Impact Project

Impact Centre
The University of Melbourne
153 Barry Street, Carlton
Vic. 3053 Australia

Phones: (03) 341 7417/8
Telex: AA 35185 UNIMEL
Telegrams: UNIMELB, Parkville

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A WAGE-RESPONSIVE DISAGGREGATION OF LABOUR SUPPLY

by

Alan A. Powell   Dean J. Parham
Dennis C. Sams   Russell J. Rimmer

Impact Project Staff

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1. INTRODUCTION

Craigie, Parham and Ryland (1979) have proposed a model of labour supply, disaggregated by occupations, which explicitly recognizes two important aspects of the labour market; namely :

(i) the fact that, at any given set of occupational wage relativities, persons with differing educational attainments will have different probabilities of being in each occupation;

(ii) the conjecture that the occupations followed by a set of persons with given educational attainment profiles will exhibit some degree of flexibility in response to changes in occupational wage relativities.

In this paper we present an operational version of Craigie *et al.*'s model and report on its estimation. Our account is organized as follows. Section 2 contains the basic structural form of the model (before the introduction of stochastic terms). The stochastics of the system and the data base are each treated briefly in Sections 3 and 4 respectively. Then follows in Section 5 a discussion of how the various constraints coming out of the theoretical analysis might best be implemented in estimation. Results are presented in Section 6 and a perspective for further work, and concluding remarks, are given in Section 7.

2. THE BASIC MODEL

We commence by demonstrating the origins of our approach within the neo-classical literature on the labour/leisure choice. We will, however, finesse one potentially difficult class of aggregation problems by positing the existence of one representative agent for the whole labour market. Unlike an individual, this fictional representative agent can work part-time as a brain surgeon and part-time as a garbage collector. The skill endowments of the total labour market will determine the relative ease with which the representative agent can reallocate his time budget among occupations as the relative rewards for them change.

2.1 The Labour-Leisure Choice

Our starting point is Williams (forthcoming 1983). Suppose the representative agent's utility function can be written

(1) $$U = V(A,B) \quad ,$$

where A is an index of wage income and B is an index of 'exertion' (the opposite of 'leisure'). We assume that

(2) $$A = \sum_{i=1}^{n} W_i L_i \quad ,$$

where W_i and L_i respectively are the real wage, and the quantity of labour supplied, in occupation i. (Real wages notionally will, in our treatment, already have been adjusted to take account of the wage equivalents of non-wage benefits, and any employment risk differentials among occupations.) Further, we assume that

(3) $$B = -G_\theta(\underline{\underline{L}}) \quad ,$$

where G is an aggregator function whose value is a single scalar index of 'exertion' associated with the production of the occupational supply vector, $\underline{\underline{L}} \equiv (L_1,\ldots,L_n)'$. The parameters of G, denoted $\theta \equiv \{\theta_1,\ldots,\theta_g\}$, are discussed later.

Next, suppose that m levels of skill, or different types of educational/vocational background, are distinguished. Let the m-vector of stocks of persons in the population possessing these skills be denoted $\underline{\underline{N}}$, and define the working-age population P by

$$(4) \qquad P \equiv \underline{\underline{1}}_m'\underline{\underline{N}} = \underline{\underline{1}}_n'\underline{\underline{L}} ,$$

where $\underline{\underline{1}}_h$ is a vector containing h units. We require the representative agent to maximize

$$(5) \qquad U = V(\underline{\underline{W}}'\underline{\underline{L}}, \; - G_\theta(\underline{\underline{L}}))$$

with respect to $\underline{\underline{L}}$, subject to (4). To derive the behavioural supply functions relating $\underline{\underline{L}}$ to $\underline{\underline{W}}$ and θ, several assumptions are required.

<u>Assumption 1</u> The utility function V is homogeneous of degree zero in $\underline{\underline{L}}$ given any fixed parameter set θ. That is, only wage income <u>per head</u> and exertion <u>per head</u> affect the welfare of the representative agent.

<u>Assumption 2</u> The function G is homogeneous of first degree in $\underline{\underline{L}}$ at fixed θ.

Assumption (1) and Euler's Theorem jointly imply

$$(6) \qquad \sum_{i=1}^{n} L_i \, \partial V/\partial L_i = 0 .$$

The Lagrangean for the constrained maximization of (5) is

$$(7) \qquad \mathcal{L} = V(\underline{\underline{W}}'\underline{\underline{L}}, \; - G_\theta(\underline{\underline{L}})) - \lambda(\underline{\underline{1}}_n'\underline{\underline{L}} - P)$$

(in which λ is a Lagrangean multiplier), whose first order conditions include

(8) $$\lambda = V_A W_i - V_B(\partial G_\theta/\partial L_i) = \partial V/\partial L_i, \quad i=1,\dots,n,$$

where V_A and V_B, respectively, are the marginal utilities of labour income and of negative exertion ($V_A \equiv \partial V/\partial A$, $V_B \equiv \partial V/\partial B$). Multiplying (8) by L_i, and summing over occupations, we obtain

(9) $$\lambda P = V_A \sum_{i=1}^{n} W_i L_i - V_B \sum_{i=1}^{n} L_i(\partial G/\partial L_i) = \sum_{i=1}^{n} L_i \, \partial V/\partial L_i .$$

However, the final term on the right of (9) vanishes because of Assumption 1. Since $P \neq 0$, it follows that $\lambda = 0$, and the constraint $\underline{\underline{1}}'\underline{\underline{L}} = P$ on the maximization of (5) is non-binding.

2.2 The Transformational Approach

With the help of the above insights from Williams (forthcoming 1983), we are now able to reinterpret the above utility maximizing problem as an equivalent transformation problem, which is the approach adopted directly in Craigie et al. (1979). We can see this by noting that the first order conditions (8), after eliminating the redundant constraint by setting $\lambda = 0$, are consistent with the first order conditions for the following transformation problem,

(10) $$\max_{\underline{\underline{L}}} \; \underline{\underline{W}}'\underline{\underline{L}} \quad \text{subject to} \quad G_\theta(\underline{\underline{L}}) = \hat{G} ,$$

provided that $\hat{G}$ (which is exogenous from the viewpoint of (10)) is chosen to equal its optimal value from the problem {Max (5) subject to (4)}. This can be seen as follows. The first order conditions for (10), are

(11) $$W_i = \mu \, \partial G/\partial L_i$$

and

$$(12) \qquad G_\theta(\underline{\underline{L}}) = \hat{G} \ ,$$

where μ is a Lagrange multiplier on the constraint (12). Multiplying (11) by L_i and summing over occupations, we have

$$(13) \qquad \sum_{i=1}^{n} L_i W_i = \mu \sum_{i=1}^{n} L_i \, \partial G/\partial L_i$$

$$= \mu G$$

(from Assumption 2).

Thus

$$(14) \qquad \mu = \sum_{i=1}^{n} L_i W_i/G$$

$$= -A/B \ ,$$

where A and B are the arguments of V. Substituting from (14) into (11), we see that (11) is consistent with (8) (in which λ, keep in mind, is zero) provided that

$$(15) \qquad \frac{V_B}{V_A} = -\frac{A}{B} \ ,$$

where the partial derivatives V_A and V_B are evaluated at the solution of (5), and A and B are evaluated at the solution of (10). A sufficient condition to ensure (15) is that

$$(16) \qquad \frac{\partial V}{\partial A}\frac{A}{V} = -\frac{\partial V}{\partial B}\frac{B}{V}$$

globally. In (16) $A, (\partial V/\partial A)$ and $(\partial V/\partial B)$ are all positive numbers, while B is negative. Taking this complication of sign into account, (16) says that the partial elasticity of the utility function V with respect to wage income is equal to its partial elasticity with respect to exertion. This is a direct consequence of Assumptions 1 and 2. At fixed $\underline{\underline{W}}$ a one per cent increase in all elements of $\underline{\underline{L}}$ must leave V unaffected; since $\underline{\underline{W}}'\underline{\underline{L}}$ and $G_\theta(\underline{\underline{L}})$ are both homogeneous of first degree in $\underline{\underline{L}}$, the one per cent increase in $\underline{\underline{L}}$ leads to changes of plus and minus one per cent respectively in A and B. The only way these changes can leave V unaffected is if (16) holds.

What the above has demonstrated is that the transformational approach (10) is equivalent to the labour-leisure choice approach (5) except that in the former the quantity of exertion is treated exogenously. Although not ideal, the data and estimation problems encountered in the implementation of the transformational approach are sufficiently severe and numerous to make this simplification acceptable (albeit it at the risk of the introduction of some simultaneity bias).

2.3 The CRETH Transformation Frontier

We now introduce a further assumption on the transformation function $G_\theta(\underline{\underline{L}})$.

Assumption 3 The parameters θ are completely determined by $\underline{\underline{N}}$. That is, the difficulty associated with producing any particular vector of occupational supplies, and of reconfiguring that bundle, is wholly determined by the stocks $\underline{\underline{N}}$ of educational attainments embodied in the population.

Craigie et al. (1979) propose the following (implicit) functional form for G :

$$(17) \qquad \sum_{i=1}^{n} \frac{Q_i(\tilde{\underline{\underline{\theta}}})}{r_i} \left[\frac{L_i}{G_\theta(\underline{\underline{L}})}\right]^{r_i} = 1 \qquad (\text{all } r_i > 1),$$

where the sets Θ and $\{r_1,\ldots,r_n;\tilde{\underline{\underline{\theta}}}\}$ are the same. Thus the parameters, $r_1,\ldots,r_n$, are assumed not to depend on stocks of educational attainments, $\underline{\underline{N}}$, while the contrary assumption is made about the parameters contained in the vector $\tilde{\underline{\underline{\theta}}}$.

The functional form (17) is CRETH (constant ratio of elasticities of transformation, homothetic) (Dixon (1976)) which is the analogue in the product-product space of Hanoch's (1971) CRESH production function. The Allen-Uzawa partial transformation elasticities between occupations i and j in (17) are

$$(18) \qquad \tau_{ij} = - \frac{\left[\sum_{\ell=1}^{n} S_\ell/(r_\ell-1)\right]^{-1}}{(r_i-1)(r_j-1)} ,$$

which depends only on $\underline{\underline{r}} = (r_1,..,r_n)'$. The parameters r_i of (17) are thus termed the transformation parameters. In our analysis below we make the following assumptions :

Assumption 4 The occupational transformation frontier is CRETH (i.e., of form (17)); and as well :

Assumption 5 The transformational parameters $\underline{\underline{r}}$ are independent of educational attainments $\underline{\underline{N}}$.

The remaining parameters of G are the Q_i in (17). These in turn are functions of $\tilde{\underline{\underline{\theta}}}$ and (via Assumption 3) of $\underline{\underline{N}}$.

