IMPACT PROJECT

A Commonwealth Government inter-agency project in co-operation with the University of Melbourne, to facilitate the analysis of the impact of economic demographic and social changes on the structure of the Australian economy

THE DEMOGRAPHIC CORE OF THE IMPACT PROJECT : AN OVERVIEW

by

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THE DEMOGRAPHIC CORE OF THE IMPACT PROJECT : A PRELIMINARY OVERVIEW

by

Dennis Sams*

1. PREAMBLE

The purpose of this paper is to provide a brief description of the demographic core of the BACHUROO model within IMPACT's framework for analysing the effects of demographic, economic and social changes on the structure of industry in Australia.[1] Such a description cannot be attempted outside the context of the IMPACT project as a whole. Necessary background information on the structure and purpose of IMPACT's medium term model is therefore given in section 2 before details concerning the demographic core itself are presented in section 3. In the fourth and final section the earlier material is summarized, and a progress report on the implementation of the demographic core is given.

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* The author would like to thank Alan Powell, Pam Williams and Clive Brooks for their assistance with this paper, and Alan R. Hall for his assistance with the Australian Demographic Databank.

1. For a full, non-technical discussion of the IMPACT project, see Alan A. Powell, The IMPACT Project : An Overview, First Progress Report of the IMPACT Project, Volume 1 (Canberra : AGPS, 1977).

2. THE MEDIUM TERM MODEL

IMPACT's medium term model will consist of three linked modules : a macroeconomic module MACRO; an industry structure module ORANI;[1] and a labour supply module BACHUROO.[2] The interconnections between the three modules are illustrated in Figure 1 taken from Powell and Parmenter.[3]

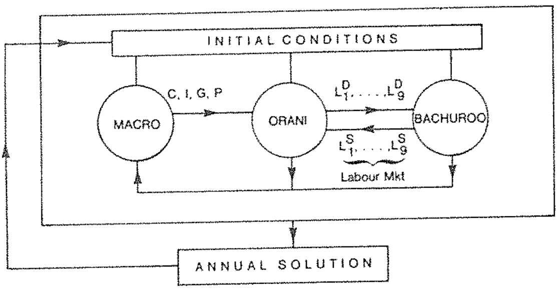

Figure 1

SIMPLIFIED DIAGRAM OF MEDIUM TERM MODEL. The levels of real consumption C, real investment I, real government spending G, and the general price level P, may be thought of as originating in MACRO. ORANI disaggregates these into 109 input-output industries and determines relative prices of commodities, imports and exports by I-O industry, and (if wage relativities are given), labour demands by nine occupations. BACHUROO determines the supplies of labour by occupation. Excess demand for or supply of labour can feed back into the macroeconomic environment. Interactively the three models will, given a set of initial conditions, produce an annual solution which then determines a new set of initial conditions for a second annual solution; and so on.

1. See Peter B. Dixon, B. R. Parmenter, G. J. Ryland and John Sutton, ORANI, A General Equilibrium Model of the Australian Economy : Current Specifications and Illustrations of Use for Policy Analysis, First Progress Report of the IMPACT Project, Volume 2 (Canberra : AGPS, 1977).

2. See Powell, op. cit., pp. 92-104.

3. Alan Powell and Brian Parmenter, "The IMPACT Project as a Tool for Policy Analysis : Brief Overview," Australian Quarterly, March 1979, pp. 62-74.

Each of the three modules of the medium term model is, in its own right, an annual model which determines, as appropriate, the annual flow or the stock at the end of the year of each endogenous variable appearing in that model. The medium term model as a whole will be executed by supplying initial values for the endogenous variables at the beginning of the projection period and the values of the exogenous variables during the period. Some of the exogenous inputs to particular modules are to be supplied as outputs from other modules. Thus a variable which is exogenous to any one module may nevertheless be endogenous to the medium term model as a whole.

Within any one period the equations within each module and within the medium term model as a whole are solved simultaneously, but for successive periods the medium term model will be operated dynamically with the values calculated for each variable at the end of a year of projection becoming the initial values of these variables for the next year. Dynamic execution, of course, requires a time series projection of values for the exogenous variables to be supplied externally. The medium term model is thus a dynamic system which responds to external conditions. The latter include policy variables (e.g., immigration, tariffs) and variables determined outside the context of the Australian economy (the world parity price of oil, for example).

The demographic and labour-force module, BACHUROO, is designed to determine the size and skill composition of the labour force[1] according to the nine occupational categories given in Table 1 and to map the functional

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1. See R. Craigie, D. Parham and G. J. Ryland, "Educational Attainment and Occupational Supply : A Theoretical Outline," IMPACT Preliminary Working Paper No. BP-16, Industries Assistance Commission, Melbourne, January 1979.

distribution of incomes, supplied by ORANI, into the personal and household distribution of incomes.[1] The projection of the skill composition of the workforce is integrated into a framework of demographic accounting designed to supply consistent estimates of the Australian population, family formation, fertility and work force participation.[2]

Table 1 : Major Occupation Groups Used in IMPACT

1.	Professional White Collar
2.	Skilled White Collar
3.	Semi and Unskilled White Collar
4.	Skilled Blue Collar (Metal and Electrical)
5.	Skilled Blue Collar (Building)
6.	Skilled Blue Collar (Other)
7.	Semi and Unskilled Blue Collar
8.	Rural Workers
*9.	Armed Services

* Armed Services are included for completeness of coverage. They are usually excluded from labour force statistical collections and are modelled largely exogenously in IMPACT, although they impinge indirectly by employing persons who would otherwise be available for the civilian labour force.

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1. See Ashok Tulpulé, "Estimation and Mapping of the Distribution of Income in Australia for the IMPACT Model," IMPACT Preliminary Working Paper No. BP-05, Industries Assistance Commission, Melbourne, November 1976.

2. For a broad outline of the approach, see R. Filmer and R. Silberberg, "Fertility, Family Formation and Female Labour Force Participation in Australia, 1922-1974," IMPACT Preliminary Working Paper No. BP-08, Industries Assistance Commission, Melbourne, December 1977.

BACHUROO also contains a submodule which endogenizes household headship rates which, when combined with the population projections, provide projections of the number of households.[1] Household formation is of interest in its own right but is also potentially important in linking occupational income to household expenditure.

In the amount of attention devoted within BACHUROO to the supply side of the labour market, and to the demography of the household sector, IMPACT's medium term model differs from the majority of existing economy wide models around the world. The major exception is the ILO's series of BACHUE models of developing countries, after which BACHUROO is named.[2]

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1. See P. J. Williams and R. C. Brooks, "An Econometric Model of Household Headship," IMPACT Preliminary Working Paper No. BP-14, Industries Assistance Commission, Melbourne, July 1978.

2. World Employment Programme, International Labour Office, Geneva, "Economic-Demographic Modelling Activities of the World Employment Programme," July 1973 (mimeo), pp. 31 ;

   G. B. Rodgers and R. Wery, "Population and Employment, A Strategy for Research," World Employment Programme, International Labour Office, Geneva, March 1974 (mimeo), pp. 13 ;

   Richard Blandy, Rene Wery, with others, "BACHUE-1 : The Dynamic Economic-Demographic Model of the Population and Employment Project of the World Employment Programme," International Labour Review, Vol. 107, No. 5 (May 1973), pp. 441-449 ;

   R. Wery, G. B. Rodgers, and M. D. Hopkins, "BACHUE-2 : Version 1 : A Population and Employment Model for the Philippines," World Employment Programme, International Labour Office, Geneva, Population and Employment Working Paper No. 5 (July 1974) (mimeo), pp. 129.

3. THE DEMOGRAPHIC CORE

The demographic core of BACHUROO is composed of three submodules; a population submodule, a labour force submodule and a household headship submodule. The structure of the demographic core is illustrated in Figure 2.

The population submodule of BACHUROO provides projections of the Australian population by sex, age and marital status. These may be of direct interest to policy discussions; alternatively they may be used as inputs to the other submodules of BACHUROO (and thence into the medium term model at large). The outputs of the population submodule are :

(a) the <u>population</u> by

sex,

single years of age from 0 years to 100+ years,

and

four marital status categories; namely,

never married,

married,

divorced

and

widowed,

at the beginning of a given year ;

(b) the <u>marital status flows</u>; namely, numbers of

first marriages,

remarriages,

divorces

and

widowings

by sex and single years of age (15 - 100+) within a given year;

(c) <u>births</u> by sex (within a given year);

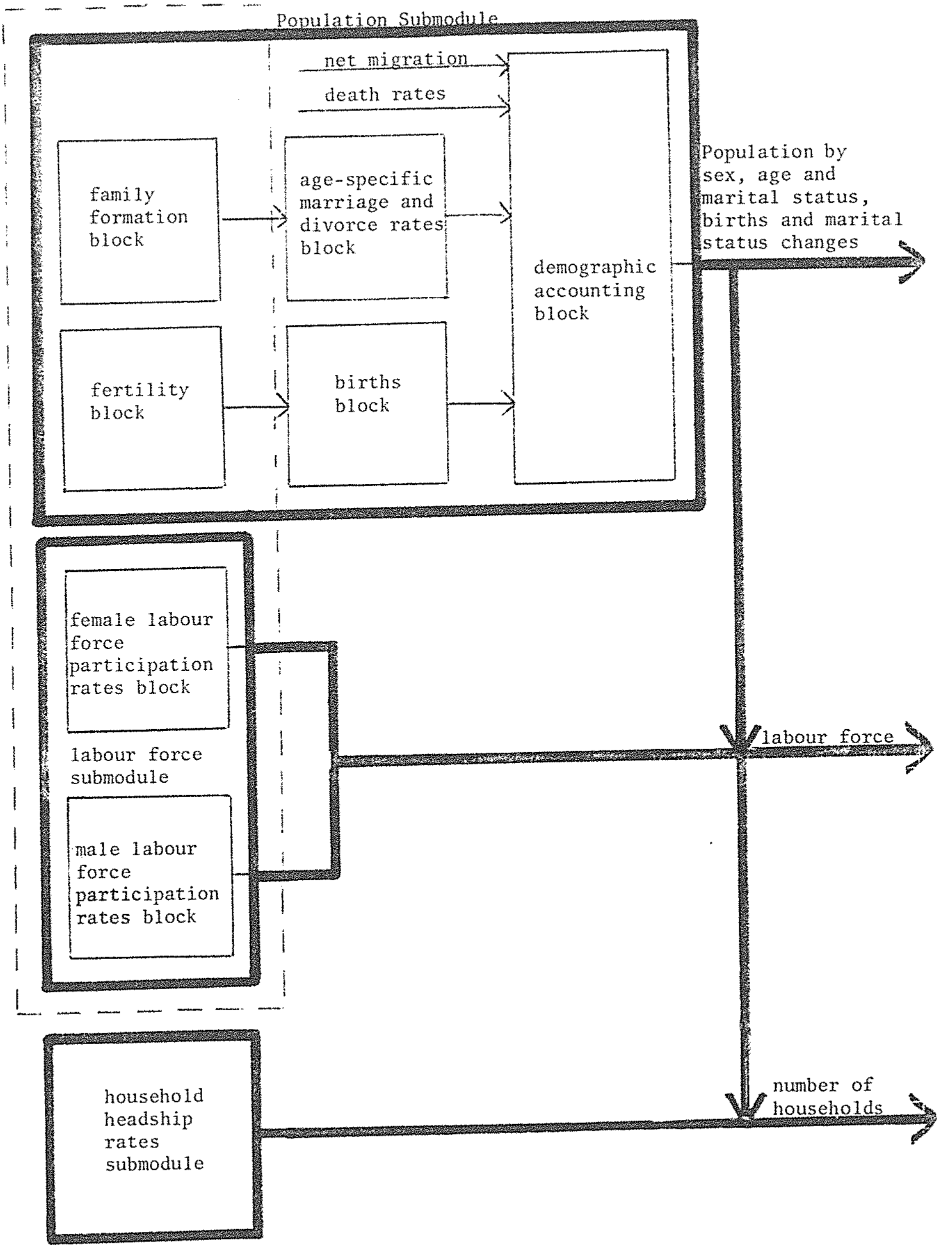

Figure 2 : Schematic diagram of the demographic core of BACHUROO.

