_t, the vector of input prices w_t, output y_t and the previous period inputs x_{t-1}, system (4) may be estimated econometrically. If a density function for u_t is given then maximum likelihood estimation could be undertaken. It should be noted that A and the parameters of C will necessarily enter (4) nonlinearly, even if $\nabla_w C$ is linear in parameters, and that both sets of parameters will have elements appearing in more than one equation. Accordingly, the equations of (4) must be estimated jointly. Another reason for joint estimation is that the individual disturbances are likely to be jointly distributed for reasons outlined in Section 2.

This model has been estimated by Nadiri and Rosen (1969, 1973) and Coen and Hickman (1970). Both use a Cobb-Douglas production function to get the derived demand functions (1) which turn out to be linear in the logs of the variables. They then postulate that (3) holds with respect to the logs of variables and so, combining (1) and (3) and adding disturbances they obtain the estimating equations (4) where all variables are to be interpreted as logs of the underlying variables. The adjustment mechanism in original variables is

$$(5)\quad x^*_{it}/x_{i\,t-1} = \prod_{j=1}^{N} (x^*_{jt}/x_{jt-1})^{a_{ij}} \qquad i = 1,\ldots,N$$

Despite the compelling reasons for joint estimation, Nadiri and Rosen did not estimate (4) as a complete system of equations but, instead, estimated each equation separately by ordinary least squares. Thus they took no account of the covariances between the disturbances and, moreover, since no account was taken of the fact that the same parameters appear in different equations they obtain multiple estimates of these parameters, specifically those of the underlying production function. In addition, they were forced to omit the user cost of labour due to lack of data. The results, which must therefore be interpreted with extreme caution, show that the adjustment mechanism is an important part of the model specification and that some of the off diagonal elements of A are significantly different from zero.

Thus, the adjustment of an input to its desired level depends on the gap between the desired and actual levels for that input and other inputs.

Coen and Hickman emphasize the desirability of estimating the system jointly and employ maximum likelihood methods. However, they assume that the matrix of adjustment coefficients is diagonal ruling out the interdependence of the adjustment paths considered so important by Nadiri and Rosen.

The adjustment paths must be interdependent. If we observe y_t as the output and x_t as the vector of inputs in period t then it must be true that $f(x_t) \geq y_t$ where f is the production function. The adjustment path must respect this constraint, otherwise we might observe inputs which could not produce the observed output, a situation inconsistent with the theory. This restriction means, for example, that if output increases such that the desired inputs all go up then if one input adjusts slowly another input must overadjust. Even if one recognizes this constraint as being important, it is another matter to impose it on the model and still maintain equations which may be estimated without undue difficulty. In the Cobb-Douglas model employed by Nadiri and Rosen these conditions can be imposed on the parameters in the form of bilinear and determinantal conditions. These are nontrivial to impose in practice and for more general technologies the problems would appear to be too complex to warrant consideration.

Once system (4) has been estimated the results may be used to describe the response of inputs to changes in output and input prices, as done in the above-mentioned papers. The short run responses are, of course, of vital importance when simulating the effects of a change in the economic policy which affects output and input prices. The long run response is also of interest and it may be calculated as the solution (1). Thus this model is very useful in that it permits a detailed description of the time paths of inputs as they adjust to external shocks.

Because such a model can be so used, it is important that it be properly estimated and interpreted. It is relatively easy to set up and estimate a lagged adjustment model, but it is not so easy to be confident that it is a reasonable model of behaviour and not simply an ad hoc descriptive device that seems to perform well in the sense that it describes the time series fairly accurately.

It has been argued that the adjustment path should respect the constraint imposed by the production function. One possible way of arguing against this position is to assume, as Coen and Hickman do, that the measured inputs do not accurately reflect the services yielded since, for example, labour and/or machinery may be worked more intensively if output is to increase quickly. This implies that the intensity or rate of utilization is a choice variable permitting increased production from given measured inputs. Certainly this assumption permits the firm to appear to be off its production function when expressed in measured units and hence supports the position that (4) may be a "good" model without any further condition to ensure that the production function is respected. However, it implies that the model is incomplete or incorrectly specified and suggests that the rate of utilization of inputs should be modelled explicitly. The literature on the theory of capital utilization has recently been surveyed by Whinston (1974).

ii) <u>Adjustment Cost Theory</u>. Why don't firms adjust fully to changes in circumstances within the period? One explanation is that at the going prices the market supply of an input is insufficient to meet the demand, that prices are somehow sticky and do not adjust to clear the market in the period; and hence that some firms cannot attain desired increases in employment of factors. Also contracts may prevent reductions in employment of labour. Or it just takes time to install new equipment or get rid of old equipment. This explanation thus rests on the time element involved in change or upon some rationing assumption.