The parameters $\underset{=}{Q} = (Q_1,\ldots,Q_n)'$ determine the general shape, and location, of the occupational transformation schedule; they do not, however, affect the 'ease' of transformation as encapsulated in the transformation elasticities.

2.4 The First Order Conditions

We now consider the problem (10) of maximizing the wage bill subject to supplying an exogenously given amount $\hat{G}$ of generalized labour services in the special case in which the transformation function $G_\theta(\underset{=}{L})$ is as defined by (17). Our treatment follows Dixon, Vincent and Powell (1976). The Lagrangean apposite to our problem may be written

$$(19) \qquad L' = \underset{=}{W}'\underset{=}{L} + \Gamma_1\left(1 - \sum_{j=1}^{n} \frac{Q_j}{r_j}\left[\frac{L_j}{G_\theta(\underset{=}{L})}\right]^{r_j}\right) + \Gamma_2\left(\hat{G} - G_\theta(\underset{=}{L})\right) ,$$

in which Γ_1 and Γ_2 are Lagrangean multipliers on the constraints which respectively constrain the technology to be CRETH and set the supply of generalized labour services exogenously. Unnecessary complications can be avoided by substituting out the second of these constraints directly. This allows us to replace (19) by

$$(20) \qquad L = \underset{=}{W}'\underset{=}{L} + \Gamma\left(1 - \sum_{j=1}^{n} \frac{Q_j}{r_j}\left[\frac{L_j}{\hat{G}}\right]^{r_j}\right) .$$

The first order conditions hence are

$$(21) \qquad 0 = \frac{\partial L}{\partial L_i} = W_i - \Gamma\left[\frac{L_i}{\hat{G}}\right]^{r_i} \frac{Q_i}{L_i} \qquad (i=1,\ldots,n) ,$$

plus equation (17).

2.5 Log Linearization of the First Order Conditions

Equations (20) and (17) may be linearized in logarithmic differentials, yielding

$$(22) \qquad w_i = \gamma + r_i(\ell_i - \hat{g}) - \ell_i + q_i = 0 \qquad (i=1,\ldots,n),$$

and

$$(23) \qquad \sum_{j=1}^{n} \left[\frac{L_j}{\hat{G}}\right]^{r_j} Q_j \left(\ell_i - \hat{g} + \frac{q_j}{r_j}\right) = 0 ,$$

where lower case letters denote logarithmic differentials of the corresponding upper case variables. Notice that here we have treated the Qs as variables so as to be able at a later point to allow them to respond to exogenously changing stocks of educational attainments ($\underline{\underline{N}}$). Next we eliminate the unobservable multiplier, γ. The procedure closely follows Dixon, Vincent and Powell (1976, pp. 17-18); here we simply state the result that

$$(24) \qquad \gamma = \sum_{i=1}^{n} S_i^*(w_i - q_i) + \hat{g} ,$$

where the $\{S_i^*\}$ are modified wage bill shares :

$$(25) \qquad S_i^* = \left[S_i/(1 - r_i)\right] \Big/ \left[\sum_{j=1}^{n} S_j/(1 - r_j)\right] ,$$

in which

$$(26) \qquad S_i = W_i L_i \Big/ \left(\sum_{j=1}^{n} W_j L_j\right)$$

is the ordinary wage-bill share of occupation j.

2.6 The CRETH Supply System

Back substitution of (24) into (22) yields

$$\ell_i = \hat{g} + \frac{1}{r_i - 1}\left[w_i - \sum_{j=1}^{n} S_j^* w_j\right] - \frac{1}{r_i - 1}\left[q_i - \sum_{j=1}^{n} S_j^* q_j\right] \qquad (i=1,\ldots,n). \tag{27}$$

This is the structural form of the CRETH supply system in logarithmic differentials -- notational differences aside, it is the system proposed by Craigie, Parham and Ryland (1979, equation (5)).

We next must consider the mapping from educational stocks to the time-varying parameters Q_j . Before doing so, however, it is convenient to consider two exceptional labour categories.

2.7 Teachers and Those not in the Workforce

There are two "occupations" to which we would not expect the above theory to apply. Teaching is a highly administered profession lying almost entirely outside the market economy. The authorities typically have set both the number of teachers employed and the wage rate relative to other occupations. The market then has been made to clear by endogenous movements in the qualifications acceptable for employment as a teacher.

The case of those opting out of the labour market is one which might, if the data were better, be handled in an extension of the current framework. Explicit recognition would be needed of non-wage rationing and discouraged workers' withdrawal from the labour market. As well, a host of variables affecting the shadow price of broadly defined "leisure" would need explicit inclusion. At IMPACT we have chosen to treat these topics separately (see, e.g., Brooks, Sams and Williams (1982)). This explains, in part, why we have exogenized $\hat{G}$ above. Our data on stocks of attainments, however, refer to the entire working age population, not just to those in the workforce. The accounting of our system must recognize this.

In order to be able to deal with these complications in the development below, we introduce two background supply functions for "teachers" (i=9) and for those "not in the workforce" (i=10) :

$$\ell_9 = \sum_{k=1}^{m} \eta_{9k}\, n_k + \underline{\underline{\beta}}_9'\, \underline{\underline{x}}_9 \quad , \tag{28}$$

and

$$\ell_{10} = \sum_{k=1}^{m} \eta_{10k}\, n_k + \underline{\underline{\beta}}_{10}'\, \underline{\underline{x}}_{10} \quad , \tag{29}$$

where ηs and $\underline{\underline{\beta}}$s are parameters, $\underline{\underline{x}}_9$ is a set of exogenous variables related to demography and to educational policy, and $\underline{\underline{x}}_{10}$ is a set of exogenous variables related to demography, the state of the macro-economy, the level (and style of administration) of unemployment benefits and the level and the nature of training allowances for full-time students.

As noted above in equation (4), the data base satisfies the following accounting identity

$$\sum_{j=1}^{n} L_j \equiv \sum_{k=1}^{m} N_k \equiv P \quad , \tag{30}$$

where P is the working age population (15 years or over). It follows that

$$p = \sum_{j=1}^{n} \frac{L_j}{\left(\sum_{i=1}^{m} L_i\right)}\, \ell_j = \sum_{k=1}^{m} \frac{N_k}{\left(\sum_{i=1}^{m} N_i\right)}\, n_k \quad , \tag{31}$$

i.e.,

$$p = \sum_{j=1}^{n} F_j \ell_j = \sum_{k=1}^{m} H_k n_k \quad , \tag{32}$$

where the Fs and Hs respectively are the shares (on a number of persons basis) of the occupations, and of the levels of educational attainment, in the population.

In the application below n, the number of occupations distinguished, is 10, and m, the number of skill types, is 5 (although ideally, disaggregation by sex would double the latter number). Exogenizing the supplies of teachers and of those not in the workforce reduces the number of occupations to be modelled to 8.

The total labour supply system (in the levels) may now be envisaged as

$$\underset{10\times 1}{\underline{\underline{L}}} = \underset{10\times 1}{\underline{\underline{\chi}}} \, (\underset{5\times 1}{\underline{\underline{N}}} , \underset{8\times 1}{\underline{\underline{W}}} , \underline{\underline{x}}_9 , \underline{\underline{x}}_{10}) \quad . \tag{33}$$

$\hat{G}$ does not appear among the arguments of χ since G itself will be determined by the variables $\underline{\underline{N}}$, $\underline{\underline{x}}_9$ and $\underline{\underline{x}}_{10}$. (Note that $\underline{\underline{x}}_{10}$ is assumed to contain the <u>absolute</u> wage level; W, on the other hand is only required to capture <u>relative</u> wages.) We assume (<u>Assumption 6</u>) that each χ_i is homogeneous of first degree in $\underline{\underline{N}}$ at fixed values of $\underline{\underline{W}}$, $\underline{\underline{x}}_9$ and $\underline{\underline{x}}_{10}$. Thus a uniform increase in the endowments of all stocks of attainments is assumed, <u>ceteris paribus</u>, to lead to an equal uniform percentage increase in all occupations (i=1,...,10).

Equation (33) is the fully reduced form of the system. As already explained, in the application reported below we treat L_9 and L_{10} as exogenous. Thus we work with 8 partially reduced-form equations of the type

(34) $\quad L_i = \Xi_i(\underline{\underline{N}}, \underline{\underline{W}}, L_9, L_{10}) \qquad (i=1,\ldots,8).$

How does the assumed first-degree homogeneity of χ affect the functions Ξ_i? By η_{ik} denote the partial derivative of $(\ell n\ \chi_i)$ with respect to $(\ell n\ N_k)$.

Then Assumption 6 implies

(35) $$\sum_{k=1}^{5} \eta_{ik} = 1 \qquad (i=1,\ldots,10).$$

Let $\varepsilon_{ik}(k=1,\ldots,5)$, ψ_i and ϕ_i stand respectively for $\partial \ell n\ \Xi_i/\partial \ell n\ N_k$ $(k=1,\ldots,5)$, $\partial \ell n\ \Xi_i/\partial \ell n\ L_9$ and $\partial \ell n\ \Xi_i/\partial \ell n\ L_{10}$. The partials η and ε are related to each other by

(36) $$\eta_{ik} = \varepsilon_{ik} + \psi_i\ \eta_{9k} + \phi_i\ \eta_{10k} \qquad (i=1,\ldots,8).$$

Summing (36) over k, and using (35), we obtain

(37) $$\sum_{k=1}^{5} \varepsilon_{ik} + \psi_i + \phi_i = 1 \qquad (i=1,\ldots,8).$$

The interpretation of (37) is as follows. In equation (34), changes in $\underline{\underline{N}}$ have a direct and an indirect influence on occupational supplies $L_1,\ldots,L_8$. The direct effect comes via the elasticities $\{\varepsilon_{ik}\}$, the indirect effect via the elasticities ψ_i and ϕ_i of L_9 and L_{10}. If the differentials induced in L_9 and L_{10} by a uniform differential in $\underline{\underline{N}}$ were sterilized by offsetting differentials in $\underline{\underline{x}}_9$ and $\underline{\underline{x}}_{10}$, then a one per cent increase in all educational stocks would, at fixed wage relativities, lead to an increase in the 8 occupations $(i=1,\ldots,8)$ exceeding one per cent. This is because these 8 occupations constitute only $(1 - F_9 - F_{10}) \times 100$ per cent of the working age population. To proceed further, we introduce :

<u>Assumption 7</u> At fixed $\underline{\underline{W}}$, L_9 and L_{10}, each Ξ_i (i=1,...,8) is homogeneous in $\underline{\underline{N}}$ of degree ν. Thus we have

$$(38)\qquad \sum_{k=1}^{5} \varepsilon_{ik} = 1/(1 - F_9 - F_{10}) > 1$$
$$= \nu \text{ (say)}.$$