The econometric model of Family Formation, Fertility and Labour Force Participation (3FLFP) is contained within the dashed lines.

(d) a measure of 'child quality' (viz., the average level of investment, per head, in children's education, etc., over a given year).

The labour force submodule provides labour force participation rates (and, when combined with the population submodule, the labour force)

by sex,

by three age groups (15-24, 25-54, 55+),

and

by two marital status categories (married and not married)

within a given year.

The child quality measure and the female labour force participation rates arise within the demographic core as a consequence of a consistent simultaneous treatment of family formation, fertility and labour force participation.

The household headship submodule supplies household headship ratios (and, when combined with the population submodule, the numbers of household heads)

by sex,

by fourteen quinquennial age groups from 15-19 to 80+ ,

and

by five marital status categories :

never married,

married,

permanently separated,

divorced

and

widowed.

The variables included within the demographic core are listed in Table 2.

The population submodule consists of 5 blocks :

(a) a demographic accounting block,

(b) a block to calculate age specific marriage and divorce rates,

(c) a births block,

(d) two of the blocks of the econometric model of family formation, fertility and labour force participation; viz. : the family formation block and the fertility block.

The demographic accounting block determines the composition of the population at the end of the period from the initial composition of the population and the age specific rates of first marriage, remarriage and divorce for women, the annual number of births, the net immigration of men and women by age and marital status and death rates by sex, age and marital status. The marital status changes for men are endogenous to the demographic accounting block and are determined by the marriage and divorce behaviour of the women. Similarly, widowings of both men and women are determined endogenously by the death of the spouse.

The net immigration flows and the death rates are exogenous and have to be supplied for any application of the model. It is intended that migration will remain exogenous but the operation of the model will be simplified by employing temporally stationary death rates or ones which vary slowly with time. The age specific marriage and divorce rates block determines rates of first marriage, remarriage and divorce for each of the eighty-six ages from 15 to 100+, from information supplied by the econometric model of family formation, fertility and labour force participation (3FLFP, hereafter). The births block calculates the number of live births of each sex from information supplied by the econometric model.

The econometric model 3FLFP supplies a consistent description of family formation, fertility and labour force participation rates for women and the relationship of these and other endogenous variables to economic and social conditions. The basic model is described in detail in Filmer and Silberberg[1] and some more details are given later in this paper. A separate econometric model of male labour force participation rates[2] relates the male participation rates to economic and social conditions. In this paper the latter model will be treated as a part of 3FLFP.

In summary, the demographic sub-module provides a framework for projecting the Australian population disaggregated by sex, age and marital status. The component flows of the model are determined by an econometric model and are subject to economic and social influences. When combined with the household headship rates and the labour force participation rates, the demographic core provides projections of the number of households by age and type and the total labour force by sex and age. These outputs are then available for use

(a) in their own right as inputs to policy discussions ;

(b) as inputs to the other submodule of BACHUROO (and in particular, to the submodules which give the occupational breakdown of the workforce) ;

and

(c) as inputs, via BACHUROO as a whole, to the ORANI and MACRO models.

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1. Filmer and Silberberg (1977), op. cit..
2. To be documented by V. Manion in a forthcoming paper.

Table 2 : Variables of the Demographic Core[1]

Item		Comment
Population		
Population of men by age i and marital status k	$\tilde{X}_k(i)$	endogenous to demographic accounting block
Population of women by age i and marital status k	$X_k(i)$	endogenous to demographic accounting block
Migrants		
Migrant men by age i_m and marital status k	$\tilde{X}^M_k(i)$	exogenous
Migrant women by age i_f and marital status k	$X^M_k(i)$	exogenous
Deaths		
Death rates for men by age i and marital status k	$\tilde{\delta}_k(i)$	exogenous
Death rates for women by age i and marital status k	$\delta_k(i)$	exogenous
Marriage and Divorce of Women		
Age specific marriage and divorce rates for women	f(i),r(i),d(i)	approximated by a smooth distribution parameterised by a propensity, mean age and variance in age
Propensities to marry and divorce	P_F, P_R, P_D	endogenous to 3FLFP model[2]
Mean ages at marriage and divorce	M_F, M_R, M_D	endogenous to 3FLFP model
Variance in ages of marriage and divorce	V_F, V_R, V_D	endogenous to 3FLFP model
Marriage selection rule by age of bride i_f and age of groom i_m	$c(i_f, i_m)$	endogenous to demographic accounting block; initial value required
Widowings of Women		
Age specific widowing rates for women	w(i)	endogenous to demographic accounting block calculated via married couples matrix
Marriage, Divorce and Widowings of Men		
Age specific marriage rates for men	$\tilde{f}(i), \tilde{r}(i)$	endogenous to demographic accounting block calculated via marriage selection rule
Age specific divorce and widowing rates for men	$\tilde{d}(i), \tilde{w}(i)$	endogenous to demographic accounting block calculted via married couples matrix

cont'd.

Table 2 cont'd.

Item		Comment
Births		
Births	B, $\tilde{B}$	endogenous to births block
Mean implied completed family size	M_N	endogenous to 3FLFP model
Variance in implied completed family size	V_N	endogenous to 3FLFP model
First nuptial confinements	C_1	endogenous to 3FLFP model
Ex-nuptial birth rate	b^e	exogenous
Nuptial confinements of parity j	C_j	endogenous to births block
Child Service		
Child quality	Q	endogenous to 3FLFP model
Child services	S	endogenous to 3FLFP model
Labour force participation rates		
Female labour force participation rate by age i and marital status k	ℓ_{ik}	endogenous to 3FLFP model
Male labour force participation rate by age i and marital status k	$\tilde{\ell}_{ik}$	endogenous to 3FLFP model
3FLFP exogenous variables		
Real income per head	Y	exogenous
Female/male relative wage rate	$(W/\tilde{W})$	exogenous
Weighted first marriages	$\hat{F}$	calculated from endogenous population variables
Female education	E	exogenous
Oral contraceptive usage	Ω	exogenous
Expected female real hourly wage	W^*	exogenous
Permanent income per head	Y^P	exogenous
Unemployment rate	u	exogenous
Real old age pension	G_a	exogenous

Table 2 cont'd.

Item		Comment
Widow's pension relative to expected female wage	(G_w/W^*)	exogenous
Nuptial birth rate	b	calculated from variables endogenous to births and demographic accounting blocks
Number of dependents per married woman	K	calculated from variables endogenous to demographic accounting block
Infant mortality rate	ϕ	calculated from variables endogenous to demographic accounting block
Headship rates		
Headship rates by sex, age i and marital status k	$h_{ik},\tilde{h}_{ik}$	endogenous to household headship submodule
Households		
Households by sex, age i and marital status k	$H_{ik},\tilde{H}_{ik}$	endogenous to household headship submodule
Exogenous variables for household headship submodule		
Permanent income per head	Y^P	exogenous
Labour force participation rate by sex, age and marital status k	$\ell_{ik},\tilde{\ell}_{ik}$	to be supplied by 3FLFP model
Employment rate by sex, age i and marital status k	$u_{ik},\tilde{u}_{ik}$	exogenous
Welfare housing expenditure	G_h	exogenous
Permanent widow's pension deflated by rental price index	G_e^P	exogenous

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1. Symbols for variables used in Appendix 1 and Figure 3.

2. Family Formation, Fertility and Labour Force Participation Model (3FLFP).

NOTE: 'exogenous' (unless otherwise qualified) means exogenous to BACHUROO as a whole, but not necessarily exogenous to the medium term model.

3.1 Demographic Accounting

The equations of the demographic accounting block are given in Appendix 1. The form of the demographic equations for females is quite simple and can be written in matrix form as

$$(1) \qquad X(i+1,t+1) \;=\; (I-D(i,t))(A(i,t)\,X(i,t) + X^M(i,t)) \;,$$

where

$$X(i,t) \;=\; (X_1(i,t),\; X_2(i,t),\; X_3(i,t),\; X_4(i,t))^T$$

is an age specific vector calculated at time t whose components are :

(i) the number of never married women of age i at the start of period t, $X_1(i,t)$;

(ii) the number of married women of age i at the start of period t, $X_2(i,t)$;

(iii) the number of divorced women of age i, $X_3(i,t)$, at the start of the period t ;

(iv) the number of widows of age i, $X_4(i,t)$, at the start of the period t.

$X^M(i,t)$ is an age specific vector calculated at t whose components are the net immigration of women of each of the four marital status categories and age i at the start of period t.

The matrix A is a 4 by 4 matrix of transfer coefficients where an element $a_{k\ell}(i,t)$ is the probability that a woman who is of marital status ℓ and age i at the start of period t is of marital status k at the end of the period. For example, the element $a_{21}(i,t)$ is simply the probability of first marriage in period t of a woman of age i at the start of the period, while other examples of the use of the notation are :

$a_{23}(i,t)$ = the probability of remarriage of a divorcee;

$a_{24}(i,t)$ = the probability of remarriage of a widow;

$a_{32}(i,t)$ = the probability of divorce;

and

$a_{42}(i,t)$ = the probability of being widowed.

The diagonal elements a_{11}, a_{22}, a_{33} and a_{44} are simply the probabilities of not changing marital status within the period.

The matrix $D(i,t)$ is a diagonal matrix whose elements are the marital status specific death rates of a woman of age i at the start of the period. I is the identity matrix. The term $(I - D(i,t))$ is simply the survival rate of women of each age by their marital status at the end of the period. Thus equation (1) represents a two stage treatment of demographic change; the changes in marital status are dealt with first independently of death and then survival rates are applied according to marital status at the end of the period.

Several elements in the transition matrix A are identically zero since we assume that only one marital status change can occur to any one person in a single period. For example, the probability of transferring from never married to divorced, $a_{31}(i,t)$, is zero since we have excluded the possibility that someone might marry and divorce in the same period. But since marital status change and death are treated as independent it is possible, although unlikely, that someone will marry and be widowed in the same period. The calculation of this probability is necessary to maintain the strict identity between the number of married men who die within a period and the number of widows created within that period.

The probabilities of first marriage, remarriage and divorce are supplied explicitly by the age specific rate block of the demographic core. The probability of being widowed is calculated, as indicated above, from the probability of the death of the husband of a woman of a given age. To achieve this, the demographic accounting block maintains a "couples matrix" containing the numbers of married couples in which the wife is of one age and the husband of another age, and the probability of being widowed is calculated from the proportion of husbands of each age and their age specific death rates.

Marital status changes for men are dependent on marital status changes for women. The widowings of males parallels that of females and is calculated from the age distribution of wives and their age specific death rates. The divorcing of males is treated similarly by assuming that the probability of divorce is the product of the probability of being married to a woman of a given age and the probability of a woman of that age divorcing her husband summed over all the possible ages of the woman. Thus widowings of both men and women and the divorcing of men are determined via the "couples matrix" and the probability of loss of the partner.

The marriages of men are determined from the marriage rates of women and a "marriage selection rule" which assigns a probability that a bride of one age will have a groom of that or another age. The "marriage selection rule" plus the age specific rate of marriage of women determines the age specific rate of marriage of men.

Births enter the demographic accounting block at age 0 , and with allowance for survival, progress to older ages over successive periods. Marital status changes are specifically excluded before age 15. The flows within the demographic accounting block are illustrated in Figure 3, in which the structure and flows of the whole demographic core are shown in some detail.