An alternative explanation is that there are "costs of adjustment" which discourage rapid change. For example, there are the costs of advertisement, interviewing, etc. associated with hiring new workers. There are the costs of foregone output and hence profits associated with having the plant idle while additions are made. Several economists have recognized the importance of adjustment costs as an explanation for slow adjustment and have set up formal models.

One such model is as follows. Let the output rate be y and the input price vector be w, both fixed, and let $g(\dot{x}(t))$ be the cost of adjustment where $\dot{x}(t)$ is the rate of change in $x(t)$. If the firm faces a fixed interest rate r then the problem posed is that the firm should minimize the discounted costs of production and adjustment given the initial employment of inputs. In continuous time this problem is to choose a time path $\dot{x}(t)$ and $z(t)$ to

$$(6) \quad \min \int_0^\infty e^{-rt}[wx(t) + g(z(t))]dt$$

$$\text{subject to} \quad \begin{aligned} \dot{x}(t) &= z(t) \\ y(t) &= f(x(t)) \\ x(0) &= x_0 , \end{aligned}$$

which is an optimal control problem. An alternative formulation might be to specify a price for output and allow the time path for output $y(t)$ to be endogenous in which case the problem would be to maximize the present value of profits. In both cases w is interpreted as the usual opportunity or user cost.

Following on from a one factor model examined by Eisner and Strotz (1963), Lucas (1967b) considered a model with a concave production function with a subset of the inputs being subject to adjustment costs. He showed that in the neighbourhood of the stable solution to the optimization problem the optimal path may be approximated by a linear differential equation system of the form

$$(7) \quad \dot{x}(t) = A[x^* - x(t)]$$

where the matrix of adjustment coefficients is a matrix function evaluated at the stationary solution x^* for non perfectly adjusting inputs. Thus in the neighbourhood of a stationary solution the flexible accelerator model discussed in the previous subsection would appear reasonable. However, away from this solution A cannot be assumed constant. Furthermore, if some basic data change (e.g. interest rate) not only will x^* change but the matrix A will also change. Thus for most empirical researchers this result (7) may be of limited applicability since over the sample period the basic data must change if any estimation is to be undertaken.

The Eisner-Strotz and Lucas results have been examined by Gould (1968) who points out that the assumption of stationary prices is crucial to their analyses. Treadway (1971) treates the multivariate flexible accelerator in a quite general way and points out that the Lucas results are based upon the assumption that the variables x and $\dot{x}$ are separable in the production process.

Schramm (1970) makes use of Lucas' results and estimates an adjustment cost model using US data on investment in three types of capital. He considers a model with a quadratic production function yielding the investment equations

$$(8) \quad x_t - x_{t-1} = Ac + ARw_t - Ax_{t-1}$$

where A is the adjustment coefficient matrix, w_t is the vector of user costs divided by the input price, R is a symmetric negative definite matrix and c is a vector of constants. He uses a result of Lucas (1967) which is that AR is symmetric negative definite. However he estimates only one of the equations of (8), that for depreciable capital.

iii) Ex Ante and Ex Post Substitution. Another approach to the question of time responsiveness of inputs to price and output changes is to assume that ex ante and ex post substitution possibilities are different. This approach emphasizes the long run versus short run options facing a firm. In the short run there is a particular technique or set of techniques available depending upon past decisions regarding investment and labour training decisions so substitution possibilities are limited. In the long run they are only limited by the state of technology.

Recently Fuss (1972) has set up a model differentiating between the short and long run and has estimated it using data on the generation of electricity. He was able to test various hypotheses including the putty-clay hypothesis which could not be rejected by the data.

c) Price Expectations

i) Models. While we have just given attention to the adjustment paths of inputs to current prices, imperfect adjustment may take the form of adjusting "planning prices" to current prices. Thus we get price expectations models. If the prices in the current period are not known when production decisions are made it is important that expectations be formed. Furthermore, when there are costs of adjustment the future course of prices will have an important influence upon how inputs respond now. For these reasons price expectations models may be used to model firm behaviour.

Quite a sizeable proportion of production function studies use price and output expectations. The most common assumption is that of adaptive expectations which hypothesizes that the expected price w^e_{it} changes according to

$$(9) \quad w^e_{it} - w^e_{it-1} = a_i[w_{it-1} - w^e_{it-1}] \qquad a_i \geq 0$$

That is, expectations increase whenever actual prices turn out to be above the expectation of the previous period. Notice that the expectations for period t are based upon past data on prices. An alternative formulation with the current price replacing last period's price on the right-hand side cannot be viewed as an expectations model since if w_{it} were known no expectations would need to be formed. Rather it would have to be viewed as a "planning price" model in which planning prices are viewed as average or long term prices.