When we do allow increases in L_9 and L_{10}, these are at the expense of occupations $L_1,\ldots,L_8$. Thus ψ_i and ϕ_i (i=1,...,8) are unambiguously negative. Indeed, from equations (37) and (38) we see that

$$(39)\qquad \psi_i + \phi_i = 1 - \nu$$
$$= -(F_9 + F_{10})/(1 - F_9 - F_{10}) < 0 \qquad (i=1,\ldots,8).$$

Further relationships exist among the partials of the functions. (30) can be rewritten

$$(40)\qquad \sum_{j=1}^{8} L_j + L_9 + L_{10} = P \quad ;$$

that is,

$$(41)\qquad \sum_{j=1}^{8} \Xi_i = P - L_9 - L_{10} \quad .$$

With $\underline{\underline{N}}$ held constant, so is $P = \underline{\underline{1}}'\underline{\underline{N}}$. Hence, if we partially differentiate (41) successively with respect to L_9 and L_{10}, we obtain, respectively :

$$(42)\qquad \sum_{j=1}^{8} \frac{\partial \Xi_j}{\partial L_9} = -1 \quad .$$

and

$$(43)\qquad \sum_{j=1}^{8} \frac{\partial \Xi_j}{\partial L_{10}} = -1 \quad .$$

But

$$(44) \qquad \psi_i \overset{\text{defined}}{=} \partial \ell n\, \Xi_i / \partial \ell n\, L_9$$

and $\qquad (i=1,\ldots,8)$

$$(45) \qquad \phi_i \overset{\text{defined}}{=} \partial \ell n\, \Xi_i / \partial \ell n\, L_{10} \quad .$$

Therefore (42) and (43) can be written

$$(46) \qquad \sum_{j=1}^{8} \psi_j \frac{L_j}{L_9} = -1$$

and

$$(47) \qquad \sum_{j=1}^{8} \phi_j \frac{L_j}{L_{10}} = -1 \quad .$$

If in these last two equations we divide L_j, L_9 and L_{10} by P, we obtain

$$(48) \qquad \sum_{j=1}^{8} \psi_j F_j = -F_9$$

and

$$(49) \qquad \sum_{j=1}^{8} \phi_i F_j = -F_{10} \quad .$$

Equation (30) also has implications for the partials of the χ functions. Differentiating with respect to N_k at fixed $\underline{\underline{x}}_9$, $\underline{\underline{x}}_{10}$ and $\underline{\underline{W}}$, we obtain

$$(50) \qquad \sum_{j=1}^{10} \frac{\partial \chi_j}{\partial N_k} = \sum_{\ell=1}^{m} \frac{\partial N_\ell}{\partial N_k} = 1 \qquad (k=1,\ldots,5).$$

Keeping in mind that

$$(51) \qquad \eta_{jk} \overset{\text{defined}}{=} \frac{\partial \chi_j}{\partial N_k} \cdot \frac{N_k}{\chi_j} \quad ,$$

and that $F_j = L_j/P$ while $H_k = N_k/P$, we see that (50) implies

$$(52) \qquad \sum_{j=1}^{10} F_j \eta_{jk} = H_k \qquad (k=1,\ldots,5).$$

Next we return to (36), multiply each side by F_i, and sum over $i=1,\ldots,8$, obtaining :

$$(53) \qquad \sum_{i=1}^{8} F_i \, \eta_{ik} = \sum_{i=1}^{8} F_i \, \varepsilon_{ik} + \eta_{9k} \sum_{i=1}^{8} F_i \, \psi_i + \eta_{10k} \sum_{i=1}^{8} F_i \, \phi_i \, .$$

Now we substitute (48), (49) and (52) into (53), obtaining

$$(54) \qquad \sum_{i=1}^{8} F_i \, \varepsilon_{ik} = H_k \qquad (k=1,\ldots,5).$$

2.8 Final Form of the Supply System

Equation (27) says that the CRETH supply system in logarithmic differentials is linear. The exogenous variables are the proportional changes in :

(i) overall supply of labour services $(\hat{g})$;

(ii) wage rates for the eight relevant occupations $(w_1,\ldots,w_8)$;

and

(iii) the parameters which locate, and affect the shape of, the transformation frontier $(q_1,\ldots q_8)$.

Because in the wider framework $\hat{g}$ is endogenized by $\underline{\underline{N}}$, $\underline{\underline{x}}_9$ and $\underline{\underline{x}}_{10}$, equation (27) is a partially reduced form. What we require as our final set of equations for estimation purposes is a more fully reduced form of the type (34) which captures the essential elements in the transformation story told by (27). Let the log-differential linearization be

$$(55) \qquad \ell_i = (\text{linear function})_i \text{ of } (\underline{\underline{w}}) + \sum_{k=1}^{5} \varepsilon_{ik} \, n_k + \psi_i \, \ell_9 + \phi_i \, \ell_{10} \qquad (i=1,\ldots,8).$$

This notation is consistent with our use of the symbols ε, ψ and ϕ above, except that here we are imposing the additional assumption of constancy on these elasticities. Thus in addition to requiring that the N_ks determine the Qs, we are imposing (as an approximation) log linearity on this relationship. Adopting the CRETH specification for the linear functions of $\underline{\underline{W}}$ in (55), we obtain :

$$(56) \qquad \ell_i = \frac{1}{r_i - 1}\left(w_i - \sum_{j=1}^{8} S_j^* w_j\right) + \sum_{k=1}^{5} \varepsilon_{ik} n_k + \psi_i \ell_9 + \phi_i \ell_{10} \qquad (i=1,\ldots,8).$$

The overall population constraint (30) implies that only 7 of these equations can be linearly independent (a point which must be recognized when estimating (56)). A convenient set of 7 linearly independent equations equivalent to (56) is

$$(57) \qquad (R_j \ell_i - R_i \ell_j) = R_i R_j (w_i - w_j) + (R_j \psi_i - R_i \psi_j)\ell_9 + (R_j \phi_i - R_i \phi_j)\ell_{10} + \sum_{\ell=1}^{5} (R_j \varepsilon_{i\ell} - R_i \varepsilon_{j\ell}) n_\ell \qquad (i \neq j),$$

where

$$(58) \qquad R_i \overset{\text{def}}{=} 1/(r_i - 1) \quad ,$$

and where we choose a fixed i (say $i=8$) and let j run over the other seven permissible values ($j=1,\ldots,7$, if i is chosen to $= 8$). Equation (57) is the systematic part of the final form of the CRETH system estimated below. The nominal parameter count is eight Rs, forty ($=8\times5$) εs, eight ψs and eight ϕs, a total of 64 coefficients.

Offsetting this total are twelve linearly independent constraints among the εs -- i.e., equations (38) and (54) -- plus nine linearly independent constraints among the ψs and ϕs -- i.e., equation (39) and one of (48) or (49). These are referred to below in Section 5 as the equality constraints. Additionally there are constraints on the signs of the elasticities ε, ψ and ϕ. We have already seen in the discussion following equation (38) that

$$(59) \qquad -\psi_i,\ -\phi_i \geq 0 \qquad (i=1,\ldots,8).$$

The ε_{ik}, recall, are the elasticities of the supplies of occupations i (i=1,...,8) with respect to stocks k (k=1,...,5) of educational attainments. An example of such an ε would be ε_{42}, the elasticity of the supply of skilled blue collar metal and electrical workers (i=4) with respect to the number of people in the working age population possessing as their highest educational attainment a diploma (k=2) from a CAE or TAFE institution. Keeping in mind that such elasticities are evaluated at fixed wage relativities and with fixed numbers of persons in the teaching and not-in-the-workforce groups, it is clear that we should require

$$(60) \qquad \varepsilon_{ik} \geq 0 \qquad (i=1,\ldots,8;\ k=1,\ldots,5).$$

Below, constraints like (59) and (60) are referred to as the non-negativity or half space constraints.

3. THE STOCHASTICS OF THE SYSTEM

The systematic component of the final form (57) contains as right-hand variables $\underline{\underline{w}}$, ℓ_9 and ℓ_{10}. All three could be regarded as stochastic. In the story as we have told it above, both ℓ_9 and ℓ_{10} would be endogenized in a wider framework (and therefore should be regarded as stochastic), while of course an even more ambitious reference frame would also provide an explanation of $\underline{\underline{w}}$. Given the institutional aspects of the labour market, and the rigidities characterizing relative wages, we feel on relatively safe ground in the case of $\underline{\underline{w}}$. The variables ℓ_9 and ℓ_{10} are more problematical.

Suppose we now explicitly recognize zero-mean additive random disturbances v_9, v_{10} and u_i $(i=1,\ldots,8)$ respectively in (28), (29) and (56). Our treatment of ℓ_9 and ℓ_{10} as exogenous requires (<u>Assumption 8</u>) that

$$(61) \qquad \underset{T\to\infty}{\mathrm{P}\ell im}\ \frac{1}{T}\sum_{t=1}^{T} v_{jt}\,u_{it} = 0 \qquad \text{for} \quad (j=9,10 \text{ and } i=1,\ldots,8),$$

where T is the sample size and the second subscript t denotes the t^{th} realization. Assumption 8 is strong. It implies zero contemporaneous correlation in the probability limit between the disturbances in each of ℓ_9 and ℓ_{10} on the one hand, and the disturbances in $\ell_1,\ldots,\ell_8$ on the other. Nevertheless, the structure of the system is sufficiently recursive to allow of (61) as a logical possibility.

In order to apply maximum likelihood techniques below we require (<u>Assumption 9</u>) that the zero mean additive disturbances u_i

(i=1,...,8) which we have appended to (56) be joint normally distributed, free from serial correlation, homoscedastic, with contemporaneous variance-covariance matrix of rank seven. These properties are preserved under the full rank linear transformation mapping (56) on to (57).