3.2 Age Specific Marriage and Divorce Rates

The age specific first marriage, remarriage and divorce rates are modelled within the demographic core by the age specific rate block. To model the age specific rate individually for each single year of age is not possible and we employ a smooth distribution across ages as an approximation to the actual age specific rates. The age specific rates in each case are low at young ages, rise sharply to a peak and then decrease slowly for older ages. To illustrate the approach, actual age specific rates for first marriage, divorce and remarriage for three typical years, together with the smooth approximation to their distribution, are shown in Figures 4a, 4b and 4c. We are able to describe the approximating curve for each of the age specific rates in any one year by three parameters :

> an index of propensity (to first marry, remarry, or divorce), that is, the area under the curve ;
>
> the mean of the age distribution of the age specific rates ;

and

> the variance of the age distribution of the age specific rates.

The first parameter can be thought of as providing a measure of the scale of the curve, the second as a measure of the location of the curve and the third as a measure of the width of the curve. The three parameters vary from year to year and capture movements in the scale and

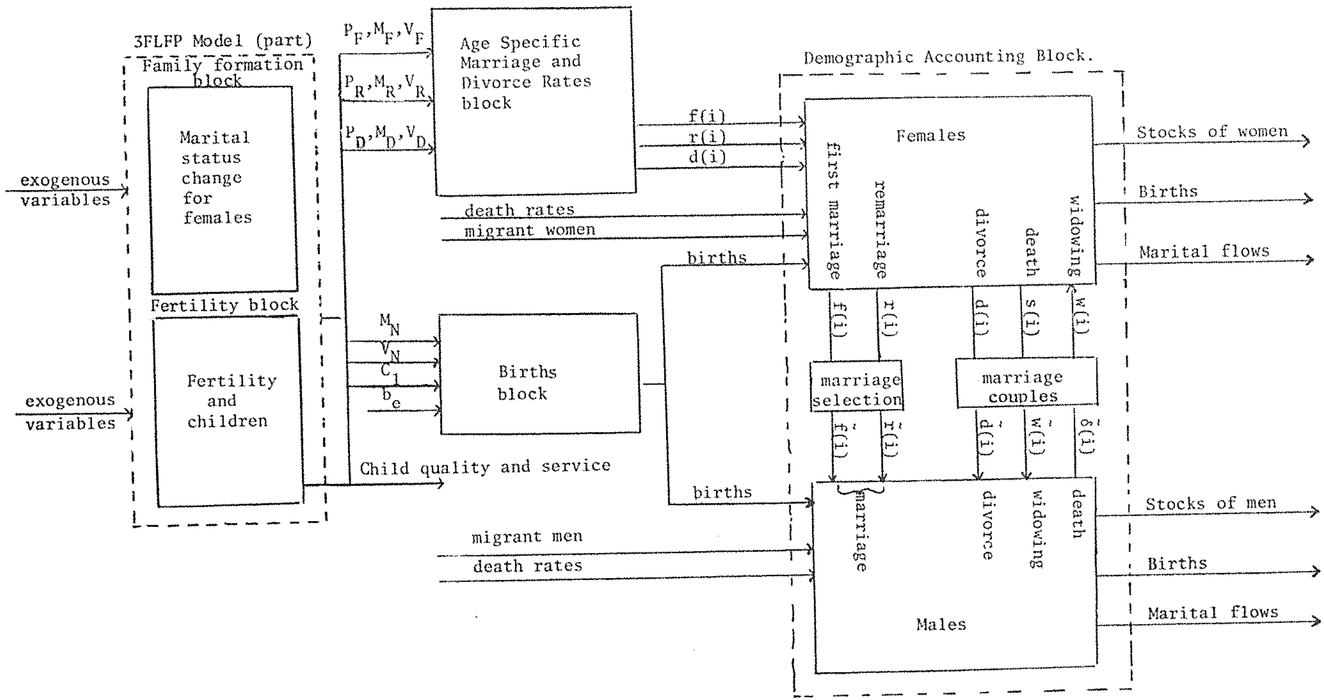


Figure 3: More detailed view of information flows in the population submodule of BACHUROO.

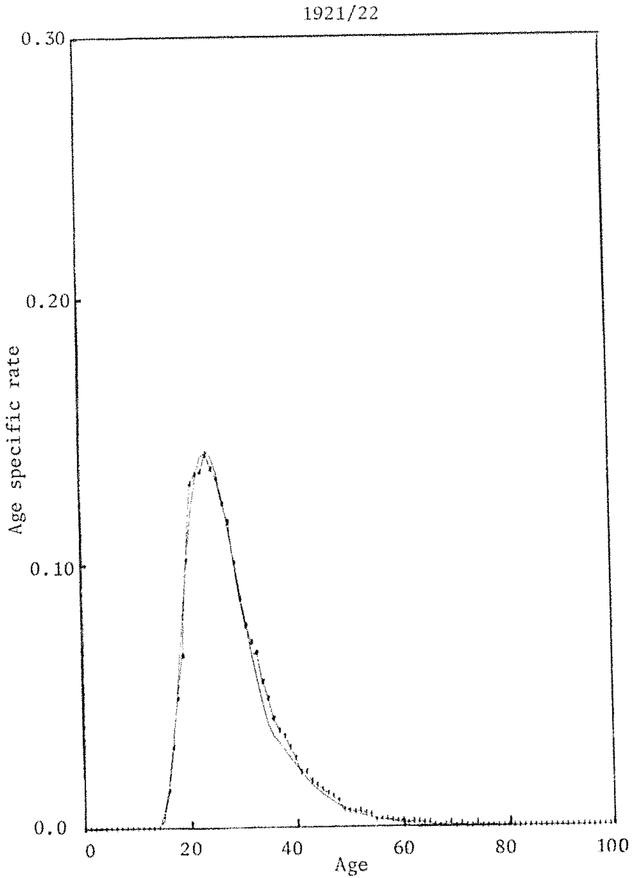
shape of the age specific rates. A fourth but stationary parameter is also required for the specification of the curves and that is the location of the origin. For first marriage, we specify that marriage cannot occur before age 15 years. For remarriage and divorce we specify that this cannot occur before age 16. These restrictions omit some very young marriages and thus the model will say nothing about these. Within the sample period (1921-22 to 1964-65), the parameters of each curve for each year were obtained by minimising the sum of the squares of the difference between the actual number of marriages at each age and that given by the approximating curve, given the number of people "at risk" of marriage or divorce in each age group. The values of the parameters for the period 1921 to 1965 for divorce are illustrated as an example in Figures 5a, 5b and 5c. The approximating curve is a gamma distribution and some further details are given in Appendix 2.

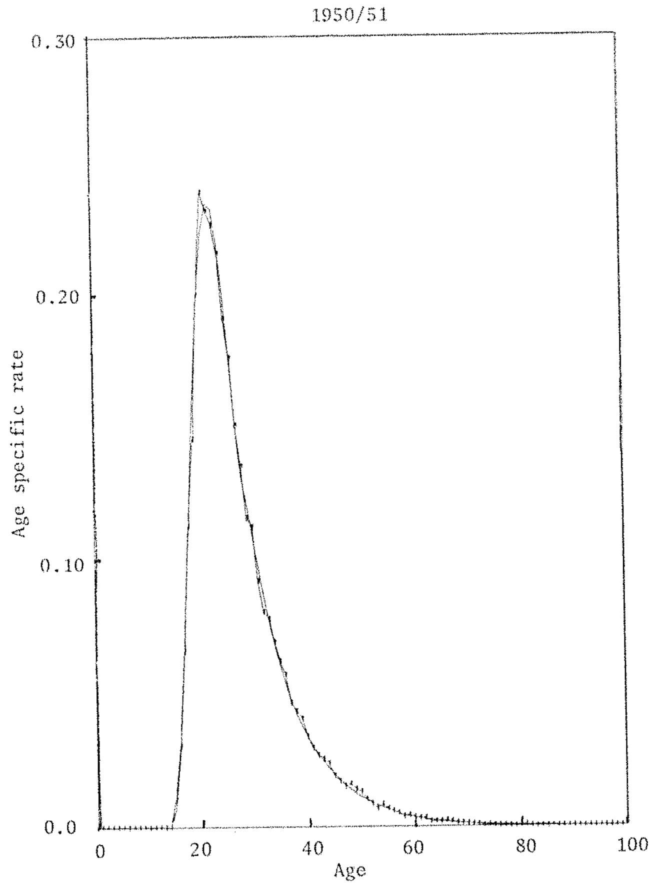
The parameters of the age specific rates are related to the economic and social environment by the econometric model (3FLFP) of family formation. The economic environment can influence the parameters of the age specific rates of marriage and divorce; these rates in their turn influence the marital status changes within the demographic accounting block and thereby the calculation of the succeeding populations of men and women.

3.3 Births

The births block supplies the number of births, by sex, to the demographic accounting block. Within the demographic core, the decision to have children is treated as a sequential decision making

First Marriage





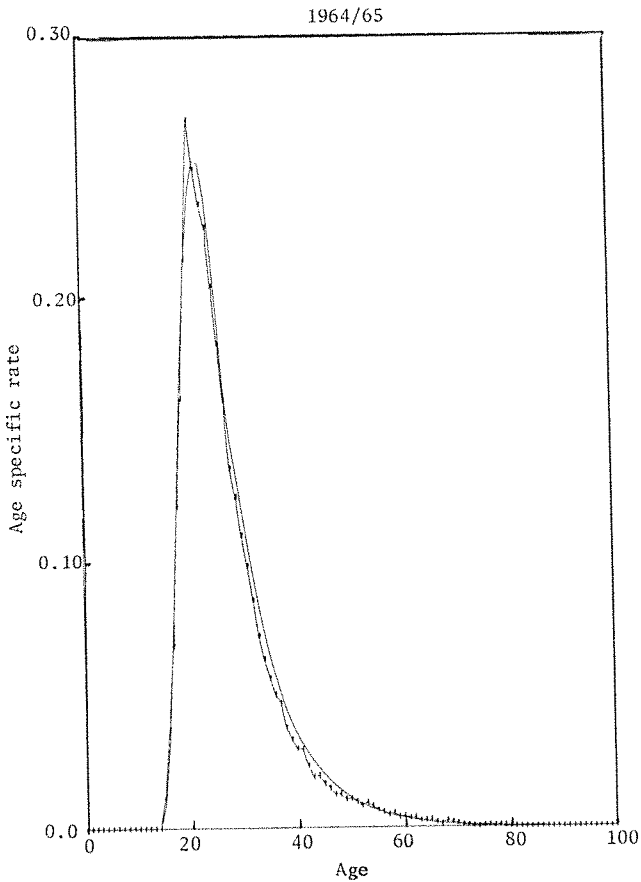
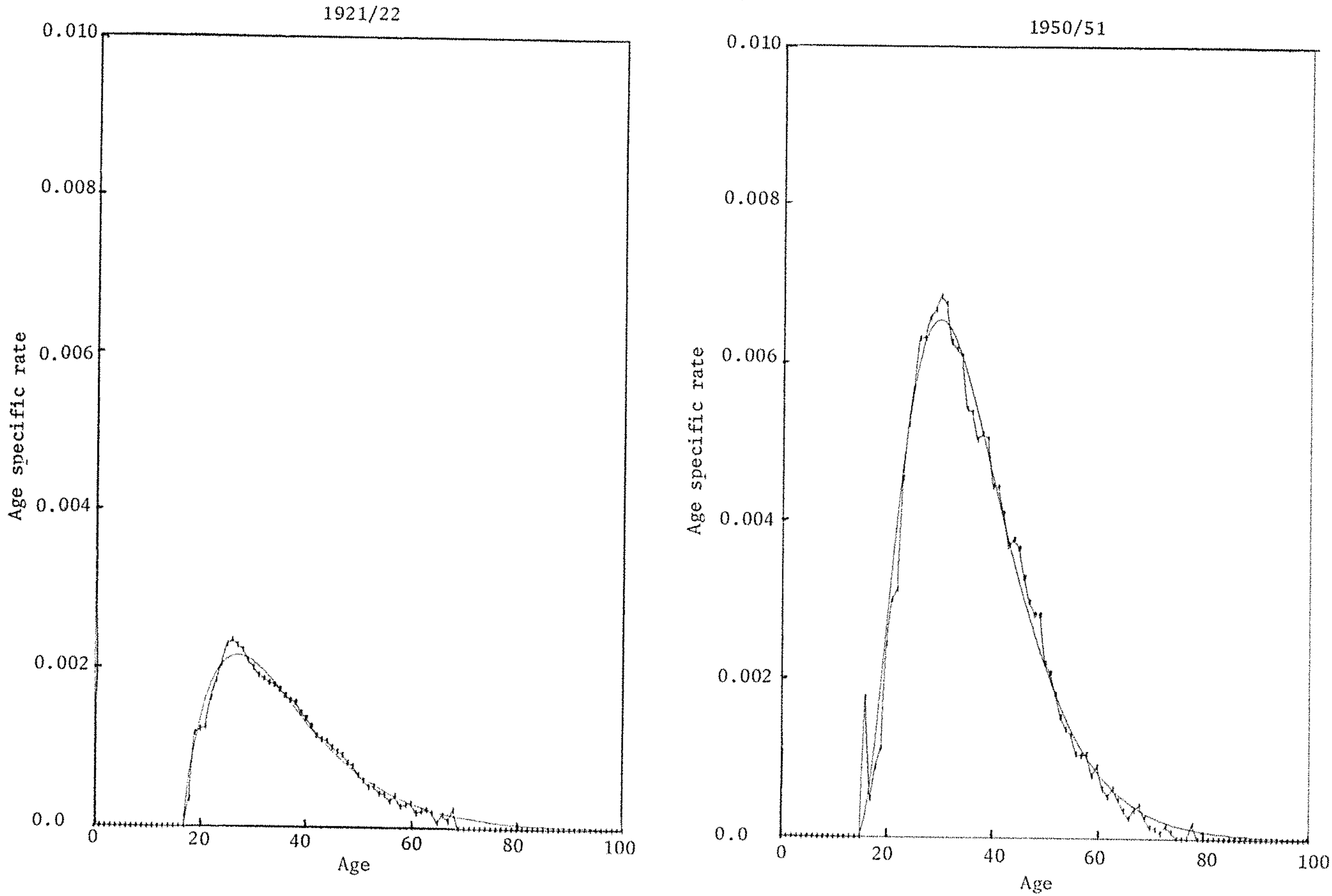