The problem with (9) is that expected prices are not observed. However, we can make (9) operational in various ways, one of which is to rewrite it as

$$(10) \quad w^e_{it} = \sum_{j=1}^{t} a_i(1 - a_i)^{j-1} w_{i,t-j-1} + (1-a)^t w^e_{i0}$$

which is a linear function of past prices and the base period expected price. The latter may be regarded as a parameter of the model since it depends upon the parameter a_i and the complete history of prices prior to period 0.

The model is completed by assuming that the firm (industry) minimizes the cost of production given these expectations. The factor demand functions are

$$(11) \quad x_{it} = \partial C(w^e_t, y_t)/\partial w^e_{it} \qquad i = 1,\ldots,N$$

Substituting the expression (10) for the non-observed expected prices w^e_{it} we find that the input demand functions are functions of the past history of prices summarized beyond the base period by w^e_{i0} which we treat as a parameter. In principle we can add random disturbances to (11) and estimate the parameters of the cost function and the parameters $a_i = (a_1,\ldots,a_N)$ and $w^e_0 = (w^e_{10},\ldots,w^e_{N0})$ of the price expectations formulae.

Just as with the quantity of adjustment models, this model may be used to simulate the time path of the response of inputs to changes in current input prices. The expectations model (10) will be stable if and only if $|1 - a_i| < 1$ for all i, that is, if and only if $0 < a_i < 2$. If $a_i = 1$ then we have the case of unit elastic expectations where we expect last year's price to occur this year; if $a_i < 1$ then expected prices adjust monotonically to actual prices and if $a_i > 1$ the adjustment is cyclical.

Estimation of (11) is nontrivial since (11) is a set of N equations which, while linear in the parameters of the cost function using a standard flexible functional form, is extremely nonlinear in the parameters of the expectations formulae. For $N > 2$ a grid search method, whereby linear-regressions are carried out conditional on the parameters a and w_0^e and the best one chosen, is likely to be inefficient. A direct approach seems to be required but even here the computational burden increases rapidly as N increases. For an example of the estimation of a similar model see Woodland (1974).

A much simpler formulation is

$$(12) \quad w_{it}^e = a_i w_{it} + (1 - a_i) w_{it-1}$$

in original variables or in logs of variables. The log form was used by Coen and Hickman (1970) but eventually they specified that $a_i = 0$ or 1 depending on i, when their estimates of the a_i turned out to be negative. A similar assumption to (12) was used by Sato (1967) to model the response of actual inputs of capital to desired inputs and last period inputs. The advantage of these formulations is that in (12) the expected price is a linear combination of current and past prices and so is a simple relationship which is relatively easy to estimate. The one disadvantage is that, being simple, it may be too restrictive to be an approximation to reality. Also, if current prices are known we must interpret w_{it}^e as a planning price.

ii) <u>Duality</u>. It turns out that there is duality or equivalence between certain quantity adjustment paths and price expectations paths. To illustrate this duality consider a firm in equilibrium producing output $y°$ with inputs $x°$ such that $f(x°) = y°$ and $w°x° = C(w°, y°)$ where $w°$ is the input price vector. There is an external shock in the form of a change in w from w° to w'. According to the quantity adjustment hypothesis the input vector will not move immediately from $x^{\circ} = \nabla_w C(w^{\circ}, y^{\circ})$ to $x' = \nabla_w C(w', y^{\circ})$ but will move along the path $x(t)$ which begins at x° and ends (asymptotically, perhaps) at x'. At all stages the path satifies the condition $f(x(t)) = y^{\circ}$ if indeed y° is produced as we assume is the case.

Now it is evident that for each t we can calculate the normal to $f(x)$ at $x = x(t)$ which we write as $w(t)$ and, furthermore, that $x(t) = \nabla_w C(w(t), y^{\circ})$ for all t. Thus there is a path $w(t)$ and such that $x(t)$ is the path of cost minimizing inputs. Therefore the path $x(t)$ derived from a quantity adjustment model is identical to that which occur if the firm faced the price path $w(t)$ and minimized the cost of production given y° and $w(t)$ at each t.

Conversely, for any given expected or planning price path $w(t)$ going from w° to w' there is an input path $x(t)$ going from x° to x' which may be interpreted as a path due to a quantity adjustment model. The upshot of these observations is that in principle the quantity adjustment and expected or planning price models are observationally equivalent and hence are indistinguishable using observed data on input quantities and prices.