The errors in (57) are

$$\tilde{u}_{ij} \overset{\text{defined}}{=} R_j u_i - R_i u_j \qquad (i \neq j). \tag{62}$$

Let $\underset{=}{\Sigma}_8$ be the variance-covariance matrix of the vector $(\tilde{u}_{81},\ldots,\tilde{u}_{87})$. The reason that $\underset{=}{\Sigma}_8$ (or any other choice $\underset{=}{\Sigma}_j$ (j=1,...,7)) has rank 7 rather than 8 is the linear dependence among the non-stochastic parts of the supply system. This is easier to see in (56) than in its 1:1 transform (57). Writing (56) with the errors u_i explicit, multiplying each side of the resultant equation by F_i and adding over i=1,...,8, we obtain :

$$\begin{aligned}\sum_{i=1}^{8} F_i \ell_i = {} & \sum_{i=1}^{8} F_i R_i (w_i - (\underset{=}{S}^*)'\underset{=}{w}) \\ & + \ell_9 \sum_{i=1}^{8} F_i \psi_i + \ell_{10} \sum_{i=1}^{8} F_i \phi_i \\ & + \sum_{i=1}^{8} F_i \sum_{k=1}^{5} \varepsilon_{ik} n_k + \sum_{i=1}^{8} F_i u_i .\end{aligned} \tag{63}$$

Using (48), (49) and (32), we obtain

$$\begin{aligned}\sum_{i=1}^{8} F_i u_i = {} & \sum_{k=1}^{5} H_k n_k - \sum_{i=1}^{8} F_i R_i (w_i - (\underset{=}{S}^*)'\underset{=}{w}) \\ & - \sum_{i=1}^{8} F_i \sum_{k=1}^{5} \varepsilon_{ik} n_k .\end{aligned} \tag{64}$$

On the left of (64) we have a non-stochastic linear combination of the stochastic elements $(u_1,\ldots,u_8)$; given our treatment of $\underline{\underline{n}}$ and $\underline{\underline{w}}$, the right of (64) is non-stochastic. Hence the domain of stochastic variation among $u_1,\ldots,u_8$ is restricted. Any seven u_i may be assigned arbitrary realizations; the remaining u_i is then fully and non-stochastically determined by (64). The rank of the variance-covariance matrix of $\underline{\underline{u}} = (u_1,\ldots,u_8)'$ is thus 7; this property is shared by $\underline{\underline{\Sigma}}_8$ (and by other $\underline{\underline{\Sigma}}_j$ corresponding to a full-rank subset of (57)).

4. A BRIEF DESCRIPTION OF THE DATA

In common with many other labour market studies, the estimation phase for this project was hampered by lack of suitable data. Considerable effort therefore was expended to produce an information series on the required variables that was internally consistent and which made the best of the various sources available.

Our prime interest is in structural information; that is, in the proportional distribution of the workforce across occupations, in movements in relative wages, and in the proportional distribution of the population across educational attainments. However all variables have been appropriately scaled to ensure that the series recognize underlying identities, such as the equality of the total population by attainments with the total population by occupations (including those not in the Labour Force). The working age population was taken to consist of those of age 15 or more. Separate information on males and females was collected. Wherever possible, data were selected to conform as closely as possible to the 30th June in each year. The longest series that we could hope to build spanned 1966 to 1976.

4.1 Occupational Supplies of Labour

The supply of labour in each occupation consists of persons employed and persons unemployed. Series for these two components were constructed separately, since (as will be seen below) the unemployment rate is used in the calculation of certainty equivalent wage rates.

Data for 1966 to 1976 were derived from special tabulations of the ABS Labour Force Survey (see Australian Bureau of Statistics (a)) in which survey data were grouped inter alia into occupational categories according to IMPACT specification. These tabulations were based on the May survey of each year, except for 1968 (where the August survey was used) and 1969 (where no survey data were available).

These data, in some cases, suffer from large errors (for further discussion see Parham and Ryland (1978)). First, it should be stated that the survey, based on less than one per cent of the population, was not designed primarily to reveal detailed information on the occupational structure of the workforce. Second, the special tabulations are based on a smaller fraction of survey returns, and population estimates were generated with a single expansion factor, rather than a set of expansion factors geared to demographic and regional benchmarks. Nevertheless, these data represent the best available series on labour supplies grouped according to a meaningful occupational classification.

Several glaring weaknesses in the data were adjusted by Williams (1980). These included:

(i) the miscoding of Meat Cutters, Canners and Preservers (employed males, 1967 to 1971);

(ii) the allocation by sex of Lecturers and Teachers in 1971;

(iii) the allocation between rural and non-rural occupations in 1972 (corresponding to survey design change); and

(iv) over-estimates of the numbers employed in rural occupations.

Two further adjustments were made to the data. First, "unemployed-not previously employed" and "unemployed-armed services" were allocated to the Semi- and Un-skilled White Collar and Semi- and Un-skilled Blue Collar occupations on the basis of the relative size of unemployment in these latter two occupations. Finally, the proportion of persons employed and unemployed in each occupation for 1969 was estimated as a weighted average of proportions in neighbouring years. For example, the proportion of employed persons in occupation i in 1969 was estimated by

$$PE_i^{69} = 0.15\ PE_i^{67} + 0.40\ PE_i^{68} + 0.30\ PE_i^{70} + 0.15\ PE_i^{71} .$$

The calculated proportions employed and unemployed could then be combined to provide estimated proportions of labour supply by each IMPACT major occupational group. These are shown, for various years, in Table 1. Scaling according to published totals for employed and unemployed males and females provided the data series used in model estimation.

4.2 Certainty Equivalent Wage Rates

The state of the available Australian labour market data is such that a satisfactory continuous time series of occupational earnings does not exist. The only option for the required time period is to piece together some (hopefully sensible) composite of :

(i) average weekly earnings of all male employees (ABS (d)) -- not disaggregated by occupation;

TABLE 1 : OCCUPATIONAL DISTRIBUTION OF TOTAL LABOUR SUPPLY (PERSONS) BY SEX FOR VARIOUS YEARS

SEX	MALES				FEMALES				PERSONS			
OCCUPATION[2]	1966	1971	1976	Growth Rate % p.a.[1]	1966	1971	1976	Growth Rate % p.a.[1]	1966	1971	1976	Growth Rate % p.a.[1]
PROF	.041	.045	.049	3.4	.009	.011	.016	11.0	.031	.034	.037	4.3
SWC	.121	.123	.120	1.6	.097	.093	.096	4.0	.114	.113	.111	2.3
USWC	.153	.159	.159	2.1	.481	.490	.506	4.8	.250	.267	.281	3.7
SBC-ME	.151	.160	.151	1.7	.008	.007	.005	0.7	.108	.110	.100	1.7
SBC-B	.071	.075	.072	1.9	.001	.002	.001	6.3	.050	.051	.047	1.9
SBC-O	.032	.029	.028	0.6	.035	.029	.023	-0.0	.033	.029	.027	0.4
USBC	.322	.315	.331	2.0	.289	.289	.269	3.5	.312	.307	.309	2.4
RUR	.092	.073	.065	-1.8	.027	.022	.023	2.9	.073	.057	.050	-1.2
TEACH	.018	.020	.026	5.6	.053	.057	.060	5.5	.028	.032	.038	5.6
TOTAL	1.000	1.000	1.000	1.7	1.000	1.000	1.000	4.2	1.000	1.000	1.000	2.5

1. Compound growth rate between 1966 and 1976.
2. Occupational Key :
   - PROF = Professional White Collar
   - SWC = Skilled White Collar
   - USWC = Semi- and Un-skilled White Collar
   - SBC-ME = Skilled Blue Collar - Metal & Electrical
   - SBC-B = Skilled Blue Collar - Building
   - SBC-O = Skilled Blue Collar - Other
   - USBC = Semi- and Un-skilled Blue Collar
   - RUR = Rural Workers
   - TEACH = Lecturers and Teachers.

(ii) Income Distribution Surveys which provide annual earned income by occupation for full year, full time workers in financial years 1968/9 and 1973/4 (ABS (b) and (c));

(iii) annual May surveys of Earnings and Hours of Employees (ABS (e)) which provide, from 1974 on, estimates of occupational weekly earnings of full time non-managerial employees.

Craigie (1980a) constructed a series, based on the first two sources. Annual earned income in 1968/69 and 1973/74 was converted to weekly income by simple division by 52. The ABS has provided special tabulations of the Income Distribution Surveys (IDS), categorized according to IMPACT occupational classifications (see Parham and Ryland 1978). The rate of growth in earnings in each occupation over the period was then calculated and used to extrapolate and interpolate for the 1966 to 1976 period. These occupation-specific wage rates were then adjusted to reflect the all-occupations estimates published in Average Weekly Earnings (ABS (d)).

Additional information was incorporated from the Earnings and Hours Surveys (EHS). Although there are differences in coverage between the IDS and the EHS, the most important feature of the wage data for the purposes of estimating our model is not the level of the wages, nor even the wage relativities, but <u>changes</u> in wage <u>relativities</u>.

Special tabulations of the EHS provided by the ABS have been analysed by Tulpulé (1981). The changes over 1974 to 1976 in occupational wages relative to the all-occupations average were calculated from the EHS data as tabulated by Tulpulé. The Craigie series was then modified in years 1975 and 1976 to reflect this additional information on changes in wage relativities.

Finally, as foreshadowed in Craigie, Parham and Ryland (1979), the estimated wage rates were adjusted for risk of unemployment as follows:

$$W_i^* = W_i(1 - u_i) \quad , \tag{65}$$

where

W_i^* = certainty equivalent wage rate in occupation i,

W_i = average wage rate for employed persons in occupation i,

u_i = rate of unemployment in occupation i.

4.3 Educational Attainments

The Population Census is the only source of information on the educational attainments of the population. Formerly infrequent (now annual) surveys provide some data on attainment amongst employed persons (ABS (f)). It was decided therefore to estimate annual flows to/from educational attainments and to apply these to the intercensal years to provide a series on the population by sex and by educational attainment from 1966 to 1976. This took advantage of all available detailed information.

The categories Degree, Diploma, Technician, Trade and Other were selected for the attainment series. The methodology for preparing the data is detailed in Parham (1982). In broad outline it involved the following steps. First, the available census data were adjusted to be as comparable with each other as possible. Chiefly, this involved distribution to other attainment categories of those in the "Not Stated" category in the 1976 census. Second, the educational attainment distributions of the component annual flows were estimated as below.

(a) Graduations, i.e., net movements between categories were taken from the work on Student Statistics by Craigie (1980 (b)).

(b) New entrants, i.e., people who merely by turning 15 years of age entered the population of working age, were all assumed to belong to the "Other" attainment category.

(c) Numbers of immigrants, defined to be total permanent plus long-term arrivals, were obtained from IMPACT tabulations of ABS migration statistics. Rough estimates of the attainment distribution of these immigrant flows were made. Prior to June 1971, the source used for this distribution was the 1971 Census data on attainments amongst migrants by years of residence in Australia. For the post-1971 period, heavy reliance was put on ABS (g), Migrants in the Workforce 1972 to 1976.

(d) Numbers of deaths and emigrants were obtained from ABS sources. Given the complete lack of data on the attainment distribution of these two flows one possible approach would have been to assume that the attainment distributions of death and emigration were in the same proportion as existed in the total population. However, this would have produced rather biased estimates since death, emigration and attainment differ greatly in their incidence or distribution across ages. Thus an age dimension was included in the construction of the data series; for example in year t

(66) $$d^t = A^t D^t ,$$

where

d^t = a 5×1 vector of deaths by educational attainment in year t,

A^t = a $5 \times n$ matrix of the distribution of the population by attainment and by 5-year age groups in year t,

D^t = a $n \times 1$ vector of deaths by age groups in year t.