Figure 4a : The historical (•——•) and fitted (——) age specific rates of first marriage for women for the financial years 1921/22, 1950/51 and 1964/65.

Divorce


1921/22
Age specific rate
0.010
0.008
0.006
0.004
0.002
0.0
0
20
40
60
80
100
Age
1950/51
Age specific rate
0.010
0.008
0.006
0.004
0.002
0.0
0
20
40
60
80
100
Age

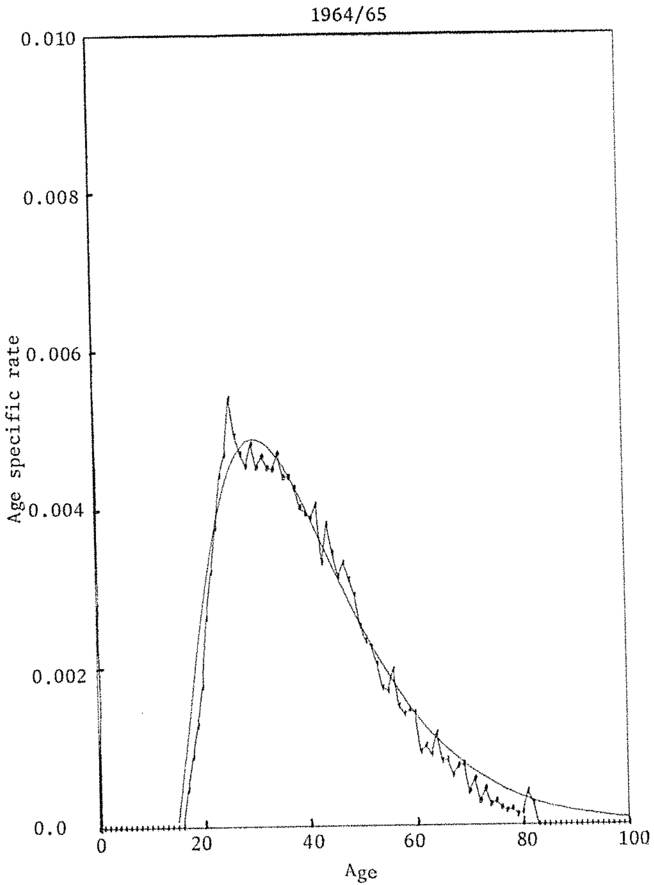
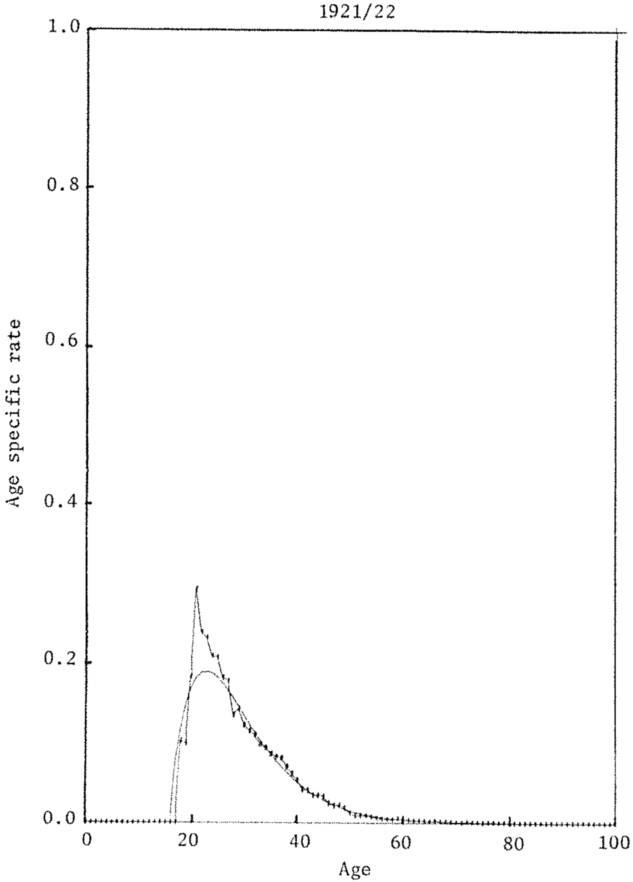
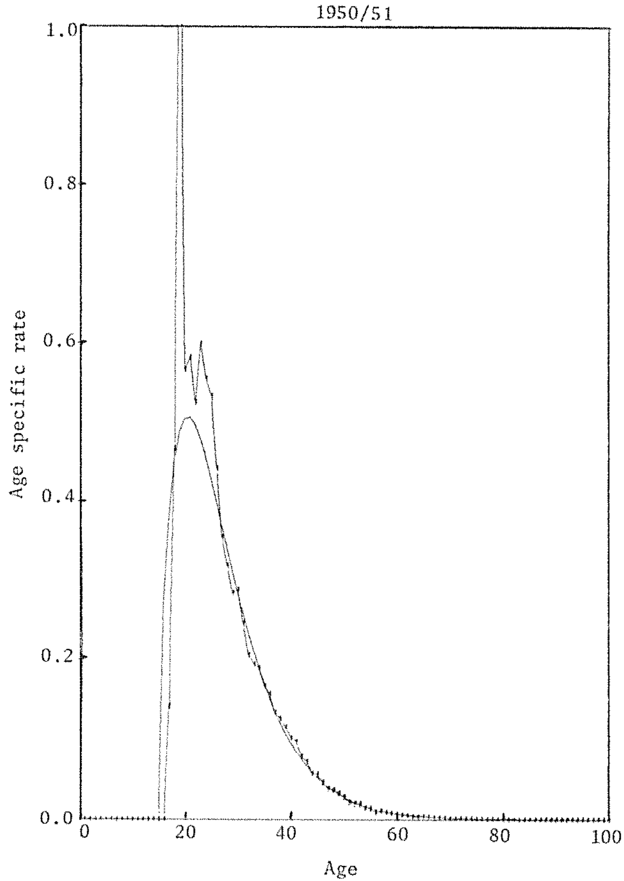


Figure 4b : The historical (•——•) and fitted (——) age specific rates of divorce for women for the financial years 1921/22, 1950/51 and 1964/65.

Remarriage


1921/22
Age specific rate
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
Age


1950/51
Age specific rate
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
Age

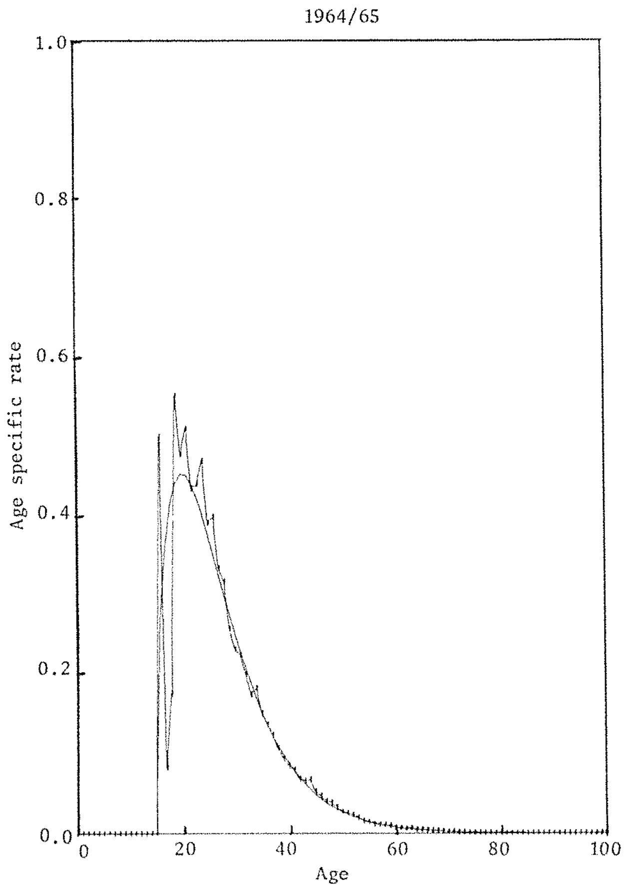


Figure 4c : The historical (•——•) and fitted (——) age specific rates of remarriage for women for the financial years 1921/22, 1950/51 and 1964/65.