6. Estimating the Production Sector of ORANI

a. The Production Sector of ORANI

In what follows I indicate suggestions regarding the estimation of the parameters of the technology within the general context imposed by Dixon (1975). That is, I do not concern myself with model formulations which are alternatives to the basic Dixon model.

Each industry is assumed to produce its (single) output subject to the available technology which is described by a production function. This production function is a Leontief production function in that all effective "inputs" enter in fixed (different) proportions to output. This extremely restrictive assumption, which involves zero substitutability and constant returns to scale, is presumably made to permit use of the Leontief input-output table which provides the only comprehensive source of information on interindustry commodity flows.

The effective "inputs" are all functions of more basic inputs so that the zero substitutability assumption refers only to effective inputs not to the basic inputs. That is the production function is weakly separable in its arguments with the aggregator being Leontief

$$(1) \quad z_0 = F(f_1(z_1),\ldots,f_N(z_N)) \equiv \min\{f_1/a_1,\ldots,f_N/a_N\}$$

where z_0 denotes output, $z_1,\ldots,z_N$ are vectors of basic inputs, $f_1(z_1),\ldots,f_N(z_N)$ are the "effective inputs" and $a_1,\ldots,a_N$ are non-negative constants or "effective input"-output coefficients.

The cost function corresponding to (1) is

$$(2) \quad C(w_1,\ldots,w_N, z_0) \equiv \min_{z_1,\ldots,z_N} \{\sum_{i=1}^{N} w_i z_i \; : \; F(f_1(z_1),\ldots,f_N(z_N)) \geq z_0\}$$

and due to the separability of F this minimization problem may be solved in two stages to get

$$(3)\quad C(w;z_0) = \min_{\lambda}\{\sum_{i=1}^{N} C_i(w_i, \lambda_i) \;:\; F(\lambda_1,\ldots,\lambda_N) \geqq z_0\}$$

where

$$(4)\quad C_i(w_i, \lambda_i) \equiv \min_{z_i} \{w_i z_i \;:\; f_i(z_i) \geqq \lambda_i\}$$

is the minimum cost of producing λ_i units of "effective input" i. Given that $F(\lambda)$ is Leontief as indicated by (1) and given that $\partial C_i(\cdot)/\partial\lambda_i > 0$ it follows that the solution to (3) will be such that $\lambda_i = a_i z_0$ and consequently we have that

$$(5)\quad C(w;z_0) = \sum_{i=1}^{N} C_i(w_i, a_i z_0) \;.$$

If we assume that $f_i(z_i) = \phi_i(g_i(x_i))$ where ϕ is a strictly monotonic function and $g_i(x_i)$ is positive linear homogeneous, that is $f_i(z_i)$ is homothetic, then in general (without assuming F is Leontief) the cost function may be written

$$(6)\quad C(w;\, z_0) = C^*(c_1(w_1),\ldots,c_N(w_N),\, z_0)$$

where

$$(7)\quad c_i(w_i) \equiv \min_{z_i} \{w_i z_i \;:\; g_i(x_i) \geqq 1\}$$

and

$$(8)\quad C_i(w_i,\lambda_i) = c_i(w)\; \phi_i^{-1}(\lambda_i) \;.$$

If, in addition, F is Leontief then we get that

$$(9)\quad C(w,z_0) = \sum_{i=1}^{N} c_i(w_i)\, a_i z_0.$$

These results show that the cost function for the industry as a whole may be written in special ways depending upon the assumptions on the technology. First, weak separability and homotheticity of the micro functions f_i yield a cost function (6) which is weakly separable in the price indices $c_i(w_i)$ so that two stage minimization is valid. Only when such indices exist can we validly talk of the price of an "effective input" such as primary factors. Second, the assumption that F is Leontief in the micro functions yields the cost function (5) which is additive in the micro cost functions defined by (4). Here there are no price indices for effective inputs. Third, if we assume that F is Leontief and that the micro functions are homothetic then the cost function is (9) which is additive <u>and</u> there exist price indices defined by (7).[12]

Therefore, given the Leontief structure for F, we will concern ourselves with the estimation of the micro cost functions $C_i(w_i, a_i z_0)$. Actually, it is interesting to note that, since we are presumably interested in the response of each basic input to changes in input prices and output we may treat a_i as a parameter of the ith micro cost function and estimate $C_i^*(w_i, z_0) \equiv C_i(w_i, a_i z_0)$ which yield the input demand functions $z_i = \partial C_i^*(w_i, z_0)/\partial w_i$. It will turn out that a_i will not be identified but this is not required if we are interested only in the <u>basic</u> inputs not the "effective inputs".