The A^t matrices were estimated on the basis of trends between the census years (for which such detailed information was available).

Working forwards and backwards from 1971, the flows of attainments were used to estimate stocks of attainments for the intercensal years. Everywhere, the latest available, consistent population estimates were employed. Some summary information is presented in Table 2.

TABLE 2 : EDUCATIONAL ATTAINMENT DISTRIBUTION OF THE POPULATION AND ANNUAL AVERAGE RATE OF GROWTH IN ATTAINMENTS BY SEX

SEX / EDUC. ATTAIN-MENT	MALES				FEMALES				PERSONS			
	1966	1971	1976	Growth Rate % p.a.	1966	1971	1976	Growth Rate % p.a.	1966	1971	1976	Growth Rate % p.a.
DEGREE	.019	.029	.043	10.7	.005	.010	.020	16.5	.012	.020	.031	12.2
DIPLOMA	.032	.033	.036	3.5	.027	.030	.037	5.5	.029	.032	.037	4.5
TECHNICIAN	.029	.034	.040	5.3	.035	.034	.036	2.5	.032	.034	.038	3.8
TRADE	.174	.179	.179	2.3	.014	.015	.016	3.6	.094	.097	.097	2.4
OTHER	.746	.725	.702	1.5	.918	.910	.891	1.8	.832	.818	.797	1.7
TOTAL	1.000	1.000	1.000	2.1	1.000	1.000	1.000	2.1	1.000	1.000	1.000	2.1

5. THE SCOPE FOR CONSISTENT SHRINKAGE OF THE PARAMETER SPACE

5.1 Three Types of Parameter Restrictions

At the conclusion of sub-section 2.8 we saw that the final form (57) of the model involves 43 "free" parameters : eight transformation parameters R_i; 28 attainment-to-occupation elasticities ε_{ik}, and seven others ψ_i and ϕ_i. The data available, on the other hand, are limited to ten realizations of each of the eight occupational supply equations. Without further prior restrictions it is therefore highly unlikely that the system could be estimated successfully on the basis of available data.

Prior considerations suggest that some of the ε_{ik} values are zero. We would not, for instance, expect more than a negligible number of degree holders to form part of the supply of the occupation "Unskilled blue collar workers". Whilst we lack data on the flows of persons from different educational attainment categories to different occupations, we do have data on the stocks of persons cross-classified by attainment and occupation at the 1971 Census. Table 3, from Craigie (1979), reproduces these data. The figures suggest putting ε_{41}, ε_{51}, ε_{61}, ε_{71}, ε_{81}, ε_{52}, ε_{62} and ε_{72} to zero. Constraints of this type will be termed exclusion restrictions. The implementation of these restrictions is not so straightforward as it might at first seem. Such restrictions must be imposed simultaneously with the equality restrictions (38) and (54) and with the non-negativity restrictions (59) and (60). An initial set of exclusion restrictions

TABLE 3 : AUSTRALIAN LABOUR FORCE 1971 BY OCCUPATION AND HIGHEST QUALIFICATION OBTAINED

IMPACT Major Occupation Group	Post-secondary Qualifications Held, k =							Nos. Employed in Each Occupation	Proportion of Employed Workforce
	1 Degree	2 Diploma	3 Technician	4 Trade	5 None	Not Classified*	Total		
1. Professional White Collar	.54	.32	.04	.01	.08	.01	1.00	193294	.04
2. Skilled White Collar	.03	.18	.16	.12	.48	.03	1.00	665824	.13
3. Semi and Unskilled White Collar	.01	.02	.03	.03	.85	.06	1.00	1352324	.25
4. Skilled Blue Collar - Metal & Electrical	-	.01	.06	.54	.39	-	1.00	505563	.09
5. Skilled Blue Collar - Building	-	-	.02	.57	.41	-	1.00	211884	.04
6. Skilled Blue Collar - Other	-	-	.01	.34	.64	.01	1.00	133327	.03
7. Semi and Unskilled Blue Collar	-	-	.01	.11	.87	.01	1.00	1468387	.28
8. Rural Workers	-	.01	.03	.04	.91	.01	1.00	403906	.08
* Armed Services	.02	.03	.06	.21	.66	.02	1.00	65196	.01
* Other (n.e.c.)	.01	.01	.02	.07	.88	.01	1.00	240570	.05
Total Employed Workforce		.04	.04	.14	.72	.03	1.00	5240275	1.00
Total Population 15 Years and Over		.03	.03	.10	.80	.02	1.00	9085582	not available

* Not part of the classification scheme adopted in this paper.
Source : Craigie (1980b), from Census data.

formulated by us turned out to lie completely outside the overlap of the equality and the non-negativity constraints.

The equality and non-negativity restrictions fall naturally into two sets, those relating to ε and those relating to ψ and ϕ. We consider the former set first.

Equations (38) and (54) constitute 13 equations relating the εs to each other. Twelve of these equations are linearly independent. It follows that at most 28 εs can be estimated independently. Let $\underline{\underline{\varepsilon}}$ be the 40×1 vector containing the $\{\varepsilon_{ik}\}$ as elements. The existence of 12 linearly independent equation constraints means that it is possible to order the elements of $\underline{\underline{\varepsilon}}$ in such a way that the first twelve elements (collectively, $\underline{\underline{\varepsilon}}_1$) and the remaining 28 elements (collectively, $\underline{\underline{\varepsilon}}_2$) satisfy

$$(67) \qquad \underset{12 \times 12}{\left[\underline{\underline{A}}_1 \right.} \;\Big|\; \underset{12 \times 28}{\left. \underline{\underline{A}}_2 \right]} \underset{40 \times 1}{\begin{pmatrix} \underline{\underline{\varepsilon}}_1 \\ \underline{\underline{\varepsilon}}_2 \end{pmatrix}} = \underset{12 \times 1}{\underline{\underline{k}}} ,$$

in which $\underline{\underline{A}}_1$ is a non-singlular matrix. In (67) the elements of $\underline{\underline{A}}_1$, $\underline{\underline{A}}_2$ and $\underline{\underline{k}}$ come from equations (38) and (54). $\underline{\underline{\varepsilon}}_1$ is thus related to $\underline{\underline{\varepsilon}}_2$ by

$$(68) \qquad \underline{\underline{\varepsilon}}_1 = \underline{\underline{A}}_1^{-1} \underline{\underline{k}} - \underline{\underline{A}}_1^{-1} \underline{\underline{A}}_2 \underline{\underline{\varepsilon}}_2$$

$$= \underline{\underline{a}} + \underline{\underline{B}} \, \underline{\underline{\varepsilon}}_2 \qquad \text{(say)}.$$

5.2 Linear Programming Approach

Keeping in mind that each element of $\underline{\underline{\varepsilon}}_1$ is non-negative, (68) may be rewritten

$$(69) \qquad -\underline{\underline{B}}\ \underline{\underline{\varepsilon}}_2 \leq \underline{\underline{a}} \ ,$$

which, together with the requirement

$$(70) \qquad \underline{\underline{\varepsilon}}_2 \geq 0 \ ,$$

i.e., that every element of $\underline{\underline{\varepsilon}}_2$ be non-negative, forms a set of feasibility requirements for a linear program (LP) in $\underline{\underline{\varepsilon}}_2$. Indeed, to have a standard linear programming problem, all that need be added to (69) and (70) is a linear maximand

$$(71) \qquad \mu = \underline{\underline{\beta}}'\ \underline{\underline{\varepsilon}}_2 = \sum_{j=1}^{28} \beta_j\ \varepsilon_j^{(2)} \ ,$$

where $\varepsilon_j^{(2)}$ is the j^{th} element of $\underline{\underline{\varepsilon}}_2$.

What does the LP approach have to offer in the estimation of $\underline{\underline{\varepsilon}}$? Firstly, it provides an algorithm to check that the equality and non-negativity constraints can both be satisfied : any feasible solution to the LP : (69), (70), (71) definitionally satisfies these constraints. Secondly, it provides a way of introducing desired exclusion restrictions when these do not violate feasibility.

Some basic results from the theory of LP are necessary at this stage (see, e.g., Dantzig (1963) or Hadley (1969)). The first is that a solution to a problem of 12 linear constraints (such as (69)) in 28 variables (such as the elements of $\underline{\underline{\varepsilon}}_2$) can have at most 12 non-zero elements. The second is that a scalar constraint within a set of inequality restrictions like (69) does not bind at a solution point if and only if the corresponding 'slack' variable is non-zero. In our application these slack variables, from (68), are just the elements of $\underline{\underline{\varepsilon}}_1$. It follows then that any solution of (69) - (71) contains exactly 12 non-zero elements. Finally, any convex combination of any two feasible (not necessarily optimal) solutions of the LP is also a feasible solution.

Suppose that we use the LP approach to generate p feasible solutions (p < 28) lying in the subspace of the $\underline{\underline{\varepsilon}}$- space believed on a priori grounds to be the most relevant part of this space. If these solutions are the 40-element vectors $\underline{\underline{\varepsilon}}^1, \underline{\underline{\varepsilon}}^2, \ldots, \underline{\underline{\varepsilon}}^p$, then the estimation of $\underline{\underline{\varepsilon}}$ reduces to finding (p - 1) convex coefficients $\hat{\theta}_1, \ldots, \hat{\theta}_{p-1}$ such that the finally estimated $\underline{\underline{\varepsilon}}$ is

$$(72) \qquad \hat{\underline{\underline{\varepsilon}}} = \sum_{i=1}^{p} \hat{\theta}_i \underline{\underline{\varepsilon}}^i ,$$

where

$$(73) \qquad \hat{\theta}_i \geq 0 \qquad (i=1, \ldots, p)$$

and

$$(74) \qquad \hat{\theta}_p = 1 - \sum_{i=1}^{p-1} \hat{\theta}_i .$$

There are some practical and conceptual problems with this approach to estimation. The most obvious conceptual problem is that for the approach to be feasible with our meagre supply of data, p must be restrained to an integer much less than 28 (p ≤ 5, say). This means that a sizeable proportion of the $\underline{\underline{\varepsilon}}$ space is ruled out of bounds and consequently much is staked on the reliability of the exclusion restrictions.