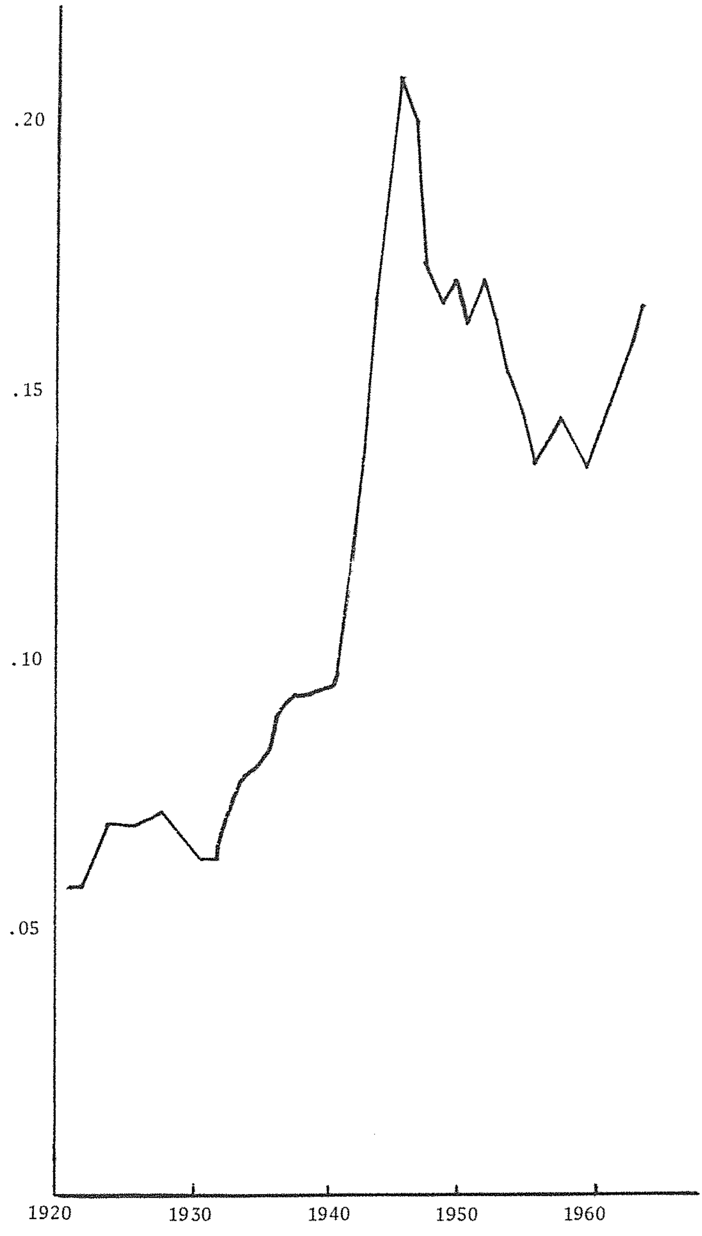


Figure 5a: Time series of estimated index of propensity to divorce.

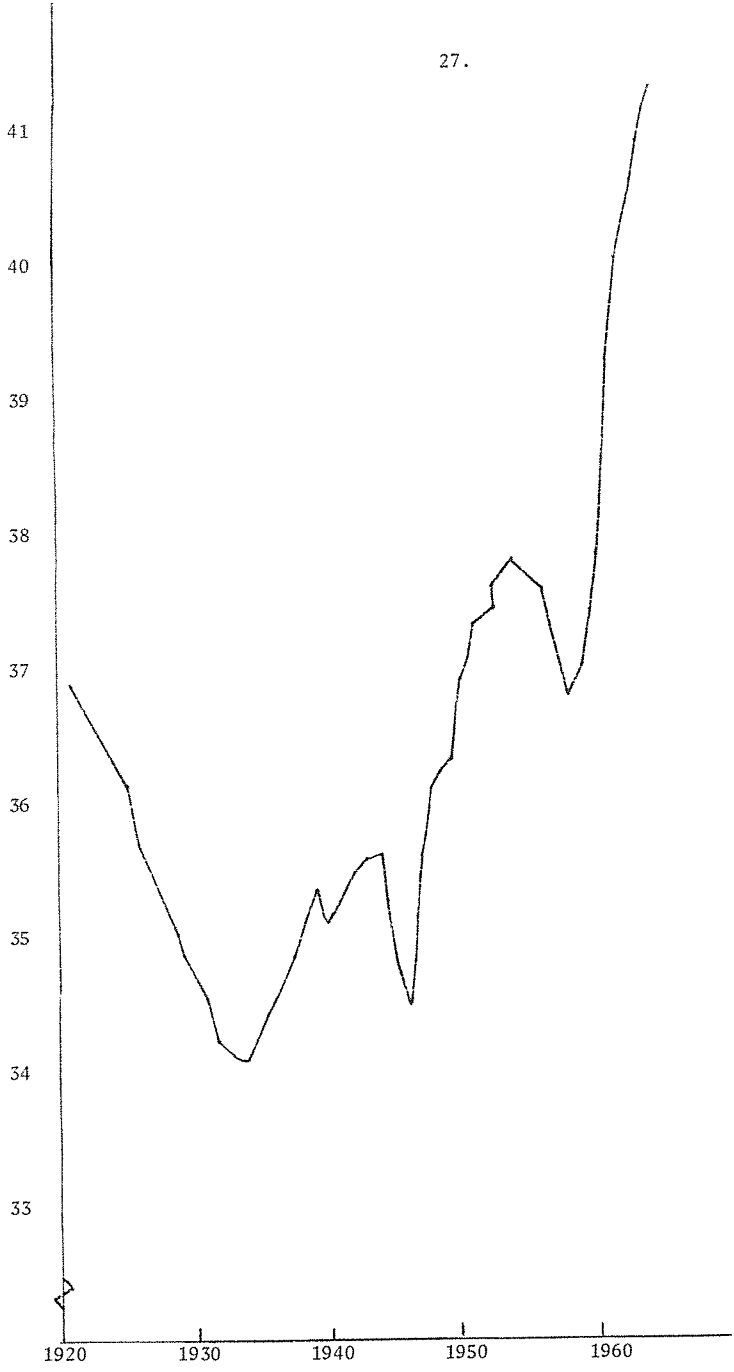


Figure 5b: Time series of estimated mean of the age distribution of the age specific divorce rates.

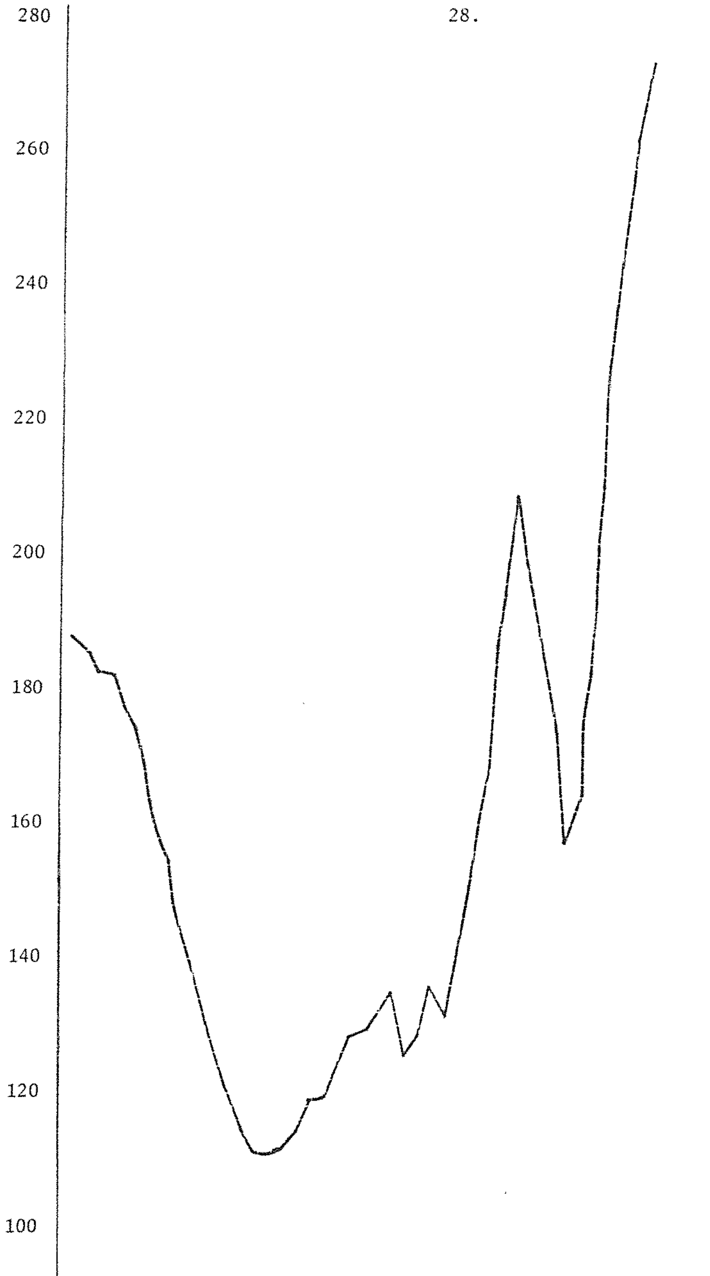


Figure 5c: Time series of estimated variance of the age distribution of the age specific divorce rates.

process. The econometric model 3FLFP supplies to the births block both the number of first nuptial confinements and also a measure of completed family size. From the former we can obtain the number of first births and from the latter we obtain parity progression ratios[1] for the probability of the second, third and higher order births. This allows us to model separately the decision to have children (that is, to have a first nuptial confinement) and the decisions to have additional children. The parity progression ratios for second and subsequent confinements are approximated by a smooth distribution which is characterized by two parameters, called the "mean implied completed family size" and "the variance in the implied completed family size." Further details are available in Appendix 3. These time subscripted parameters are respectively the mean and the variance of the stationary distribution of family sizes that would occur if the parity progression ratios for the year in question were to hold indefinitely.

The number of live nuptial births of each sex is determined as a constant multiple of the number of confinements. Ex-nuptial births are proportional to the number of nuptial births and currently that proportion is set exogenously.

To summarize, the age specific rate block and the births block accept inputs from the econometric model 3FLFP which are translated into the parameters of the age distributions of marriage, remarriage, divorce, and first nuptial confinements, together with a measure of implied completed family size. These two blocks then supply the necessary inputs of vital and marital events to the demographic accounting block.

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1. Parity progression ratios are the probabilities that a woman with a given number of previous confinements will have at least one further confinement.

3.4 The Econometric Model (3FLFP)

The econometric model is grounded in a microeconomic approach which assumes that decisions concerning marriage, divorce, child bearing and labour force participation involve a strong component of economic rationality. The rates of marriage and divorce and the ages at which these occur are modelled to be influenced by the costs and benefits of marriage and divorce respectively.

The variables endogenously determined within the econometric model are year-specific values of the 9 parameters of the marital status block and the 3 parameters of the births block plus a measure of child quality and the labour force participation rate for men and women for each of six groups obtained by cross-classifying three age groups by two marital status categories, "married" and "not married." These variables are listed in Table 3 along with the explanatory variables which determine their values. All equations are linear in the logarithms of the variables with the exception of the first nuptial confinement equation which is linear in all variables. The specification of the econometric model is outlined in Filmer and Silberberg[1] and further details should be sought there.

Above we have indicated that the treatment of births allows the decision to have children to be characterized as a sequential process in which first nuptial confinements and higher order births are modelled independently. This permits a distinction in the model between the decision to become parents and the subsequent decision on the total number of children to have. A distinction is also made between the desired number of children and

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1. Filmer and Silberberg (1977), op. cit..

the desired quality of life for the children. Thus a change in the demands for children can be manifest either as a change in the number of children or a change in the (material) quality of their life. The demand for increasing quality with regard to children can influence both the rates of marriages and the number of women willing to participate in the workforce. Participation within the workforce can influence the demand for children and child quality.

Married women can choose between work in the home, in the labour force or leisure; the choice is modelled as being influenced by the opportunities, costs and benefits to be expected from each of these within a simultaneous framework. Thus within the demographic core, the decisions concerning family formation and fertility are closely related to decisions affecting the supply of labour.

The variables which are exogenous to (strictly speaking, predetermined from the viewpoint of) 3FLFP consist of two categories, those which are endogenous to some other block of the demographic core, and those which are either exogenous to the demographic core at the current time sub-script, or else predetermined by the demographic core as a whole at some earlier period of projection. As an example of the last mentioned, consider the output 'marital flows' shown at the right of Figure 3. With a lag these re-enter the core via the exogenous variables input into 3FLFP on the left of Figure 3.