An alternative way of looking at this is to think of estimating the parameters of $f_i(z_i)/a_i$ in which a_i will not be identified unless we can observe the values taken by f_i.

b) Estimation Procedures and Data Requirements

The fundamental difficulty in estimating the micro production functions $f_i(z_i)$ and the input-output coefficients a_i is that the quantity of the effective input f_i is not observable. What are observable are the output level z_0, the basic input quantities $z_1, \ldots, z_N$ and the respective prices p and $w_1, \ldots, w_N$. If observations on these are available over a sufficiently long period, say annual data for 1950-1974, then we can use regression methods to estimate the parameters of the micro cost functions (adjusted for a_i) $C_i^*(w_i, z_0)$ given a functional form and a stochastic specification.

Are these data available? The Dixon (1975) specification shows several different interpretations of the $f_i(z_i)$ functions. First, for each intermediate input there are two basic inputs - domestically supplied and foreign supplied. Here we need time series on the price of each source of the input and a time series on output z_0 to estimate the cost function, and of course if demand functions are to be estimated we also require series on quantities from each source used in the industry. I suspect that the latter are not available at all while the former are unlikely to be available. If this suspicion is correct then it is not possible to estimate the cost functions for these intermediate inputs. If the only sources of information are the input-output tables for domestic inputs and foreign inputs then it would appear that fixed coefficients would have to be assumed (given by the single sample observation) unless some *a priori* information is available.

Second, the primary factors (land, labour and capital) combine to form an effective input which we might term real value added. Again to estimate this function or its dual the micro cost function, it is necessary to have time series on the output of the industry and either the quantities of the basic primary factor inputs or their prices. And if demand functions are to be estimated all these series are required. It is likely that these series are available or that they could be constructed since government agencies usually keep relatively good records on primary factors. It should be noted however that the prices for the factors are the opportunity cost or user cost prices or rentals not the asset prices. The importance of correctly defining the rental for inputs (especially capital and land) within this context has been emphasized by Jorgenson and Griliches (1967), Christensen and Jorgenson (1969, 1970) and others. For examples of research involving the calculation of rental prices and aggregate quantity and price indices of capital and their use in the estimation of the technology see, for example, Berndt and Christensen (1973, 1974) and Woodland (1972, 1975) who estimated value added production functions for 10 major industries in Canada. For recent work on index number formulae see Diewert (1976).

If these data are available then the technology may be estimated under alternative assumptions regarding the functional form, stochastic specification, technical change, behaviour, and the adjustment process. These areas have been discussed in some detail in the other sections of this report. As a practical matter I would suggest that the estimation should begin in a fairly simple way leading up to more sophisticated specifications as circumstances warrant. We now consider these possible specifications.

Regarding technical change it would seem appropriate, particularly if data are poor, to not consider embodied technical change and induced technical change at least until simpler models have been estimated. Embodied technical change models involve nonlinearity in the technical change parameters and their usefulness has not been established. Induced technical change models have not been subject to many empirical tests and the estimating model suggested in section 4 is complex and very nonlinear. Thus we consider only disembodied autonomous technical change. If we assume Hicks neutrality together with homotheticity then factor shares (and ratios) are independent of scale and technical change and so these factors can be ignored and estimates of substitution elasticities obtained by estimating share equations. Such an exercise may be quite relevant for answering some questions, such as what is the effect of an increase in the rental price of capital upon the cost share of labour, but may only be part of the answer to other questions such as what is the effect of an increase in the price of energy upon the share and amount of labour used in manufacturing. The latter question involves scale and technical change factors and presumably is what the ORANI model is all about.

If we assume that the technology is linear homogeneous then our estimates will be consistent with long run competitive equilibrium in which zero profits are earned. If we allow increasing returns to scale we must interpret these scale effects as being external to the firm to maintain the competitive assumptions. If decreasing returns are allowed we imply the existence of fixed factors. These comments have implications for output pricing. For those groups referring to materials from different sources I see no reason for supposing non-constant returns to scale. For primary factors scale effects are not so easily ruled out.

Thus, for those "effective inputs" i which depend upon the source of an intermediate input we could assume linear homogenity in output. For the real value added function (primary inputs) scale effects might be estimated and special cases tested. We are left with the problem of how to estimate the cost function for the industry with price, scale and technical change effects. As a first approach technical change could enter as a time variable, in the absence of suitable indices of technical change. The simplest approach is to let t enter the individual cost functions either in the form of factor augmenting technical change or in a general way.