The enforcement of the exclusion restrictions itself presents some practical difficulties. A number of routes might be taken, but the one which seemed most practical to us involved selecting valid partitions $\{\underline{\underline{\varepsilon}}_1, \underline{\underline{\varepsilon}}_2\}$ which assigned to $\underline{\underline{\varepsilon}}_1$ twelve ε_{ik} elements on which we did not hold strong priors. (By 'valid' here we mean that the partition must produce a non-singular $\underline{\underline{A}}_1$ in (67).) Elements which we held strongly to be either zero or non-zero were assigned to $\underline{\underline{\varepsilon}}_2$ (subject to the validity criterion just mentioned). The vehicle for our influencing which particular set of ε_{ik}s would appear in $\underline{\underline{\varepsilon}}_2$ as non-zero values was the set of weights $(\beta_1, \ldots, \beta_{28})$ in the objective function (71). In any one solution $\underline{\underline{\varepsilon}}^j$ $(j=1,\ldots,p)$, at most twelve non-zero values could occur in total, and therefore at most twelve in the $\underline{\underline{\varepsilon}}_2$ part of the solution. Our informal algorithm for generating the $\underline{\underline{\varepsilon}}^j$ therefore assigned non-zero weights β_i to exactly twelve ε_{ik}s at a time; namely, those particular epsilons which it was desired to bring into the solution. Zero weights were assigned to the remaining 16 (=28 - 12) elements of $\underline{\underline{\varepsilon}}_2$. The non-zero weights β_i in each case were arbitrary, but their ranking was chosen to reflect the subjective importance of particular ε_{ik}s ending up with estimated values differing from zero. More details of this selection procedure are offered below in the Results section. What we should stress here is that the procedure provides a method of obtaining plausible estimates of the ε_{ik}s which are consistent with all three classes of restriction, and which concord with prior conjecture. We cannot, however, claim that these estimates are unique. Even for investigators holding identical priors, enough arbitrary elements remain in the procedure described above to result in at least minor differences in the results.

One final technical point concerns constraining the coefficients θ in (72) - (74) to convexity. This is conveniently handled by repeated applications of the identity $\sin^2\alpha + \cos^2\alpha = 1$. If, for example, $p = 3$, we would reparameterize $\theta_1,\ldots,\theta_3$ on α_1 and α_2, where

$$(75)\quad \begin{cases} \theta_1 = \sin^2\alpha_1 \sin^2\alpha_2 , \\ \theta_2 = \sin^2\alpha_1 \cos^2\alpha_2 , \\ \theta_3 = \cos^2\alpha_1 . \end{cases}$$

These coefficients were generated by the identities

$$(76)\quad \begin{cases} \sin^2\alpha_1 + \cos^2\alpha_1 = 1 \\ (\sin^2\alpha_2 + \cos^2\alpha_2)\sin^2\alpha_1 + \cos^2\alpha_1 = 1 . \end{cases}$$

We now turn to the elasticities ψ_i and ϕ_i. The equality constraints (39) and (48), (49), and the non-negativity constraints (59) also fall naturally into the linear programming framework. As will be seen in the Results section, prior information suggests that $\psi_7 = \psi_8 = 0$. Using (39), ϕ_7 and ϕ_8 are immediately determined as $(1 - \nu)$ in each case. This leaves six ψs and six ϕs to be estimated. Equations (39), (48) and (49) provide seven linearly independent equality constraints for this purpose. Thus we have a linear program of seven linearly independent equations in twelve variables subject to half-space constraints. Using arbitrary objective functions which assign successively one to the weight in the objective function of one ψ or ϕ, and zeros everywhere else, it is possible by the solution of successive linear programs to build up a basis set of 12-vectors $(\psi_1,\ldots,\psi_6; \phi_1,\ldots,\phi_6)$, each containing exactly seven non-zero elements. Convex combinations of these lie within the part of the ψ, ϕ space consistent with the equality and the half-space constraints. As above, the estimation problem becomes one of finding appropriate convex coefficients.

As in the case of the application of the LP approach to the $\underline{\underline{\varepsilon}}$s, the above procedure fails to explore some regions of the parameter space which are consistent with the equality and half space constraints. All feasible points in the ψ, ϕ space can be generated as convex combinations of the vectors locating the vertices of the convex polyhedral cone defined by the constraints. Unfortunately, these vertices are too numerous to allow their use in practice.

6. THE RESULTS

The description of our results occurs in the following order : the generation of feasible $\underline{\underline{\varepsilon}}$ solutions, the generation of feasible $(\underline{\underline{\phi}},\underline{\underline{\psi}})$ solutions, identification of the supply system, and estimation.

6.1 Feasible $\underline{\underline{\varepsilon}}$ Sets

The list of occupations and educational attainments used in this study is as given in Table 3, except that the 'occupations' are extended to include $i = 9$ (teachers) and $i = 10$ (not in the workforce). Our priors on the elasticities of supply of occupations i $(i=1,\ldots,8)$ with respect to stocks of persons holding highest attainment k $(k=1,\ldots,5)$ at fixed wage relativities and at fixed supplies L_9 of teachers and fixed stocks L_{10} of people not in the workforce, can be classified into four groups :

(i) those ε_{ik}s whose values are strongly believed to have positive values;

(ii) those ε_{ik}s whose values are strongly believed to be zero, or so close to zero that they may be so taken without introducing appreciable errors;

(iii) those ε_{ik}s (not included in (i)) where a positive value is preferred to a zero value;

and

(iv) those ε_{ik}s where no prior beliefs are held.

The maintained set of priors is given in Table 4.

TABLE 4 : MAINTAINED PRIOR ASSUMPTIONS ABOUT ATTAINMENT-TO-OCCUPATION MAPPING*

IMPACT Major Occupation Group j =	Post-secondary Qualifications Held, k =				
	1 Degree	2 Diploma	3 Technician	4 Trade	5 None
1 Professional White Collar	+ 5	+11		Z10	Z9
2 Skilled White Collar	+10	+ 6	+ 7	p1	p2
3 Semi and Unskilled White Collar		p4	p3		+1
4 Skilled Blue Collar - Metal & Electrical	Z1		+12	+3	
5 Skilled Blue Collar - Building	Z2	Z6	Z11	+4	
6 Skilled Blue Collar - Other	Z3	Z7	Z12	+8	
7 Semi and Unskilled Blue Collar	Z4	Z8			+2
8 Rural Workers	Z5				+9

* The contents of the Table is read as follows. A Z followed by an integer indicates that a particular ε_{ik} is believed to be zero, or very close to zero (where the ranking of the strengths of this belief among such ε_{ik}s is indicated by the integer, 1 indicating strongest); + followed by an integer indicates an ε_{ik} strongly believed to be positive (with the integers again indicating the ranks); p followed by an integer indicates an ε_{ik} which, while not in the + set, would be preferred to have a positive value (where the integer following the p indicates the ranking of such preferences); a blank space indicates an ε_{ik} on which there is no maintained prior. The + , "blank" and p sets have been chosen to ensure that the former two have exactly 12 elements each.

In what follows, the equality constraints developed above are always applied at sample mean values of the occupational and attainments share vectors ($\underline{\underline{F}}$ and $\underline{\underline{H}}$). The first step in applying the linear programming approach to the generation of feasible $\underline{\underline{\varepsilon}}$s is to select a valid partition ($\underline{\underline{\varepsilon}}_1$, $\underline{\underline{\varepsilon}}_2$), such that $\underline{\underline{A}}_1$ in (67) is of full rank. In the implementation of a linear program in $\underline{\underline{\varepsilon}}_2$, it is possible to exercise some more-or-less direct control over which elements are non-zero by an appropriate choice of weights $\underline{\underline{\beta}}$ in the objective function (71). On the other hand, there is very little control over which scalar constraints in (69) will be satisfied as inequalities (and therefore lead to a positive value of a corresponding element of $\underline{\underline{\varepsilon}}_1$). It therefore seemed preferable to us to allocate those elements about which we had the vaguest priors (i.e., set (iv), the 'blank' set of Table 4) to $\underline{\underline{\varepsilon}}_1$, and those about which we had the strongest priors (sets (i) and (ii), the + and Z sets of Table 4) to $\underline{\underline{\varepsilon}}_2$. The structure of the LP problem in turn required us to allocate exactly 12 elements to the set (iv). This limitation is imposed by the dimension of $\underline{\underline{\varepsilon}}_1$ (i.e., the number of linearly independent equalities developed in Section 2). Twelve is a suitable dimension for set (i), on the other hand, because it is the maximum mumber of non-zero elements of $\underline{\underline{\varepsilon}}_2$ which can be found in any single LP solution. Some scope for reallocation among the sets of course exists and indeed becomes necessary for reasons which we now discuss.

It turned out that the prior information contained in Table 4 is inconsistent with the equality and half space constraints in the following sense : if all the ε_{ik}s in class (iv) are allocated to the $\underline{\underline{\varepsilon}}_1$ vector, the corresponding $\underline{\underline{A}}_1$ is singular. General principles for detecting such inconsistencies prior to computation are not easily formulated. In fact, in a process of trial and error it became clear that we would have to contrive partitions and objective function weighting schemes in order to obtain a small set of feasible solutions which jointly contain at least one non-zero value for each ε_{ik} assigned to class (i).

These LP problems were solved using the subroutine H01AEF of the Numerical Algorithms (NAGS) library available on the CSIRONET system. In order to keep down the dimensionality of the $\underline{\underline{\theta}}$ vector in the estimation to follow, we restricted ourselves to five feasible solutions for the ε matrix. These solutions are shown in Table 5. The sub-space to be searched in the estimation phase, therefore, was restricted to convex combinations of these solutions. It will be noted that for given i, ε_{ik}s sum across k to 1.7195, which is the value of ν in the data base (see equation (38)).

The adoption of Table 5 as the basis for parameterization immediately restricts 18 ε_{ik}s to zero. These contain 9 of the 12 assigned to class (ii) in Table 4, the exceptions having ranks 4, 9 and 10 within this class.

6.2 Feasible ($\underline{\underline{\phi}}$, $\underline{\underline{\psi}}$) Sets

The parameters $\phi_1,\ldots,\phi_8$ and $\psi_1,\ldots,\psi_8$ are also subject to half-space constraints (59) and equality constraints (48) and (49). Using techniques similar to those described above, it was possible to generate several feasible joint ($\underline{\underline{\phi}}$, $\underline{\underline{\psi}}$) solutions. As foreshadowed above, before computing these solutions an additional element of prior

TABLE 5 : FIVE SETS OF ATTAINMENT-TO-OCCUPATION ELASTICITIES SATISFYING EQUALITY AND HALF-SPACE RESTRICTIONS*,†

Solution identifier	ε_{11}	ε_{12}	ε_{14}	ε_{15}	ε_{21}	ε_{22}	ε_{23}	ε_{24}	ε_{25}
G	.9942	.7253	0	0	0	.2581	.5168	0	.9446
H	.9942	0	.7253	0	0	.4825	.5168	.7203	0
I	.1599	1.5596	0	0	.2581	0	.5168	0	.9446
J	0	1.5596	0	.1599	.3076	0	0	0	1.4120
O	0	1.5596	.1599	0	0	0	0	0	1.7195

	ε_{35} (a)	ε_{43}	ε_{44}	ε_{45}	ε_{54}	ε_{55}	ε_{64}	ε_{65}
G	1.7195	0	.2179	1.5016	1.7195	0	1.7195	0
H	1.7195	0	0	1.7195	0	1.7195	0	1.7195
I	1.7195	0	.2179	1.5016	1.7195	0	1.7195	0
J	1.7195	.5362	.2179	.9654	1.7195	0	1.7195	0
O	1.7195	0	.1666	1.5529	1.7195	0	1.7195	0

	ε_{71}	ε_{73}	ε_{75}	ε_{84}	ε_{85}
G	0	0	1.7195	0	1.7195
H	0	0	1.7195	.9201	.7994
I	0	0	1.7195	0	1.7195
J	0	0	1.7195	0	1.7195
O	.1099	.1847	1.4249	0	1.7195

* ε_{ik} is the elasticity of supply of occupation i with respect to the stock of persons whose highest attainment is k at fixed occupational wage relativities and at fixed levels of the supply of Teachers ($i = 9$) and of the number of people of working age not in the workforce ($i = 10$). Elasticities ε_{ik} not appearing explicitly above have value zero in all solutions. For key to occupation index (i) and attainment index (k) see Table 4.