Table 3 : The Endogenous Variables of the Econometric Model (3FLFP) and their Determinants

Propensity to first marriage is estimated as a function of:

- Female-male relative wage rate
- Real income per head
- Child services[(a)]
- Female education
- Contraception

Age at first marriage is estimated as a function of:

- Female-male relative wage rate
- Real income per head
- Child services[(a)]
- Female education
- Contraception
- Age at first marriage (lagged one period)

Variance of age at first marriage is estimated as a function of:

- Female-male relative wage rate
- Real income per head
- Child services[(a)]
- Female education
- Contraception

Propensity to remarriage is estimated as a function of:

- Female-male relative wage rate
- Real income per head
- Child services[(a)]
- Female education
- Contraception

... *cont'd.*

Table 3 cont'd.

Age at remarriage is estimated as a function of:

- Female-male relative wage rate
- Real income per head
- Child services[(a)]
- Female education
- Contraception

Variance of age at remarriage is estimated as a function of:

- Female-male relative wage rate
- Real income per head
- Child services[(a)]
- Female education
- Contraception

Propensity to divorce is estimated as a function of:

- Female-male relative wage rate (lagged two periods)
- Real income per head (lagged two periods)
- Dependents per married female (lagged two periods)
- Age at marriage (lagged two periods)

Age at divorce is estimated as a function of:

- Female-male relative wage rate (lagged two periods)
- Real income per head (lagged two periods)
- Dependents per married female (lagged two periods)

Variance of age at divorce is estimated as a function of:

- Female-male relative wage rate (lagged two periods)
- Real income per head (lagged two periods)
- Dependents per married female (lagged two periods)

... *cont'd.*

Table 3 cont'd.

Implied completed family size is estimated as a function of:

Real female hourly wage (lagged one period)

Real income per head (lagged one period)

Real old-age pension (lagged one period)

Infant mortality rate (lagged one period)

Contraception (lagged one period)

Variance of implied family size is estimated as a function of:

Real female hourly wage (lagged one period)

Real income per head (lagged one period)

Real old-age pension (lagged one period)

Contraception (lagged one period)

Child quality (lagged one period)

First nuptial confinements are estimated as a function of:

Real female hourly wage (lagged one period)

Real income per head (lagged one period)

Real old-age pension (lagged one period)

Infant mortality rate (lagged one period)

Weighted first marriages

Contraception (lagged one period)

Child quality is estimated as a function of:

Real female hourly wage (lagged one period)

Real income per head (lagged one period)

Infant mortality rate (lagged one period)

Contraception (lagged one period)

... *cont'd.*

Table 3 cont'd.

Labour force participation rates of married women aged 15-24 and 25-54 are estimated as functions of:

Expected real female hourly wage

Unemployment rate

Permanent income per head

Nuptial birth rate

Child quality[a]

Contraception

Labour force participation, age-specific (lagged one period)

Labour force participation rate of married women aged 55 and over is estimated as a function of:

Expected real female hourly wage

Unemployment rate

Permanent income per head

Labour force participation (lagged one period)

Labour force participation rate of other women aged 15-24 is estimated as a function of:

Expected real female hourly wage

Unemployment rate

Permanent income per head

Labour force participation (lagged one period)

... *cont'd.*

Table 3 cont'd.

Labour force participation rates of other women aged 25-54 and 55 and over are estimated as a function of:

Expected real female hourly wage

Unemployment rate

Permanent income per head

Widow pension relative to the expected female wage

Labour force participation (lagged one period)

The labour force participation rates for men have not been finalised and will be documented in a forthcoming publication.

Note : (a) These explanatory variables are endogenous. 'Child services' is the product of child quality and implied completed family size.

Source : Filmer and Silberberg, op.cit. (modified). The list above excludes those variables specific to particular historical events (e.g., dummy variables for conscription and World War II) which are not relevant to the use of the model for forward-looking projections.

4. SUMMARY AND PROGRESS REPORT

The demographic core of the IMPACT medium term model provides projections of the Australian population, labour force and households by sex, age and marital status within a tightly integrated framework which allows projections to be made on the basis of changing economic and social conditions. The econometric model 3FLFP within the demographic core relates decisions concerning the occurrence of marital status changes, fertility and labour force participation to each other and to economic and social variables.

The modelling of age-specific rates of marriage and divorce enables the population submodule to separate the effects of changes in behaviour from changes in the demographic structure of the population due to the mere effluxion of time (i.e., ageing). This simplifies the task of the econometric model by limiting its role to capturing the behavioural changes and not those arising from mechanical changes in the population structure. Estimates of the labour force participation rates are among the behavioural variables modelled in 3FLFP, and when these are combined with the population estimates, the demographic core provides estimates of the total Australian labour force.

The births block provides estimates of the number of births and allows for changing fertility patterns arising from family formation and fertility decisions modelled in 3FLFP. The demographic accounting block imposes a rigorous accounting discipline on the stocks and flows variables relating to vital, marital and migration related events. Finally, the household headship submodule provides estimates of household headship rates which, when combined with the population estimates, enable the calculation within the demographic core of the total number of Australian households.

The role of the demographic core is not to supply just a single forecast or best estimate of the Australian population over some future period. Rather, its purpose is policy analytical. Not only does the demographic core provide specific population estimates for a range of settings of the exogenous variables, but it also provides insights into the mechanisms linking changing economic conditions to the future level, composition and workforce participation of the Australian population. In designing the computer implementation of the demographic core, care has been taken to maintain flexibility so that the model can be used at various levels of integration both internally and within the BACHUROO module. For example, we can foresee developments within the econometric model which will be implemented without unduly disturbing the structure of the rest of the demographic core. Likewise, we foresee linking the other BACHUROO sub-modules to the demographic core without the need for major modification to the software.

At this time, the demographic core has been implemented in an initial form and we are currently testing the within sample performance of the model. In brief summary, we can say that the female side of the model is functioning satisfactorily. The age specific marriage and divorce rate model supplies an adequate description of these rates and the demographic accounting block is functioning correctly. The performance of the econometric model 3FLFP is described in Filmer and Silberberg.[1] While some areas need improvement in 3FLFP, it provides a satisfactory starting point for projection. The births block performs well over the sample period but we are not satisfied at this stage that it has been sufficiently integrated with the demographic accounting block.

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1. Filmer and Silberberg (1977), op. cit..

The male side of the model is considered unsatisfactory. In particular, our attempt to use a fixed "marriage selection rule" seems to have failed. We are currently developing a model in order to overcome this limitation.

The household headship model as described in Williams and Brooks[1] has been implemented. Initial testing indicates that the model as initially estimated cannot cope with the wide discrepancy between within sample unemployment levels and recent experience. The model is being respecified. The fact that data from the 1976 Census recently became available offers an opportunity to re-estimate the model over a wider range of labour market conditions. The headship model will not be linked to the rest of the core until both are performing satisfactorily in independent trials. The results of these trials will be reported in additional papers as results come to hand. It is planned to incorporate these results (and others), plus a full technical description of the demographic core, and illustrative policy-oriented projections, into a single volume. Hopefully this will be completed within calendar 1979.

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1. Williams and Brooks (1978), op. cit..

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Appendix 1

The Equations of the Demographic Accounting Block of the Demographic Core

The following equations specify relationships between marital status changes, deaths and immigration, and the adjustment of the size and composition of the population from the beginning of one period to the next.

The equation specifying the adjustments to the population of women can be written as

$$X(i+1,t+1) = \Big(I - D(i,t)\Big)\ \Big(A(i,t)\ X(i,t) + X^M(i,t)\Big)$$

as described in the text of this paper.

In this appendix, the transition coefficients for women (the elements of the $A(i,t)$ matrix) are written as

$$a_{k\ell}(i,t) = \tfrac{1}{2}\Big(\alpha_{k\ell}(i,t) + \alpha_{k\ell}(i+1,t)\Big)\ , \qquad k,\ell = 1,2,3,4\ ,$$

where $a_{k\ell}(i,t)$ is the probability of a woman of age i at the start of the period changing from marital status ℓ to marital status k within the period, and

$\alpha_{k\ell}(i,t)$ is defined similarly except that it refers to the probability for a woman of age i at the time of the transition.

The distinction between age at the start of the period and age at the time of the transition is important. Within the demographic accounting equations the population variables and the immigration flows are determined at the beginning of each period, while the age specific marriage and divorce rates, the death rates and the marriage selection rule are measured by age at the time of the event. Also, the flows, such as $X_{k\ell}(i,t)$ which measures the number of women changing status within the period, are by age at the beginning of the period. The necessity of the distinction can be illustrated by the case of first marriage. The vital statistics for first marriage are collected by age at the time of marriage, whereas the population of never married females is measured at a particular time, which we can define as the beginning of the period. The equation above accounts for the possibility that a person who is aged i at the beginning of the period and who marries during the period may be aged i or $(i+1)$ at the time of the marriage. Hence we specify that the probability of marriage of a woman who is of age i at the start of the period is the sum of half the probability that she will be of age i at the time of the marriage, and half the probability that she will be of age $(i+1)$ at that time.

The marital status changes for men are determined by the corresponding changes for women and linked via "the couples matrix" and "the marriage selection rule". The male equations sum over the number of women who change a particular status times the probability that this status change affects the status of the male.

Below are listed the equations for the individual marital status changes, then the equations for the adjustment of the population for these flows and migration, and finally the adjustment of the population for deaths.

Marital Status Change Equations

Variables

$X_k(i_f)$ = number of women of age i_f and status k at the start of the period

$\tilde{X}_k(i_m)$ = number of men of age i_m and status k at the start of the period

$X_{k\ell}(i_f)$ = number of women of age i_f and status ℓ at the start of the period who change to status k within the period

$\tilde{X}_{k\ell}(i_m)$ = number of men of age i_m and status ℓ at the start of the period who change to status k within the period

$X^M{}_k(i_f)$ = net migration of women of age i_f and status k at the start of the period

$\tilde{X}^M{}_k(i_m)$ = net migration of men of age i_m and status k at the start of the period

$C(i_f,i_m)$ = number of married couples with wife of age i_f and husband of age i_m, both ages at the start of the period

$C^M(i_f,i_m)$ = net migration of married couples with wife of age i_f and husband of age i_m, both ages at the start of the period

$\alpha_{k\ell}(i_f)$ = proportion of women of status ℓ and age i_f at the time of the event who change to status k with the period

$\tilde{\delta}_k(i_m)$ = proportion of men of status k and age i_m at the time of the event who die within the period

$\delta_k(i_f)$ = proportion of women of status k and age i_f at the time of the event who die within the period

$c_k(i_f,i_m)$ = proportion of women, who are marrying, of age i_f at the time of the event who marry a man of marital status k and age i_m at the time of the event

Identities

$$\sum_{k=1,3,4} c_k(i_f,i_m) = c_.(i_f,i_m)$$

$$\sum_{i_m} \tfrac{1}{2}\left(c_.(i_f,i_m) + c_.(i_f,i_m+1)\right) = 1.0$$

$$\sum_{i_f} C(i_f,i_m) = \tilde{X}_2(i_m)$$

$$\sum_{i_m} C(i_f,i_m) = \tilde{X}_2(i_f)$$

Marriages

$$X_{2\ell}(i_f) = X_\ell(i_f)\ \tfrac{1}{2}\left(\alpha_{2\ell}(i_f) + \alpha_{2\ell}(i_f+1)\right),\ \ell = 1,3,4$$

$$\tilde{X}_{2k}(i_m) = \sum_{i_f}\sum_{\ell=1,3,4} X_\ell(i_f)\ \tfrac{1}{2}\Big[\alpha_{2\ell}(i_f)\ \tfrac{1}{2}\left(c_k(i_f,i_m) + c_k(i_f,i_m+1)\right)$$
$$+ \alpha_{2\ell}(i_f+1)\ \tfrac{1}{2}\left(c_k(i_f+1,i_m) + c_k(i_f+1,i_m+1)\right)\Big],$$

$$k = 1,3,4$$

Divorce

$$X_{32}(i_f) = X_2(i_f)\ \tfrac{1}{2}\left(\alpha_{32}(i_f) + \alpha_{32}(i_f+1)\right)$$

$$\tilde{X}_{32}(i_m) = \sum_{i_f} C(i_f,i_m)\ \tfrac{1}{2}\left(\alpha_{32}(i_f) + \alpha_{32}(i_f+1)\right)$$