Let us drop the i subscript and denote the inputs by $x = (x_1, \ldots, x_N)$. Then our micro cost function is $C(w, y, t)$ where $y = az$ is the effective input or the micro output. We can specify a functional form for C and then estimate the input demand functions, or the share equations and/or the cost function itself. For example, if we add terms to the GCD cost function in equation (17) of section 3 to get a second order approximation in t as well as w and y we get

$$(10) \quad \ln C = b_{oo} + \sum_{ij}\sum b_{ij} \ln(w_i + w_j) + \ln y \sum_i b_{yi} \ln w_i + b_{0y} \ln y + b_{yy} (\ln y)^2 + t \sum_i d_{ti} \ln w_i + d_{0t} t + d_{tt} t^2 + d_{yt} \ln y \, t$$

where $\sum_{ij}\sum b_{ij} = 1$, $\sum_i b_{yi} = 0$ and $\sum_i d_{ti} = 0$. Thus there are an extra N+2 parameters due to the inclusion of t as a variable representing technical change. The share equations are

(11) $s_i = 2\sum_j b_{ij} w_i (w_i + w_j)^{-1} + \ln y\, b_{yi} + t\, d_{ti}$ $\quad i=1,\ldots,N$

which are also linear in parameters. If $d_{ti} = 0$, $i=1,\ldots,N$ then shares are independent of technical change which is one type of neutrality that might be assumed. The effects of technical change may be summarized by calculating $\partial C/\partial t/C = \partial \ln C/\partial t$. A similar formulation is possible for the TL form. And, of course, the second order approximation property may be given up by adding restrictions on the d parameters to reduce the number of parameters. For example, "the" rate of technical progress $\partial \ln C/\partial t = d_{0t}$ a constant might be imposed.

If factor augmenting technical change is assumed with constant rates of augmentation then the cost function is $C(w^*, y)$ where $w_i^* = w_i e^{-\lambda_i t}$ and λ_i is the rate of augmentation for input i. Then in the cost functions (16)-(18) of section 3 we replace w by w* which in general makes the resulting forms nonlinear in parameters reducing the attractiveness of this assumption. For the TL form the cost function reduces to (18) of section 3 in terms of w plus the term

(12) $d_{ot}t + d_{tt}t^2 + d_{yt} \ln y\, t + t \sum_i d_{ti} \ln w_i$

where

(13) $d_{ot} \equiv -\sum_i \alpha_i \lambda_i$, $d_{tt} \equiv \frac{-1}{2} \sum_{ij}\sum b_{ij} \lambda_i \lambda_j$, $d_{yt} \equiv -\sum_i b_{yi} \lambda_i$

$d_{ti} \equiv -\sum_j b_{ij} \lambda_j$, $i=1,\ldots,N$.

If we set up a second order approximation to a general cost function $C(w, y, t)$ using TL we get the usual TL given by (18) of section 3 plus (12) where the d parameters are independent of the b parameters and satisfy $\sum d_{ti} = 0$. Thus there are N+2 extra parameters to deal with t just as with the GCD (10). But under the factor augmenting hypothesis (12) and (13) indicate that there are only N extra parameters; the d parameters depend on the b and λ parameters and so are not independent and cannot validly be treated as such in the estimation of the cost function. Thus the appearance of the d parameters in (12) in linear form is deceptive. In other words the factor augmentation assumption for the TL cost function imposes restrictions on the parameters which need to be respected during estimation. Otherwise we are back with the second order approximation specification so there has been no gain.

Thus in the interests of simplicity of estimation the linear form implied by the second order approximation would appear to be the appropriate form. The cost is an extra 2 parameters. For the case of the GCD, equations (10) and (11) with random disturbances $u_c, u_1, \ldots, u_N$ added from the estimating equations. Note that only N-1 of the share equations are independent so any one may be dropped and the remaining N (1 cost, N-1 share) equations may be estimated by maximum likelihood methods using a standard program such as Chow and Fair (1971).

For the GL form we can obtain a second order approximation to C(w, y, t) by taking equation (16) of section 3 (a second order approximation in w and y) and adding the term

$$y(t \sum_i d_{ti} w_i + d_{tt}\, t^2 \sum_i w_i + d_{yt}\, t \sum_i w_i / y)$$

which involves N+2 additional parameters just as in the case of the TL and GCD forms. The input demand functions will be those in equation (19) of section 3 with the additional terms

$$y\, t\, d_{ti} + d_{yt}\, t + d_{tt}\, t^2 , \qquad i=1,\ldots,N .$$

The form used by Parks (1970) may be regarded as a special case where $d_{yt} = d_{tt} = 0$ and hence does not have the second-order-approximation property. As indicated in section 2 the Parks form may be interpreted as a specification of a linear form of factor augmentation. Again, for the general form for GL the input demand functions are linear in parameters facilitating estimation.