† Note that $\sum_{k=1}^{5} \varepsilon_{ik} = \nu = 1.7195$ for each $i=1,\ldots,8$ and that $\sum_{j=1}^{8} F_j\ \varepsilon_{jk} = H_k$ for each $k=1,\ldots,5$ where $\underline{\underline{F}}' = (.0206\ \ .0667\ \ .1593\ \ .0643\ \ .0301\ \ .0177\ \ .1867\ \ .0361\ \ .0198\ \ .3987)$ is the mean vector of occupational shares (persons basis) and $\underline{\underline{H}}' = (.0205\ \ .0322\ \ .0345\ \ .0963\ \ .8166)$ is the mean vector of attainments stocks (persons basis).

(a) The value of this elasticity is determined without further estimation.

information was introduced; namely,

(77) $\psi_7 = \psi_8 = 0$.

This implies that a change in the supply of lecturers and teachers at fixed relative wages neither adds to, nor subtracts from, the supplies of Unskilled Blue Collar workers $(i = 7)$ or of Rural workers $(i = 8)$. Using (38), and keeping in mind that ν is known from the data base, we see that ψ_7 and ψ_8 are also known.

We had little difficulty in generating distinct, feasible LP solutions. The shortage of degrees of freedom again meant that we had to restrict the number of $(\underline{\phi}, \underline{\psi})$ solutions to be used as the basis of our final parameterization; in fact we chose five solutions for which a convex combination would exist that would best satisfy our priors. The solutions chosen are shown in Table 6.

6.3 Identification of the Supply System

The identifiability of (57) as a set of supply equations has not so far been discussed. Without writing out in full a specification for the demand side of the system, it is not possible to be sure whether the estimation of (57) from a set of contemporaneous annual data on all the variables is feasible. It is likely that the demand side would be driven by specific activity variables (e.g., outputs of sectors) which do not appear in the supply equations. This would be a factor favouring the identification of the supply system as written.

TABLE 6 : FIVE FEASIBLE* SETS OF ELASTICITIES OF OCCUPATIONAL LABOUR SUPPLIES WITH RESPECT TO THE SIZES OF THE 'LECTURERS AND TEACHERS' AND THE 'NOT IN THE WORKFORCE' GROUPS†

Solution Identifier	Variable on Left: Number of Persons not in the Workforce (i=10) — Variable on Right : Supply of Persons in Occupation i, i =						Number of Lecturers and Teachers (i=9) — Variable on Right : Supply of Persons in Occupation i, i =					
	1	2	3	4	5	6	1	2	3	4	5	6
1	0	-.7195	-.7195	-.7195	-.7195	-.4432	-.7195	0	0	0	0	-.2763
2	-.7195	-.7195	-.7195	-.7195	-.4872	0	0	0	0	0	-.2323	-.7195
3	-.7195	-.4235	-.7195	-.7195	-.7195	-.7195	0	-.2960	0	0	0	0
5	-.7195	-.7195	-.5955	-.7195	-.7195	-.7195	0	0	-.1240	0	0	0
7	-.7195	-.7195	-.7195	-.4124	-.7195	-.7195	0	0	0	-.3071	0	0

* 'Feasible' means satisfying constraints (48), (49) and (59) at sample mean values of the share vectors $\underline{\underline{F}}$ and $\underline{\underline{H}}$ (detailed in the footnote to Table 5).

† For key to occupations i, see Table 3. The numbers in the left half of the table are values of ϕ_i, while those on the right are values of ψ_i. Note that $\phi_i + \psi_i = 1 - \nu = -.7195$ for each $i=1,\ldots,6$. For occupations 7 and 8, $\psi_7 = \psi_8 = 0$ and $\phi_7 = \phi_8 = -.7195$.

If the identification of the supply system failed when current relative wages were used in (57), then in principle it might well be achieved with the use of lags. The use of lagged wage data would be equivalent to the assumption that labour supply responds to last year's relative wages (whereas labour demand may respond to contemporaneous wage relativities). Operationally this does not, in the case of our data base, hold much promise, for two reasons. First, our series is pitifully short (10 years), so that lagging beyond a single year could not be contemplated. Second, the relative price signals in our data are not independent annual information, but are smoothed, and reflect at most five separate annual inputs from survey information. Lagging the wage data consequently does not markedly change their correlation structure vis à vis the endogenous variables. In the event we worked with contemporaneous data, although a single year's lag on the wage data was also tried.

6.4 Estimation

The structural form (57) was coded for FIML estimation using Wymer's (1977) RESIMUL software. The parameters to be estimated were :

(a) eight transformation parameters $R_1,\ldots,R_8$;

(b) four convex coefficients $\theta_1,\ldots,\theta_4$ (with $\theta_5 \equiv 1 - \sum_{j=1}^{4} \theta_j$) which were used to estimate the ε_{ik}s on the basis of Table 5;

(c) four convex coefficients $\mu_1,\ldots,\mu_4$ (with $\mu_5 \equiv 1 - \sum_{j=1}^{4} \mu_j$) which were used to estimate $\underline{\phi}$ and $\underline{\psi}$ on the basis of Table 6.

The final parameterization in the computer code was chosen to ensure that the r_i $(i=1,\ldots,8)$ stayed in the interpretable range (see equation (17)).

Unfortunately, this model could not be estimated because convergence (after 56 iterations) of the FIML search could not be obtained. At exit, one value of R (R_6) was tending towards zero, which possibly was causing ill-conditioning in the computations where singularity problems were being encountered ; these in turn made further iteration impossible. The model was recoded with R_6 treated as a small (but finite) constant; namely, .0001. Again convergence could not be obtained (after a further 20 iterations); moreover one of the convex coefficients at exit had gone to a corner solution. Further attempts to obtain satisfactory estimates involved one or more of the following variations :

(a) changes in starting values for parameters;

(b) the generation and use of an alternative to Table 5 as a consistent ε-generator;

(c) substitution of the one-transformation-parameter CET (constant elasticity of transformation) functional form for the 8 parameter CRETH formulation (achieved by putting each R_i in (57) to a single parameter R);

(d) the use of wage data lagged by one year;

(e) the use of arbitrary (rather than estimated) convex coefficients $\underline{\underline{\theta}}$;

(f) the addition of intercept terms to the structural form (57);

(g) changing the method of gradient search within the FIML algorithm;

and

(h) the use of an alternative to Table 6 which encompassed only 2 (rather than 5) $(\underline{\phi}, \underline{\psi})$ solutions.

Except in variations which drastically reduced the number of parameters to be estimated, it was not possible to obtain convergence in these experiments. In the case of the CET specification, convergence was obtained when the convex coefficients $\theta_1,\ldots,\theta_5$ were all set arbitrarily (leaving a total of 5 free parameters). The results of estimations using contemporaneous wages data are shown in Tables 7 and 8. There the single transformation parameter shown is τ, the transformation elasticity (which in the CET case is the same, irrespective of which pair of occupations is chosen). (In terms of equation (56), τ is $1/(1 - r)$, where the i subscript on r and the asterisk on S are no longer needed.)

Table 7 sets the ε_{ik}s equal to the simple means of the five solution values given in Table 5; Table 8 takes the means of only solutions G and H in that table. Under the tight sets of restrictions imposed, the $\underline{\phi}$ and $\underline{\psi}$ estimates are moderately stable under this sensitivity test. The same is not true, however, of the transformation elasticity. Although retaining the correct sign, the estimated value varies from .03 to .10 (with respective asymptotic $|t|$ ratios of 2.0 and 3.4). Neither value is at all close to Williams' (forthcoming, 1983) estimate of 1.3 for the components of supply flexibility due to interoccupational transfers. Taken literally $\hat{\tau} = 0.1$ implies that a ten per cent increase in the certainty equivalent

TABLE 7 : FIML ESTIMATES OF CET OCCUPATIONAL SUPPLY SYSTEM
WITH EPSILONS SET ARBITRARILY TO AVERAGE VALUES OF TABLE 5

I. Parameters Set to Arbitrary Values[a]

ε_{11}	ε_{12}	ε_{14}	ε_{15}	ε_{21}	ε_{22}	ε_{23}	ε_{24}	ε_{25}	ε_{35}	ε_{43}
.430	1.081	.177	.032	.113	.148	.310	.144	1.004	1.720	.107
ε_{44}	ε_{45}	ε_{54}	ε_{55}	ε_{64}	ε_{65}	ε_{71}	ε_{73}	ε_{75}	ε_{84}	ε_{85}
.164	1.448	1.376	.344	1.376	.344	.022	.037	1.661	.184	1.535

II. Parameters Estimates by FIML[b]

ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	
-.7105 (4.40)	-.5692 (5.85)	-.7195 (12.42)	-.7195 (6.05)	-.6052 (9.09)	-.3654 (1.37)	
ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	τ
0 (0)	-.1503 (1.55)	0 (0)	0 (0)	-.1143 (1.72)	-.3541 (1.72)	.0329 (2.03)

(a) ε_{ik}s not appearing explicitly are set to zero 0 .

(b) Asymptotic $|t|$ ratios shown in parentheses.