Widowings

$$X_{42}(i_f) = \sum_{i_m} C(i_f,i_m)\ \tfrac{1}{2}\left(1 - \alpha_{32}(i_f) + 1 - \alpha_{32}(i_f+1)\right)$$

$$\times\ \tfrac{1}{2}\left(\tilde{\delta}_2(i_m) + \tilde{\delta}_2(i_m+1)\right)$$

$$\tilde{X}_{42}(i_m) = \sum_{i_f} C(i_f,i_m)\ \tfrac{1}{2}\left(1 - \alpha_{32}(i_f) + 1 - \alpha_{32}(i_f+1)\right.$$

$$\times\ \tfrac{1}{2}\left(\delta_2(i_f) + \delta_2(i_f+1)\right) \quad .$$

Widowings of newly marrieds

$$X_{4k}(i_f) = \sum_{i_m} \sum_{\ell=1,3,4} X_k(i_f)\ \tfrac{1}{2}\left[\alpha_{2k}(i_f)\ \tfrac{1}{2}\left(c_\ell(i_f,i_m) + c_\ell(i_f,i_m+1)\right)\right.$$

$$\left. + \alpha_{2k}(i_f+1)\ \tfrac{1}{2}\left(c_\ell(i_f+1,i_m) + c_\ell(i_f+1,i_m+1)\right)\right]$$

$$\times\ \tfrac{1}{2}\left(\tilde{\delta}_2(i_m) + \tilde{\delta}_2(i_m+1)\right) \qquad k = 1,3,4$$

$$\tilde{X}_{4k}(i_m) = \sum_{i_f} \sum_{\ell=1,3,4} X_\ell(i_f)\ \tfrac{1}{2}\left[\alpha_{2\ell}(i_f)\ \tfrac{1}{2}\left(c_k(i_f,i_m) + c_k(i_f,i_m+1)\right)\right.$$

$$\left. + \alpha_{2\ell}(i_f+1)\ \tfrac{1}{2}\left(c_k(i_f+1,i_m) + c_k(i_f+1,i_m+1)\right)\right]$$

$$\times\ \tfrac{1}{2}\left(\delta_2(i_f) + \delta_2(i_f+1)\right) \quad , \qquad k = 1,3,4$$

Stocks at the beginning of the next period

$$X'_k(i_f+1) = \left(X_k(i_f) + \sum_{\ell\neq k} X_{k\ell}(i_f) - \sum_{\ell\neq k} X_{\ell k}(i_f) + X^M{}_k(i_f)\right)$$

$$\times \left[1 - \tfrac{1}{2}\left(\delta_k(i_f) + \delta_k(i_f+1)\right)\right]$$

$$\tilde{X}'_k(i_m+1) = \left(\tilde{X}_k(i_m) + \sum_{\ell\neq k} \tilde{X}_{k\ell}(i_m) - \sum_{\ell\neq k} \tilde{X}_{\ell k}(i_m) + \tilde{X}^M{}_k(i_m)\right)$$

$$\times \left[1 - \tfrac{1}{2}\left(\tilde{\delta}_k(i_m) + \tilde{\delta}_k(i_m+1)\right)\right]$$

$$C'(i_f+1,i_m+1) = \Bigg\{C(i_f,i_m)\left[1 - \tfrac{1}{2}\left(\alpha_{32}(i_f) + \alpha_{32}(i_f+1)\right)\right]$$

$$+ \sum_{\ell=1,3,4} X_\ell(i_f)\ \tfrac{1}{2}\Bigg[\alpha_{2\ell}(i_f)\ \tfrac{1}{2}\left(c_.(i_f,i_m) + c_.(i_f,i_m+1)\right)$$

$$+ \alpha_{2\ell}(i_f+1)\ \tfrac{1}{2}\left(c_.(i_f+1,i_m) + c_.(i_f+1,i_m+1)\right)\Bigg]$$

$$+ C^M(i_f,i_m)\Bigg\} \times \left[1 - \tfrac{1}{2}\left(\delta_k(i_f) + \delta_k(i_f+1)\right)\right]$$

$$\times \left[1 - \tfrac{1}{2}\left(\tilde{\delta}_k(i_m) + \tilde{\delta}_k(i_m+1)\right)\right]$$

Appendix 2

The Modelling of Age Specific Rates of Marriage and Divorce for Women

The age specific rates for first marriage, remarriage and divorce for women are, respectively, the distributions across ages of the number of first marriages of women divided by the number of never married women of the same age, the number of remarriages of women divided by the number of widowed or divorced women of the same age and the number of divorces of women divided by the number of married women of the same age. The age specific rate characteristically begin at a low value for young ages, rise rapidly to a peak and then decrease slowly with increasing age. These rates are approximated within the age specific rate block of the population submodule by the gamma distribution,

$$p(x,t) = \frac{X_p}{\beta^{\alpha}\Gamma(\alpha)} \; x^{\alpha-1} \, e^{-x/\beta} \quad ,$$

where x is the age at the time of marriage or divorce, and

$$X_p = \int_0^{\infty} p(x,t)\; dx$$

is the area under the age specific distribution and is referred to in the model as the index of propensity to first marry, remarry or divorce, as appropriate.

The parameters α and β are related to the mean and variance of the distribution by

$$\bar{x} = \alpha\beta$$

and

$$\text{var} = \alpha\beta^2 \quad ,$$

respectively;

and conversely,

$$\alpha = (\overline{x})^2/\text{var} \quad ,$$

while

$$\beta = \text{var}/\overline{x} \quad .$$

Using the gamma relationship and a knowledge of the propensity, mean, and variance of the age distribution of each of first marriages, remarriages and divorces, it is possible to construct the probability of marriage, remarriage or divorce for each single year of age within the age specific rates block.

The gamma distribution is adequate to describe remarriage and divorce rates. However, first marriage rates rose too quickly and then fell too slowly across ages for the gamma distribution to provide a satisfactory approximation. The first marriage rates are modelled by a slightly modified gamma distribution:

$$p(x,t) = \frac{X_1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} + \frac{X_0}{\beta_0^{\alpha_0}\Gamma(\alpha_0)} (x-x_0)^{\alpha_0-1} e^{-(x-x_0)/\beta_0} \quad ,$$

where X_0, α_0, β_0, x_0 are constants, and X_1, α and β are determined for each year.

The propensity to first marry is given by

$$X_p = X_1 + X_0 \quad ,$$

while the mean of the distribution is

$$\overline{x} = (X_1\alpha\beta + X_0\alpha_0\beta_0)/(X_1 + X_0)$$

and the variance is

$$\text{var} = \frac{X_1^2\alpha\beta^2 + X_0^2\alpha_0\beta_0^2 + X_1X_0\left(\alpha(\alpha+1)\beta^2 + \alpha_0(\alpha_0+1)\beta^2 - 2\alpha\beta\alpha_0\beta_0\right)}{(X_1 + X_0)^2}$$

Over the post-war period from 1945-46 to 1964-65, the value of the constants in the above equation are estimated to be

$$X_0 = 0.016 ,$$

$$\alpha_0 = 2.0 ,$$

$$\beta_0 = 6.5 ,$$

and

$$x_0 = 29.0 .$$

By using the propensity, mean and variance of the distribution and these constants, it is possible to reconstruct the age distribution of the rate of first marriage.

Appendix 3

The Calculation of the Number of Births

The births block of the demographic core calculates annual projections of the number of births of each sex based on projections of the number of confinements in each year.

The econometric model (3FLFP) provides projections of the number of first nuptial confinements and the mean and variance of "implied completed family size". The births block then calculates from these parameters the number of confinements of each birth order from

(a) the number of confinements of lower order in preceding periods,

(b) the probabilities of women of each order progressing to a higher order,

(c) the intervals between confinements of each order, and

(d) the ratio of ex-nuptial confinements to nuptial confinements in each year.

The number of births is calculated mechanically on the basis of projections of the number of confinements since it is considered that it is the decision to have a confinement which is influenced by economic factors, whereas the relationship between the number of live births of each sex and the number of confinements is determined by biological factors. Thus the numbers of births of each sex are calculated by multiplying the number of confinements by constant factors for the average number of live births per confinement and for the sex ratio of births;

(A1) $$B_t = (C_t + C_t^e)\ \beta\sigma \quad ,$$

and

(A2) $$\tilde{B}_t = (C_t + C_t^e)\ \beta\tilde{\sigma} \quad ,$$

where B_t is the number of female births in year t ,

$\tilde{B}_t$ is the number of male births in year t ,

C_t is the number of nuptial confinements in year t ,

C_t^e is the number of ex-nuptial confinements in year t ,

β is the average number of live births per confinement allowing for still births and multiple births,

σ is the proportion of births which are female ,

and $\tilde{\sigma}$ is the proportion of births which are male, where $\sigma + \tilde{\sigma} = 1.0$.

In the current implementation of the births block, the number of ex-nuptial confinements each year is determined exogenously as a proportion of the number of nuptial confinements;

(A3) $$C_t^e = x_t C_t \quad .$$

The Number of Nuptial Confinements

The model of the number of nuptial confinements in the demographic core distinguishes between the decision by a married woman to have a first nuptial confinement and the decision to have further confinements.

The number of nuptial confinements is simply the sum of nuptial confinements of all orders;

(A4) $$C_t = \sum_j C_{jt} \quad ,$$

where C_{jt} is the number of nuptial confinements of order j in year t .

Estimates of first nuptial confinements are provided directly by the econometric model and higher order confinements are calculated from the interval distribution between confinements of each order and the probability of progressing to a higher order.

Let $\rho_{n,t,\tau}$ equal the probability that a married woman who had her n^{th} confinement τ years ago has her $(n+1)^{th}$ confinement in year t . Then the number of confinements of order $n+1$ in year t is

(A5) $$C_{n+1,t} = \sum_{\tau=0}^{\infty} C_{n,t-\tau}\, \rho_{n,t,\tau} \quad ,$$

which represents the sum of the number of n^{th} order confinements in each previous period multiplied by the probability, $\rho_{n,t,\tau}$, that the next confinement occurs in year t .

The probability, $\rho_{n,t,\tau}$, can be separated into two components; the parity progression ratio of order n , and the interval distribution between confinements.

The parity progression ratio of order n, p_{nt} , is the probability that a married woman who has her n^{th} confinement in t will have at least one additional confinement in the future, if conditions prevailing in t persist. Pollard (1975)[1] proxies the n^{th} parity progression ratio determined at t by

1. A.H. Pollard, "Estimating Parity Progression Ratios from Australian Official Statistics", Research Paper No. 78, Macquarie University School of Economic and Financial Studies, Sydney, April 1975.

(A6) $$p_{nt} = \sum_{\tau=0}^{\infty} \rho_{n,t,\tau} \ .$$ [1]

The interval distribution between confinements for each parity is then

(A7) $$q_{n,t,\tau} = \rho_{n,t,\tau} / p_{nt} \ .$$

This is equal to the probability that a married woman having her $(n+1)^{th}$ confinement in t had her n^{th} confinement τ years earlier. In Pollard (1972)[2], Table 3 gives the interval distribution for the progression from the first to second confinement for the years 1960-1970. For example, the birth interval distribution $q_{1,1963,\tau}$ is illustrated in Figure A1.