For some micro group i there may be insufficient data to carry out the estimation as indicated above. Indeed, there may only exist data from one input-output table in which case any econometric estimation is impossible. If constant returns to scale and fixed coefficients (no substitution) are assumed for group i along with constant rates of input augmentation the cost function for this group is

$$(14) \quad C(w, az_0, t) = \sum_j w_i e^{-\lambda_j t} az_0 = \bar{z}_0 \sum_j w_j (ab_j e^{-\lambda_j t})$$

where the b_j are the basic input-micro output coefficients. An input-output table may be used to identify ab_j, the basic input-real output coefficients, but only if the rates of augmentation λ_j are known or assumed, say $\lambda_j = 0$.

It is clear from (14) and also true for the more general functional forms and estimation procedures that the a parameters (one for each micro group) will not be identified and can be set arbitrarily, say a = 1, a reflection of the fact that the micro production functions are ordinal not cardinal.

Perfect adjustment has been assumed so far. If lagged adjustment is required then it may be incorporated as a direct quantity adjustment using the flexible accelerator form discussed in section 5(b) or as a price expectations model such as those discussed in section 5(c). In either case the share equations do not seem to be suitable forms so the input demand functions generated from the GL should be used. The GL form yields demand functions linear in parameters so they may be written as $x^* = Bv$ where B is a matrix of parameters (not all free) and v is a vector of variables. Using the adjustment mechanism (3) of section 5 we get the estimating equations

$$(15) \quad x_t = ABv_t + (I - A)x_{t-1} + u_t$$

where u_t is a vector of disturbances. This is nonlinear in A and B. It looks as if we could replace AB by matrix C in (15) so it is linear, estimate C and A and then get the estimate for B as $B = A^{-1}C$ if A is nonsingular. However, the elements of B are not independent (e.g. symmetry restrictions for some pairs) and $A^{-1}C$ would not respect these restrictions. Thus we cannot avoid the nonlinearity in (15) in this way.

Following the price expectations or planning price approach we replace each price by a distributed lag function of past prices since this function is nonlinear in parameters and the demand functions are nonlinear in prices, they are very nonlinear in parameters. Thus imperfect adjustment models require nonlinear techniques which are certainly available but require more effort in use.

The above discussion is intended to give the menu of choices with some recommendations for estimating industry cost functions. Until the data base is known in more detail there is little point in delving deeper into the practical aspects of the estimating procedures.

c) An Alternative Approach to Modelling the Production Sector ?

The model outlined by Dixon (1975) and to which attention was directed in this report is aimed at providing a detailed description of each industry's technology. This is obviously the approach which should be taken in setting up a model to determine the effects of tariff and other policy changes upon resource allocation. The purpose of this subsection is to briefly indicate an alternative approach suggested by the possible lack of suitable time series data on industry inputs of intermediate goods distinguished by source (domestic or foreign). The main feature (advantage and disadvantage depending upon the vantage point) of the approach is that it avoids using data on and hence does not yield an explanation of interindustry flows. It does allow us to predict the effects of prices and factor endowments on net outputs and factor rewards.

In brief if we assume the production sector is perfectly competitive with the absence of externalities and faces fixed commodity prices and fixed endowments of primary factors it will behave as if it were a single price taking firm maximizing the value of net outputs subject to the technological and endowment constraints. If p is the price vector, y is the net output vector, x is the factor endowment and $T(y, - x) \geqq 0$ defines the production possibility set the competitive solution is summarized by the function

$$(16) \quad \pi(p, x) \equiv \max_{y}\{py \; : \; T(y, - x) \geqq 0\}$$

where π may be interpreted as a variable or gross profit function or a gross national product function. If π is differentiable then

(17) $y = \partial\pi(p, x)/\partial p$

and

(18) $w = \partial\pi(p, x)/\partial p$

which are the net output supply functions (17) and the shadow price functions for fixed factors (18). In sections 2 and 3 we indicated that there exist second order approximation forms for π which yield supply and price functions linear in parameters thus facilitating estimation. The parameters of π may be estimated by estimating (17) for which we require time series on the price vector p, the net output vector y and the resource vector x. No industry specific data are required!

The commodities would include all sorts of outputs, and all "variable" inputs which may be purchased outside the production sector. Included here would be imported inputs of various kinds. Thus (17) contains the import demand functions as well as the output supply functions. Furthermore the input vector might contain the total amount of available unskilled labour assuming it is perfectly mobile or for immobile factors (in short run) such as fixed capital separate elements might be given to each industry. Thus the approach is fairly general within its competitive assumptions.