TABLE 8 : FIML ESTIMATES OF CET OCCUPATIONAL SUPPLY SYSTEM WITH EPSILONS SET ARBITRARILY TO MEANS OF SOLUTIONS G AND H IN TABLE 5

I. Parameters Set to Arbitrary Values[a]

ε_{11}	ε_{12}	ε_{14}	ε_{15}	ε_{21}	ε_{22}	ε_{23}	ε_{24}	ε_{25}	ε_{35}	ε_{43}
.994	.363	.363	0	0	.370	.517	.360	472	1.720	0

ε_{44}	ε_{45}	ε_{54}	ε_{55}	ε_{64}	ε_{65}	ε_{71}	ε_{73}	ε_{75}	ε_{84}	ε_{85}
.109	1.611	.860	.860	.860	.860	0	0	1.720	.460	1.260

II. Parameters Estimated by FIML[b]

ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	
-.7195	-.4235 (5.91)	-.7195 (13.87)	-.7195 (6.04)	-.7195 (13.92)	-.7195 (3.46)	

ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	τ
0 (0)	-.2960 (4.13)	0 (0)	0 (0)	0 (0)	0 (0)	0.1059 (3.43)

(a) ε_{ik}s not appearing explicitly are set to zero.

(b) Asymptotic $|t|$ ratios shown in parentheses.

wage rate of a given occupation leads to a one per cent increase in the supply of that occupation within the same year. Allowing for the treatment of expectations in Williams' (but not in our) work, we would perhaps be justified in doubling or trebling our estimate to put it on a basis comparable to hers. A formidable gap remains.

In terms of the right-hand variables in equation (57), neither set of results fits the data well; this, however, is due to some extent to the fact that these endogenous variables are differences (between two occupations) of first differences (of logarithms in time). Quasi-R^2s (defined as the square of the simple correlation coefficient between the estimated and actual sample values of the endogenous variable in question), values of the likelihood function at exit, and sample means of residuals for each of these estimations are given in Table 9.

The supply equations for two occupations simply don't fit the data at all : Skilled Blue Collar (Metal and Electrical) $(i = 4)$ and Semi- and Unskilled Blue Collar Workers $(i = 7)$ have quasi-R^2 of less than 0.05. The fit is best for Skilled White Collar $(i = 2)$, Semi- and Unskilled White Collar $(i = 3)$ and Skilled Blue Collar (Building) $(i = 5)$, where the quasi-R^2 range from 0.5 to 0.8.

The results from Table 8 accord better with our prejudices about a reasonable order of magnitude for the elasticities of supply with respect to wages than those from Table 7. Moreover, it can be argued that the results in Table 8 are in better agreement with our priors as set out in Table 4 than those of Table 7. The descriptive statistics, unfortunately, do not support us. Whereas the Table 7

TABLE 9 : FURTHER DESCRIPTIVE STATISTICS ASSOCIATED WITH ESTIMATES REPORTED IN TABLES 7 AND 8

Estimates in Table	Occupation i, i=								Value of Log Likelihood Function
	1	2	3	4	5	6	7	8	
	Quasi-R^2 of Reduced Form Equation for Supply of Occupation i								
7	.234	.827	.696	.004	.479	.110	.043	.123	260.423
	.245	.579	.714	.005	.679	.074	.049	.036	251.466
8									
	Sample Mean Value of Residuals[a] for Reduced Form Equation for Supply of Occupation i (per cent) (a)								
7	- 1.37	2.65	4.30	5.06	4.41	16.22	4.50	-0.17	
8	-13.4	12.1	14.7	20.2	16.0	36.6	17.2	-0.17	

(a) Expressed as a percentage of sample mean of corresponding endogenous variable.

estimates result in equations which do not exhibit sizeable biases over the sample period -- with the exception of occupation 6, the average within sample bias is of the order of five per cent or less of the sample mean values of the endogenous variables concerned -- these biases are much greater in the Table 8 results, probably indicating specification error. There is a three per cent drop in the log likelihood of the Table 8 results relative to those of Table 7. Although we have not computed statistics relating to serial correlation, visual inspection of the residuals is enough to confirm that autocorrelation is a widespread problem throughout both estimated systems.

7. CONCLUSION

The results of this study are mainly negative. If we had not made intensive use of economic theory, the occupational supply system which we set out to estimate would have involved 120 parameters : sixty-four own and cross wage elasticities, forty attainment-to-occupation elasticities ε_{ik}, and eight elasticities with respect to each of (i) the supply of Teachers ($\underline{\psi}$) and (ii) the stock of persons Not in the Workforce ($\underline{\phi}$). Such a system would not have been estimable from the 80 (10 years × 8 occupations) data points available. The CRETH specification maintains reasonable flexibility but succeeds in reducing the number of unknown wage parameters from 40 to eight; equality restrictions developed from economic theory reduce the number of free ε_{ik}s from 40 to 28 and the number of other parameters (the ϕs and ψs) from 16 to 7. Yet it is still unrealistic to expect to be able to estimate the resultant 43 parameters from 80 data points (especially by FIML methods).

Constraints on the signs of the ε_{ik}s, ϕs and ψs further restrict the relevant part of the parameter space, but in a way which makes satisfactory parameterization for estimation difficult. Our approach was to use linear programming to generate a limited number of feasible ε matrices, and ($\underline{\phi}$, $\underline{\psi}$) vectors; the parameters to be found econometrically then were two sets each of 5 convex coefficients (one set each for the ε matrix, and for the ($\underline{\phi}$, $\underline{\psi}$) vectors). Because convex coefficients definitionally add to unity, the number of such free parameters involved is 8. The resultant system then had a total of 16 unknown parameters : the

8 wage (i.e., transformation) parameters as above, plus 8 free convex coefficients. Unfortunately, even such a highly constrained system as this defied econometric estimation.

It did prove possible to estimate a 5-parameter system. Such a system was obtained:

(a) by replacing the more flexible, 8-parameter CRETH specification by the less flexible, 1-parameter CET specification;

and

(b) by abandoning any attempt to estimate the attainment-to-occupation mapping econometrically.

The parameters in this stripped down system were one transformation elasticity τ, and four free convex coefficients associated with the five feasible $(\underline{\underline{\phi}}, \underline{\underline{\psi}})$ vectors generated by linear programming.

Two sets of results conditional on arbitrary ε-matrices are reported in this paper. (The data are given in the Appendix.) Although the elasticity of transformation in both cases is of the expected sign (and in at least one case nominally significantly different from zero), the values obtained (-0.03 and -0.10) are much lower in absolute value than we believe to be plausible. They do not gel well with Williams' (forthcoming 1983) estimate of a closely related parameter.

A further point needs noting here. In the original system we did not distinguish between males with attainment k and females with attainment k as a source of occupation i. Yet it is clear that such a distinction would be needed in practice -- there are still heavy sexual biases in the assignment of occupations.

We did not introduce this distinction simply because we knew the data would not allow it -- the original parameter count would have risen at least to 160.

What can be done? As at the time of our first attempts to mobilize a data base (in 1977), the most pressing need now (in 1982) is to secure more frequent collections of structurally detailed statistics on the labour market. Such collections need as their framework a skill-based occupational classification. The current ASCO (Australian Standard Classification of Occupations) project will not provide such a framework, in our view. It is high time that Australia began to collect longitudinal data which might eventually shed more light on the questions we have addressed in this paper.

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DATA APPENDIX

TABLE A1 : SUPPLIES OF PERSONS TO OCCUPATIONS

	PROF	SWC	USWC	SBC-ME	SBC-B	SBC-O	USBC	RUR
1966	152347	550484	1212606	523530	242528	158436	1512824	352651
1967	160396	537735	1248103	531784	250472	157550	1569524	354014
1968	166305	547436	1286665	553027	261109	159247	1617023	345425
1969	175825	568979	1349131	573503	269633	160992	1631583	340858
1970	190031	586884	1442876	599377	279999	166731	1643862	336321
1971	188824	629602	1489659	614390	285888	161023	1710221	315778
1972	192405	629500	1488775	619639	287931	165258	1753972	313230
1973	210966	645337	1569261	625437	291705	178931	1816335	325631
1974	221695	686698	1666833	614879	297189	161344	1852990	318386
1975	207963	691949	1680885	632938	286753	153298	1880186	314159
1976	231228	690579	1743258	619306	292773	165112	1922700	313736

Definitions and Sources : See section 4.1

Occupation key :
PROF = Professional White Collar
SWC = Skilled White Collar
USWC = Semi- and Un-skilled White Collar
SBC-ME = Skilled Blue Collar - MetaL & Electrical
SBC-B = Skilled Blue Collar - Building
SBC-O = Skilled Blue Collar - Other
USBC = Semi- and Un-skilled Blue Collar
RUR = Rural Workers

TABLE A2 : CERTAINTY EQUIVALENT WAGES FOR PERSONS IN OCCUPATIONS

	PROF	SWC	USWC	SBC-ME	SBC-B	SBC-O	USBC	RUR
1966	2.63	1.58	0.97	1.27	1.23	1.09	0.96	2.21
1967	2.75	1.64	1.04	1.38	1.30	1.16	1.03	2.44
1968	2.87	1.75	1.11	1.47	1.40	1.18	1.11	2.57
1969	3.02	1.90	1.21	1.58	1.51	1.28	1.22	2.66
1970	3.19	2.06	1.31	1.72	1.68	1.35	1.34	3.01
1971	3.58	2.40	1.55	1.91	1.88	1.61	1.58	2.96
1972	3.85	2.49	1.64	2.15	2.01	1.60	1.67	2.96
1973	3.99	2.70	1.84	2.39	2.21	1.77	1.89	3.99
1974	4.73	3.18	2.23	2.74	2.62	2.09	2.23	5.04
1975	5.98	4.00	2.79	3.11	3.05	2.56	2.61	6.12
1976	7.38	4.72	3.25	3.58	3.42	2.90	3.00	6.74

Definitions and Sources : See Section 4.2

Occupation key : See Table A1.

TABLE A3 : EDUCATIONAL ATTAINMENTS OF THE POPULATION AGED 15 YEARS AND OVER

	DEGREE	DIPLOMA	TECHNICIAN	TRADE	OTHER
1966	100232	241061	264636	774543	6852751
1967	111634	249977	273244	794330	6978936
1968	124722	259003	282020	816558	7117649
1969	140902	269181	292322	843783	7268433
1970	158135	279490	303015	874681	7405552
1971	180103	292260	314632	895722	7546735
1972	205479	304168	326072	915465	7664843
1973	232842	317464	337652	931801	7775275
1974	260846	334412	352739	950139	7904974
1975	287870	354093	370460	969440	7993413
1976	316395	373480	386069	986218	8086498

Sources : See Section 4.5.

TABLE A4 : PERSONS IN (a) LECTURING AND TEACHING OCCUPATIONS AND (b) NOT IN THE LABOUR FORCE

	LECTURERS & TEACHERS	NOT IN THE LABOUR FORCE
1966	136215	3391603
1967	152746	3445801
1968	160391	3503327
1969	167536	3576585
1970	181958	3592835
1971	181060	3653008
1972	183415	3781900
1973	197639	3733798
1974	206428	3776680
1975	209128	3918022
1976	234017	3935955