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1. Note that Pollard's definition employs the interval distribution between past confinements and the confinements occurring in t as a proxy for the interval distribution between confinements in t and future confinements.

   The parity progression ratio is defined correctly by

   $$(1 - p_{nt}) = \prod_{\tau=0}^{\infty} (1 - \rho'_{n,t,\tau}) \ ,$$

   where $\rho'_{n,t,\tau}$ equals the probability that a women who had her n^{th} confinement in t will have her $(n+1)^{th}$ confinement τ years later.

   So $$p_{nt} = \sum_{\tau=0}^{\infty} \rho'_{n,t,\tau} - \sum_{\substack{\tau,\tau' \\ \tau\neq\tau'}}^{\infty} \rho'_{n,t,\tau} \rho'_{n,t,\tau'} + \ldots \ .$$

   Thus Pollard's formula identifies $\rho'_{n,t,\tau}$ with $\rho_{n,t,\tau}$ and omits the second and higher order terms in p_{nt}.

2. A.H. Pollard, _op. cit._, p. 10.

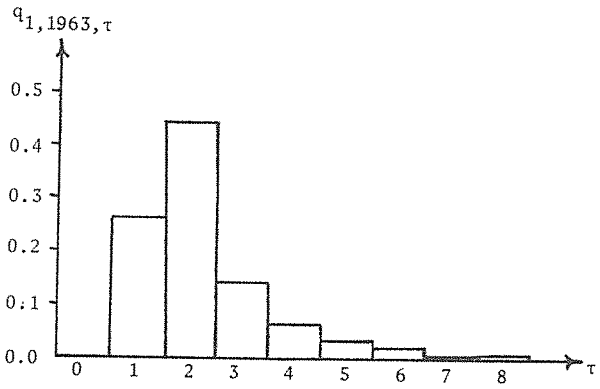

Figure A1 : Birth interval distribution between first and second nuptial confinements for 1963.

The number of nuptial confinements of order $n+1$ in t is then

(A8)
$$C_{n+1,t} = \left(\sum_{\tau=0}^{\infty} C_{n,t-\tau} \, q_{n,t,\tau} \right) p_{nt} \quad ,$$

and its calculation requires

(i) the number of confinements of lower order in previous periods,

(ii) the parity progression ratio of order n , and

(iii) the interval distribution for n^{th} order confinements .

The number of confinements in previous periods is predetermined within the births block. The parity progression ratio can be obtained from the parameters provided by the econometric model as explained in the next section. The distribution of intervals between confinements is modelled crudely in the current implementation as explained in the section after next.

The Parity Progression Ratios

The parity progression ratios, p_{nt} , are modelled via two parameters: the mean and variance of "the implied completed family size" distribution. The distribution of "implied completed family size" is not directly observable but can be calculated from;

(A9)
$$f_{1t} = 1 - p_{1t} \quad ,$$
$$f_{nt} = \left(1 - p_{nt}\right) \left(\prod_{j=1}^{n-1} p_{jt}\right) \quad , \qquad n = 2,3,\ldots,$$

where f_{nt} is the probability that a married woman who has already had her first confinement will have exactly n confinements in her lifetime, if conditions prevailing in t persist. Thus f_{nt} is an "implied completed family size" as it is dependent on conditions at time t persisting into the future. We define $f_{0t} = 0.0$ for later convenience.

and p_{nt} is the parity progression ratio of order n .

In order to estimate the coefficients of the econometric model (3FLFP), the "implied completed family size" distribution and its mean and variance were calculated from values of the parity progression ratios provided by G. Spencer.[1]

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1. Annual parity progression ratios for orders 1 to 7 were kindly supplied by Geraldine Spencer. Also see Geraldine Spencer, "Projecting Australia's Fertility", Australian Journal of Social Issues, Vol. 9, No. 4, 1974, pp. 273-284.

The calculated distribution is, by definition, zero for an implied completed family size of zero, rises sharply to a maximum family size of two and then falls slowly with increasing family size. This suggests that the frequency distribution of "implied completed family size" can be approximated by integrating a gamma density over each parity interval;[1] i.e.,

(A10) $$f_{nt} = \int_{n-1}^{n} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} dx, \qquad n = 1,2,3 \ldots .$$

where $\Gamma(\alpha)$ is the gamma function .

The mean M_N and the variance V_N of this distribution are related to the parameters α and β of the continuous gamma distribution by

(A11) $$M_N = \alpha\beta + 0.5 ,$$

and

(A12) $$V_N = \alpha\beta^2 .$$

For projections, the births block converts the values of the mean and variance of the "implied completed family size" distribution into the parity progression ratios for each order via equations (A10), (A11), (A12) and

$$P_{1t} = 1 - f_{1t}$$

(A13) $$p_{nt} = \left(1 - \sum_{j=1}^{n} f_{jt}\right) \Big/ \left(1 - \sum_{j=1}^{n-1} f_{jt}\right) , \qquad n = 2,\ldots$$

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1. For further details see Filmer and Silberberg, (1977) op. cit., pp. 65-67.

The Interval Distribution Between Confinements

In the current implementation of the births block, the interval distribution is approximated by assuming that the interval for all confinements of order n in t is simply equal to the mean interval, $\bar{\tau}_n$, of such confinements. But since confinements are spread over the whole of period t, then the previous confinement could have occurred between $t - \bar{\tau}_n$ and $t - \bar{\tau}_n + 1$. For example, if the average interval between first and second nuptial confinements is 2.39 years as in 1963 then we would assume that all the first confinements would have occurred (uniformly) between 1.39 and 2.39 years before the beginning of this period.

The next assumption is best explained in terms of our example and Figure A2. The equation for the number of second confinements in 1963 would be

$$C_{2t} = (0.61C_{1,t-2} + 0.39C_{1,t-3})\ p_{1t} \quad .$$

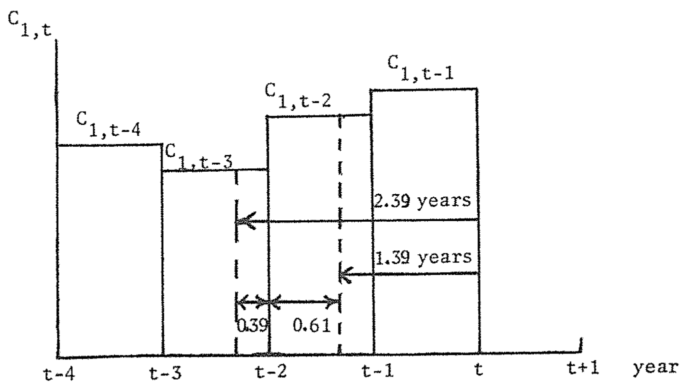

Figure A2 : The overlap of the interval $(t-\bar{\tau}_n, t-\bar{\tau}_n+1)$ with the preceding years.

The assumption is that since 61 per cent of the period between 1.39 and 2.39 years prior to t covers the year $t-2$ and 39 per cent covers year $t-3$, then 61 per cent of the second confinements in t correspond to women who had their first confinement 2 years before and the remaining 39 per cent of the second confinements in t correspond to women who had their first confinement 3 years before.

The approximation that is currently used is equivalent to replacing the interval distribution $q_{1,1963,\tau}$ illustrated in Figure A1 which has a mean interval of 2.39 years by the interval distribution;

$$
\begin{aligned}
q^*_{1,1963,1} &= 0 \\
q^*_{1,1963,2} &= 0.61 \\
q^*_{1,1963,3} &= 0.39 \\
q^*_{1,1963,\tau} &= 0.0 \quad , \quad \tau > 4 \ .
\end{aligned}
$$

This interval distribution is illustrated in Figure A3. Note that $\sum_{\tau} q^*_{1,1963,\tau}$ is still equal to 1.0 .

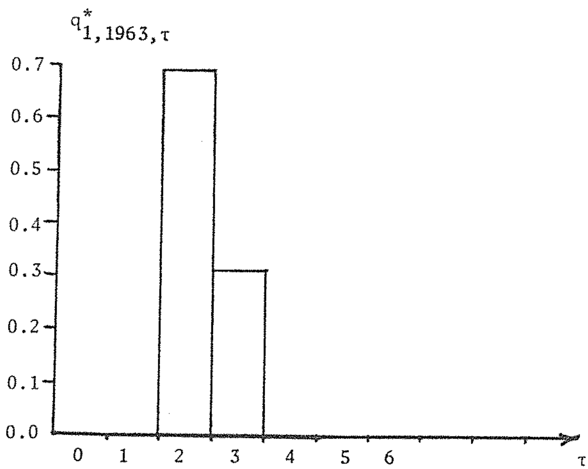

Figure A3 : The approximation in the current implementation of the births block to the birth interval distribution for 1963.

The appropriate expressions for second and higher order confinements are given in Table A1 and are assumed not to vary over time. The mean intervals, $\bar{\tau}_n$, are those calculated by G. Spencer[1] for 1974. This specification of the distribution of the interval between confinements is considered unsatisfactory and is under review.

<u>Summary</u>

The calculation of births in the demographic core attempts to incorporate mechanisms which allow the number of births to be influenced by the ex-nuptial rate of confinements and the decisions of married women concerning firstly, whether to have a family, secondly, the size of that family, and thirdly, the spacing of that family. In the present implementation the ex-nuptial rate is supplied exogenously, the number of first nuptial confinements is supplied directly by the econometric model, the decision to have further children and the size of the family is parameterized by the mean and variance of the "implied completed family size" distribution which are supplied by the econometric model and the birth interval distribution is approximated in an ad hoc manner.

Thus the modelling of births within the demographic core provides a consistent framework and a set of flexible mechanisms to influence the number of births. It is hoped that the deficiencies in relation to the birth interval distribution will be removed in the near future.

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1. Annual calculations of the mean interval between confinements of successive order were kindly supplied by Geraldine Spencer. Also see G. Spencer, <u>op. cit.</u> (1974).

Table A1

Calculation of Higher Order Confinements

$$C_{2,t} = (0.69C_{1,t-2} + 0.31C_{1,t-3})\, p_{1,t} \; ; \qquad \bar{\tau}_2 = 2.31 \text{ years}$$

$$C_{3,t} = (0.61C_{2,t-2} + 0.39C_{2,t-3})\, p_{2,t} \; ; \qquad \bar{\tau}_3 = 2.39 \text{ years}$$

$$C_{4,t} = (0.65C_{3,t-2} + 0.35C_{3,t-3})\, p_{3,t} \; ; \qquad \bar{\tau}_4 = 2.35 \text{ years}$$

$$C_{5,t} = (0.67C_{4,t-2} + 0.33C_{4,t-3})\, p_{4,t} \; ; \qquad \bar{\tau}_5 = 2.33 \text{ years}$$

$$C_{6,t} = (0.77C_{5,t-2} + 0.23C_{5,t-3})\, p_{5,t} \; ; \qquad \bar{\tau}_6 = 2.23 \text{ years}$$

$$C_{7,t} = (0.90C_{6,t-2} + 0.10C_{6,t-3})\, p_{6,t} \; ; \qquad \bar{\tau}_7 = 2.10 \text{ years}$$

$$C_{8,t} = (0.05C_{7,t-1} + 0.95C_{7,t-2})\, p_{7,t} \; ; \qquad \bar{\tau}_8 = 1.95 \text{ years}$$

$$C_{9,t} = (0.06C_{8,t-1} + 0.94C_{8,t-2})\, p_{8,t} \; ; \qquad \bar{\tau}_9 = 1.94 \text{ years}$$

$$C_{m,t} = (0.06C_{m-1,t-1} + 0.94C_{m-1,t-2})\, p_{m-1,t} \; ; \qquad \bar{\tau}_m = 1.94 \text{ years}$$

(for $m \geqslant 10$)