Obviously one would like to have commodities and inputs disaggregated as much as possible (e.g. commodities as per input-output table; capital by industry, labour by occupation, etc.) but the dimension of the estimating equations would become prohibitive. Restrictions would have to be imposed. The obvious restriction is that π is separable in groups of commodities and factors. For example one such separability assumption leads to

$$\text{(19)} \quad \pi(p, x) = \pi^*(r_1(p_1), \ldots, r_M(p_M), f_1(x_1), \ldots, f_N(x_N))$$

where $r_i(p_i)$ is a price index for commodity group i (p_i is a vector) and $f_i(x_i)$ is a quantity index for factor group i (x_i is a vector). For example, the commodity groups might be consumer goods, investment goods, energy, primary goods, and materials while the factor groups might be the industry value added functions, i.e. classified on an industry basis, or land, labour, fixed capital, and equipment.

If capital is distinguished by sector then it must be assumed immobile in the short run. If as Dixon (1975) suggests there is a fixed dollar value of investment in new capital which is to be allocated among industries the shadow price of capital in industry i (factor i, say) is obviously relevant and since it is $w_i = \partial\pi/\partial x_i$ it is readily obtained from the estimated $\pi(p, x)$. Specifically, given the rate of interest on bonds, the capital taxation schedules, the depreciation rates and the asset prices of new capital we can calculate rental prices or opportunity cost prices for capital in each industry and allocate the investment budget to maximize the return to capital. That is, we solve, assuming x is all capital for simplicity of notation,

$$\text{(20)} \quad \max_x \{\pi(p, x) - rx \ : \ q(x - \bar{x}) \leq V\}$$

where r is the vector of rentals (opportunity costs), $\bar{x}$ is the end of year capital stock vector, q is the capital asset price vector, and V is the dollar value of net investment. The solution for $x(p, \bar{x}, r, q, V)$ is obtained from

$$\text{(21)} \quad \frac{\partial\pi/\partial x_i - r_i}{\partial\pi/\partial x_j - r_j} = \frac{q_i}{q_j} \quad \text{and} \quad q(x - \bar{x}) = V$$

if all funds are spent. Thus the variable profit function approach appears useful not only in obviating the need for industry data but also in simplifying the analysis of the allocation of investment. Of course, the industry cost functions may be set up with capital fixed yielding a shadow rental in the same way.

For a first attempt at estimating a variable profit function for the production sector see Kohli (1975).

FOOTNOTES

1. The duality theorems derived by Diewert (1973b) are expressed in terms of the asymmetric from $y_{11} = t^*(\tilde{y}_1, y_2)$ $\equiv \max \{y_{11}: t(y_{11}, \tilde{y}_1, y_2) = 0\}$ where y_{11} is the first element of vector y_1, ie $y_1 = (y_{11}, \tilde{y}_1)$. This form is used since its properties are easier to characterize.

2. In terms of the transformation function we have that $t(y, -x) = 0$ from which we derive the production function
$y = f(x) \equiv \max \{y: t(y, -x) = 0\}$.

3. The function $f^*(x) \equiv \max \{y: wx \geqq C(w, y), w > 0\}$ coincides with $f(x)$ and hence indicates how to reconstruct f.

4. Since $1/r(y) \equiv \frac{\partial g(y)}{\partial y} \frac{y}{g(y)}$ we have that
$d \ln g = \frac{\partial g(y)}{\partial y} = (y\, r(y))^{-1}$ which, upon integration, yields the functional form for g: $g(y) = k(ye^{\beta y})^{1/d}$ where k is a constant of integration.

5. For a discussion of this see Diamond and McFadden (1965).

6. A detailed account of separability is provided by Blackorby, Primont and Russell (1977).

7. Since constant returns to scale is assumed.

8. The idea of "epochs" was also used by Brown (1966).

9. For a comprehensive theoretical treatment see Nordhaus (1969b).

10. We use the convention that $\hat{b}_i \equiv \dot{b}_i/b_i$ where $\dot{b}_i \equiv db_i/dt$.

11. Recall that constant returns to scale are assumed.

12. For a survey of the theory of aggregation see Blackorby, Primont and Russell (1975, 1977).

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Abbreviations of Journal Titles

EJ	The Economic Journal
AER	The American Economic Review
RE Stat	The Review of Economics and Statistics
JPE	The Journal of Political Economy
RES	The Review of Economic Studies
J. Econometrics	Journal of Econometrics
JET	The Journal of Economic Theory
JIE	The Journal of International Economics
Em	Econometrica
CJE	The Canadian Journal of Economics
IER	The International Economic Review
JE Literature	The Journal of Economic Literature
JASA	The Journal of the American Statistical Association
SEJ	The Southern Economic Journal
QJE	The Quarterly Journal of Economics